# HARQ Throughput Enhancement Using Maximum Likelihood Block Detection with Recursive QR Packet Combining and M-algorithm for Single-carrier MIMO

Tetsuya YAMAMOTO† Kazuki TAKEDA† and Fumiyuki ADACHI‡
Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
†{yamamoto, kazuki}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

*Abstract*—**For achieving very high-speed and high quality packet transmission, the joint use of multi-input multi-output (MIMO) spatial multiplexing and hybrid automatic repeat request (HARQ) is very effective. The HARQ throughput depends on the packet combining scheme. In this paper, we consider a single-carrier (SC) MIMO and propose a maximum likelihood block detection using recursive QR packet combing and M-algorithm called recursive QRM-MLBD. We will show, by computer simulation, the recursive QR packet combing provides a better throughput performance than the conventional bit-level log likelihood ratio (LLR) packet combining. We also compare the achievable throughput performance of proposed recursive QRM-MLBD to that of the frequency-domain detection based on the minimum mean square error criterion (MMSED).**

***Keywords-component; SC, MIMO, HARQ, QR decomposition, M-algorithm***

## I. INTRODUCTION

In next generation mobile communication systems, high data rate wireless communication systems are demanded. Multi-input multi-output (MIMO) spatial multiplexing technique has been gaining much attention in a band-limited wireless channel [1]. However, since the mobile wireless channel is composed of many propagation paths with different time delays, the channel becomes severely frequency-selective as the transmission data rate increases [2]. MIMO spatial multiplexing with orthogonal frequency-division multiplexing (OFDM) [3] is attractive. However, OFDM suffers from high peak-to-average power ratio (PAPR). Recently, the single-carrier (SC) MIMO spatial multiplexing using frequency-domain block detection technique has been gaining an increasing popularity because of its lower PAPR property [4-5]. The use of cyclic prefix (CP) and frequency-domain signal detection such as the frequency-domain linear detection based on the minimum mean square error criterion (MMSED) [4] can improve the transmission performance of SC-MIMO spatial multiplexing with a low complexity. However, a big performance gap from the maximum likelihood (ML) performance still exists due to the presence of residual inter-symbol interference and inter-antenna interference.

The ML detection (MLD) [6] is the optimal detection scheme in terms of transmission performance. However, its computational complexity becomes extremely high because the number of symbol candidate sequence is exponentially increased to $X^{N_tN_m}$ for $X$-QAM, where $N_t$ is the number of transmit antennas and $N_m$ is the block size. Recently, near ML-based reduced complexity signal detection was proposed [7] for SC-MIMO spatial multiplexing (we call this detection scheme as the ML block detection using QR decomposition and M-algorithm (QRM-MLBD)). In QRM-MLBD, QR decomposition is applied to a concatenation of the space and frequency-domain channel and discrete Fourier transform (DFT). It was shown [7] that QRM-MLBD can significantly improve the packet error rate (PER) performance of SC-MIMO spatial multiplexing compared to the MMSED.

Packet access will be the core technology of the next generation mobile communication systems. Very high-speed and high-quality packet transmissions in a limited bandwidth can be achieved by the joint use of MIMO multiplexing and hybrid automatic repeat request (HARQ). In this paper, we assume the type I HARQ. In type I HARQ, if any error is detected in a received packet, the same packet is retransmitted until it is correctly received [8]. The time-diversity gain can be obtained by combining retransmitted packets. The HARQ throughput depends on the packet combining scheme. How to jointly perform the QRM-MLBD and packet combining is an important technical issue.

A simple packet combining is the bit-level log likelihood ratio (LLR) packet combining [9], in which the bit-level LLRs in each received retransmitted packet are simply added to obtain the combined LLRs. This bit-level LLR packet combining is suboptimal and therefore, packet combining gain cannot be obtained sufficiently. Another packet combining is a straightforward application of the QR decomposition to a totality of the received retransmitted packets (this combining is called the expanded QR packet combining). However, this expanded QR packet combining needs to store all the retransmitted packets and the associated channel matrixes and its computational complexity increases with the number of retransmitted packets.

In [10], the packet combining scheme which incorporates the packet combining into the QR decomposition was presented for MIMO spatial multiplexing using QR decomposition aided decision feedback equalizer (DFE) (we call this packet combining scheme as the recursive QR packet combining). The recursive QR packet combining does not need to store all of the received retransmitted packets and the channel matrixes. Only the previously obtained upper triangular matrix and the channel matrix associated with the present received packet are required to carry out the QR decomposition.

In this paper, we consider SC-MIMO and propose a ML block detection using recursive QR packet combining and M-algorithm called the recursive QRM-MLBD and show, by computer simulation, that the recursive QR packet combining provides a better throughput performance than the conventional bit-level LLR packet combining. We also compare the achievable throughput performance of the recursive QRM-MLBD to that of the MMSED.

The remainder of this paper is organized as follows. In Sect. II, system model of SC-MIMO spatial multiplexing using HARQ and recursive QRM-MLBD is presented. Sect. III describes the proposed recursive QRM-MLBD for SC-MIMO. In Sect. IV, we will show some simulation results. Sect. V offers some concluding remarks.

## II. SYSTEM MODEL

### A. *Transmission System*

System model of SC-MIMO spatial multiplexing using HARQ and recursive QRM-MLBD is illustrated in Fig. 1. In this paper, we consider type I HARQ. Throughout the paper, the symbol-spaced discrete time representation is used.

At the transmitter, after channel coding and puncturing, the coded bit sequence is stored in the transmitter buffer. The coded bit sequence is transformed into a data-modulated symbol sequence. Then, the data-modulated symbol sequence is serial-to-parallel (S/P) converted to $N_t$ parallel symbol sequence, each to be transmitted from a different transmit antenna and each parallel symbol sequence is divided into a sequence of symbol blocks of $N_m$ symbols each. The data symbol block of $n_t$th transmit antenna can be expressed using the vector form as $\mathbf{d}_{n_t}=[d_{n_t}(0),...,d_{n_t}(N_m-1)]^T$, where $(.)^T$ expresses the transposition. The data symbol block is transformed by $N_m$-point DFT into the frequency-domain signal, which is then, mapped to the wider bandwidth of $N_c$ subcarriers. Finally, $N_c$-point inverse DFT (IDFT) is applied to obtain the time-domain transmit signal block (note that $N_m$<<$N_c$ in the case of SC frequency division multiple access (SC-FDMA) [11]). The last $N_g$ samples of transmit block are copied as a CP and inserted into the guard interval (GI) placed at the beginning of each transmit block and a CP-inserted signal block of $N_c$+$N_g$ samples is transmitted.

The signal block is transmitted over a frequency-selective fading channel. The received signal block after CP removal is transformed by $N_c$-point DFT into the frequency-domain signal. Then, de-mapping is performed and joint QRM-MLBD and packet combining is carried out.

### B. *Received Signal*

Consider that the $p$th retransmitted packet has been received. The frequency-domain received signal vector at the $n_r$th receive antenna $\mathbf{Y}_{n_r}^{(p)}=[Y_{n_r}^{(p)}(0),...,Y_{n_r}^{(p)}(N_m-1)]^T$ after $N_c$-point DFT and de-mapping is expressed as [7]

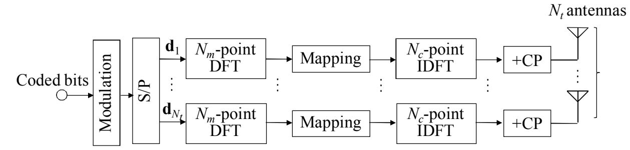
$$\mathbf{Y}_{n_r}^{(p)}=\sqrt{2E_s/T_s}\sum_{n_t=1}^{N_t}\mathbf{H}_{n_t,n_r}^{(p)}\mathbf{F}\mathbf{d}_{n_t}+\mathbf{N}_{n_r}^{(p)}, \qquad (1)$$

where $E_s$ and $T_s$ are respectively the symbol energy and duration, $\mathbf{F}$ is the DFT matrix of size $N_m$×$N_m$ and $\mathbf{H}_{n_r,n_t}^{(p)}$ is the frequency-domain channel matrix between the $n_t$th transmit antenna and $n_r$th receive antenna in the $p$th retransmission.

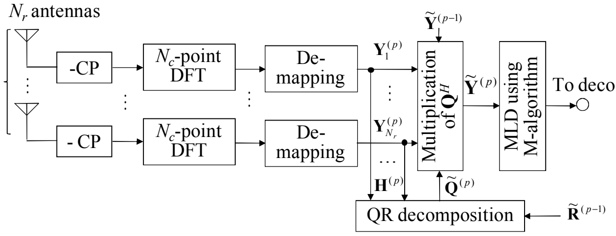
From Eq. (1), the $N_rN_m$×1 overall frequency-domain received signal $\mathbf{Y}^{(p)}=[\{\mathbf{Y}_1^{(p)}\}^T,...,\{\mathbf{Y}_{N_r}^{(p)}\}^T]^T$ is given by

$$\begin{aligned}\mathbf{Y}^{(p)}&=\sqrt{\frac{2E_s}{T_s}}\begin{bmatrix}\mathbf{H}_{1,1}^{(p)}\mathbf{F} & \mathbf{H}_{1,2}^{(p)}\mathbf{F} & \cdots & \mathbf{H}_{1,N_t}^{(p)}\mathbf{F}\\ \mathbf{H}_{2,1}^{(p)}\mathbf{F} & \mathbf{H}_{2,2}^{(p)}\mathbf{F} & \cdots & \mathbf{H}_{2,N_t}^{(p)}\mathbf{F}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{H}_{N_r,1}^{(p)}\mathbf{F} & \mathbf{H}_{N_r,2}^{(p)}\mathbf{F} & \cdots & \mathbf{H}_{N_r,N_t}^{(p)}\mathbf{F}\end{bmatrix}\begin{bmatrix}\mathbf{d}_1\\ \vdots\\ \mathbf{d}_{N_t}\end{bmatrix}+\begin{bmatrix}\mathbf{N}_1^{(p)}\\ \vdots\\ \mathbf{N}_{N_r}^{(p)}\end{bmatrix},\\ &=\sqrt{2E_s/T_s}\,\mathbf{H}^{(p)}\mathbf{d}+\mathbf{N}^{(p)}\end{aligned} \qquad (2)$$

where $\mathbf{H}^{(p)}$ is an equivalent channel matrix of size $N_rN_m$×$N_tN_m$, which is a concatenation of the space and frequency-domain channel and DFT, $\mathbf{d}$ is the $N_tN_m$×1 overall data symbol vector, and $\mathbf{N}$ is the $N_rN_m$×1 overall noise vector.



(a) Transmitter



(b) Receiver

Figure 1. System model.

## III. PACKET COMBINING SCHEMES

In this section, three packet combining schemes are presented; the bit-level LLR packet combing, the expanded QR packet combining, and the recursive QR packet combing. It is assumed that the same packet has been retransmitted $P$ times (including the first transmission).

### A. *Bit level LLR packet combining*

In the bit-level LLR packet combining, the conventional QRM-MLBD is performed at each retransmission and then, the bit-level LLRs in each received retransmitted packet are simply added to obtain the combined LLRs. QRM-MLBD consists of two steps; QR decomposition and M-algorithm. First, the QR decomposition is applied to the equivalent channel matrix $\mathbf{H}^{(P)}$ to obtain $\mathbf{H}^{(P)}$= $\mathbf{Q}^{(P)}\mathbf{R}^{(P)}$, where $\mathbf{Q}^{(P)}$ is an $N_rN_m$×$N_tN_m$ unitary matrix and $\mathbf{R}^{(P)}$ is an $N_tN_m$×$N_tN_m$ upper triangular matrix. The transformed frequency-domain received signal $\hat{\mathbf{Y}}^{(P)}=[\hat{Y}^{(P)}(0),...,\hat{Y}^{(P)}(N_tN_m-1)]^T$ is obtained as

$$\begin{aligned}\hat{\mathbf{Y}}_{bit-level}^{(P)}&=\{\mathbf{Q}^{(P)}\}^H\mathbf{Y}^{(P)}\\ &=\sqrt{2E_s/T_s}\,\mathbf{R}^{(P)}\mathbf{d}+\{\mathbf{Q}^{(P)}\}^H\mathbf{N}^{(P)}\end{aligned}. \qquad (3)$$

From Eq. (3), the ML solution can be obtained by searching for the best path having the minimum Euclidean distance in the tree diagram composed of $N_tN_m$ stages. In each stage, the best $M$ paths are selected as surviving paths by comparing the path metrics based on the squared Euclidean distance for all surviving paths and are passed to the next stage. The LLR is used for the soft-input in the decoder. When QRM-MLBD is used, however, the LLR values cannot be directly computed since surviving paths at the last stage do not necessarily contain both 1 and 0 for every coded bit. In this paper, we applied the LLR estimation method proposed in [12]. The approximate LLR values are computed at every stage by using path metric and are updated successively as tree search progress. If the LLR value cannot be computed at the last stage, the recently updated approximate LLR value at the upper stage is used. Finally, obtained bit-level LLRs in each received retransmitted packet are simply added to obtain the combined LLRs. In the bit-level LLR combining, receiver needs to store only combined LLRs and the computational complexity stay constant at each retransmission.

### B. Expanded QR packet combining

First, we define the expanded received signal vector $\overline{\mathbf{Y}}^{(P)}$ of size $PN_rN_m\times 1$ as

$$\begin{aligned}\overline{\mathbf{Y}}^{(P)}=\begin{bmatrix}\mathbf{Y}^{(1)}\\ \vdots\\ \mathbf{Y}^{(P)}\end{bmatrix}&=\sqrt{\frac{2E_s}{T_s}}\begin{bmatrix}\mathbf{H}^{(1)}\\ \vdots\\ \mathbf{H}^{(P)}\end{bmatrix}\mathbf{d}+\begin{bmatrix}\mathbf{N}^{(1)}\\ \vdots\\ \mathbf{N}^{(P)}\end{bmatrix},\\ &=\sqrt{2E_s/T_s}\,\mathbf{G}^{(P)}\mathbf{d}+\overline{\mathbf{N}}^{(P)}\end{aligned}\tag{4}$$

where $\mathbf{G}^{(P)}$ is the expanded channel matrix of size $PN_rN_m\times N_tN_m$. Then, QRM-MLBD is applied to Eq. (4) by applying QR decomposition to the expanded channel matrix $\mathbf{G}^{(P)}$ to obtain $\mathbf{G}^{(P)}=\overline{\mathbf{Q}}^{(P)}\overline{\mathbf{R}}^{(P)}$, where $\overline{\mathbf{Q}}^{(P)}$ is a $PN_rN_m\times N_tN_m$ unitary matrix and $\overline{\mathbf{R}}^{(P)}$ is an $N_tN_m\times N_tN_m$ upper triangular matrix. The transformed frequency-domain received signal $\hat{\mathbf{Y}}^{(P)}_{expaned}$ is obtained as

$$\begin{aligned}\hat{\mathbf{Y}}^{(P)}_{expand}&=\{\overline{\mathbf{Q}}^{(P)}\}^H\,\overline{\mathbf{Y}}^{(P)}\\ &=\sqrt{2E_s/T_s}\,\overline{\mathbf{R}}^{(P)}\mathbf{d}+\{\overline{\mathbf{Q}}^{(P)}\}^H\,\overline{\mathbf{N}}^{(P)}\end{aligned}.\tag{5}$$

The M-algorithm is performed in a similar fashion to the conventional QRM-MLBD and the obtained bit LLRs are used for the soft-input in the decoder.

This expanded QR packet combining can achieve optimal packet combining gain. However, the expanded QR packet combining needs to store all the retransmitted packets and the associated channel matrixes and therefore, the memory size of the receiver increases. Furthermore, the size of the expanded channel matrix to which the QR decomposition is applied becomes $pN_rN_m\times N_tN_m$ and therefore, the computational complexity required for the QR decomposition and generation of the transformed signal grows with the number of retransmissions.

### C. Recursive QR packet combining

The recursive QR packet combining is derived below. Application of the QR decomposition to the expanded channel matrix $\mathbf{G}^{(P)}$ is rewritten as

$$\mathbf{G}^{(P)}=\overline{\mathbf{Q}}^{(P)}\overline{\mathbf{R}}^{(P)},\tag{6}$$

Here, $\mathbf{G}^{(P)}$ can be expressed by using $\mathbf{G}^{(P-1)}$ and $\mathbf{H}^{(P)}$ as

$$\mathbf{G}^{(P)}=\begin{bmatrix}\mathbf{G}^{(P-1)}\\ \mathbf{H}^{(P)}\end{bmatrix}.\tag{7}$$

Furthermore, $\mathbf{G}^{(P)}$ can be modified, using the relationship $\mathbf{G}^{(P-1)}=\overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}^{(P-1)}$, as

$$\mathbf{G}^{(P)}=\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{H}^{(P)}\end{bmatrix}=\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{H}^{(P)}\end{bmatrix}.\tag{8}$$

Applying the QR decomposition to $\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{H}^{(P)}\end{bmatrix}$ in Eq. (8) is expressed as

$$\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{H}^{(P)}\end{bmatrix}=\tilde{\mathbf{Q}}^{(P)}\tilde{\mathbf{R}}^{(P)}.\tag{9}$$

Now, $\tilde{\mathbf{Q}}^{(P)}$ is an $(N_rN_m+N_tN_m)\times N_tN_m$ unitary matrix and $\tilde{\mathbf{R}}^{(P)}$ is an $N_tN_m\times N_tN_m$ upper triangular matrix. Finally, from Eqs. (6), (8), and (9), we have

$$\mathbf{G}^{(P)}=\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\tilde{\mathbf{Q}}^{(P)}\tilde{\mathbf{R}}^{(P)}.\tag{10}$$

Since $\tilde{\mathbf{Q}}^{(P)}$ and $\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}$ are both a unitary matrix, $\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\tilde{\mathbf{Q}}^{(P)}$ becomes also a unitary matrix. The expanded received signal vector $\overline{\mathbf{Y}}^{(P)}$ of Eq. (4) can be transformed, by using $\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\tilde{\mathbf{Q}}^{(P)}$, as

$$\begin{aligned}\hat{\mathbf{Y}}^{(P)}&=\left[\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\tilde{\mathbf{Q}}^{(P)}\right]^H\overline{\mathbf{Y}}^{(P)}\\ &=\{\tilde{\mathbf{Q}}^{(P)}\}^H\begin{bmatrix}\{\overline{\mathbf{Q}}^{(P-1)}\}^H\,\overline{\mathbf{Y}}^{(P-1)}\\ \mathbf{Y}^{(P)}\end{bmatrix}=\{\tilde{\mathbf{Q}}^{(P)}\}^H\begin{bmatrix}\hat{\mathbf{Y}}^{(P-1)}_{expand}\\ \mathbf{Y}^{(P)}\end{bmatrix},\end{aligned}\tag{11}$$

which implies that the transformed signal vector $\hat{\mathbf{Y}}^{(P)}$ of Eq. (5) can be obtained recursively using the previously obtained transformed vector $\hat{\mathbf{Y}}^{(P-1)}_{expand}$ and the present overall frequency-domain received signal vector $\mathbf{Y}^{(P)}$.

Equations (9) and (11) form the recursive QR packet combining. As a special case, when $P$=1, Eqs. (9) and (11) become

$$\begin{cases}\mathbf{H}^{(1)}=\mathbf{Q}^{(1)}\mathbf{R}^{(1)}\\ \hat{\mathbf{Y}}^{(1)}=\{\mathbf{Q}^{(1)}\}^H\,\mathbf{Y}^{(1)}\end{cases},\tag{12}$$

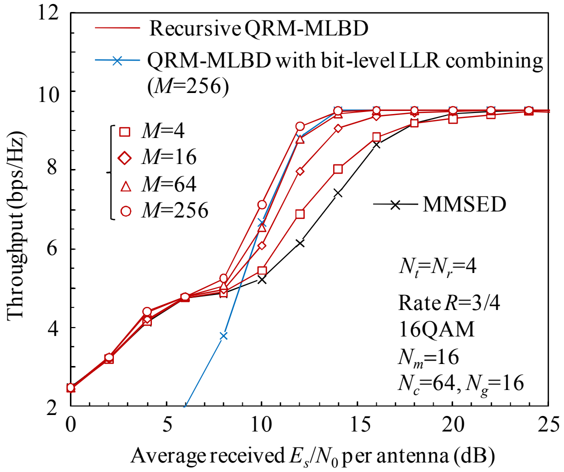
which is identical to the conventional QRM-MLBD (and also the bit-level LLR combining). The memory size can be reduced because this recursive QR packet combining does not need to store all of the received retransmitted packets and equivalent channel matrixes unlike the expanded QR packet combining. The computational complexity of QR decomposition can also be reduced compared to the expanded QR packet combining when the number of retransmissions is more than two. The size of a matrix to which the QR decomposition is applied is always $(N_rN_m+N_tN_m)\times N_tN_m$ irrespective of the number of retransmissions when $p>1$. It should be noted that this recursive QR packet combining obtains the same time diversity gain as the expanded QR packet combining without increasing the memory size and complexity. The proposed recursive QRM-MLBD is composed of the above recursive QR packet combining and ML detection using M-algorithm.

## IV. COMPUTER SIMULATION

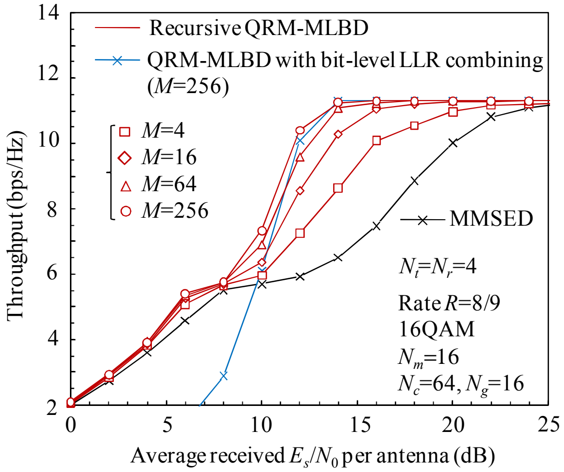
The HARQ throughput performance of SC-MIMO spatial multiplexing with the recursive QRM-MLBD is evaluated by computer simulation. 16QAM data modulation is used. We assume $N_t$=$N_r$=4, $N_m$=16, $N_c$=64, $N_g$=16, and $L$=16-path frequency-selective block Rayleigh fading channel with exponential power delay profile having the decay factor 3dB. Independent channel is assumed for each retransmission. The ideal channel estimation is assumed. We employ a rate 1/3 turbo encoder using two (13, 15) recursive systematic convolutional (RSC) component encoders. The two parity sequences from the turbo encoder are punctured to obtain rate-3/4 and 8/9 turbo codes. Log-MAP decoding with 6 iterations is assumed. The packet size is set to 4096.

Figure 2 plots the achievable throughput performances of SC-MIMO using type I HARQ and the recursive QRM-MLBD for $M$=4, 16, 64, and 256. The throughput performances using QRM-MLBD with the bit-level LLR packet combining for $M$=256 and MMSED also plotted for comparison. It can be seen from Fig. 2 that the recursive QRM-MLBD provides significantly higher throughput performance than the bit-level LLR packet combing in a low $E_s/N_0$ region where retransmissions always likely occur. This is because optimal packet combing gain can be obtained. The recursive QR combing can reduce the required $E_s/N_0$ to achieve 50% of the peak throughput by about 3.5, and 2.0dB, compared to the bit-level LLR packet combining when $R$=3/4 and 8/9, respectively. It can be also seen from Fig. 2 that in a low $E_s/N_0$ region the achievable throughput of the recursive QRM-MLBD is almost the same regardless of the value of $M$. This is because large packet combining gain can be achieved when retransmission occurs. As a result, the probability removing the correct path at early stages can be reduced. This implies that in the retransmission, the receiver can reduce the value of $M$. The recursive QRM-MLBD requires higher complexity for QR decomposition compared to the bit-level LLR packet combining. However, it can be noticed from Fig. 2 that the recursive QRM-MLBD can reduce the required value of $M$ for achieving a certain throughput, i.e., $M$=4 to achieve 50% of the peak throughput. This implies that the recursive QRM-MLBD can reduce significantly the computational complexity required for the squared Euclidean distance calculations. The overall computational complexity of the recursive QRM-MLBD for the first retransmission case ($p$=2) is only 13% ($M$=16) of that of the bit-level LLR packet combining ($M$=256).

It can also be seen from Fig. 2 that when compared to the MMSED, the recursive QRM-MLBD achieves better throughput performance. The recursive QRM-MLBD with $M$=16(64) can reduce the required $E_s/N_0$ for the peak throughput performance by about 4.0(5.8), and 5.5(7.5) dB when $R$=3/4 and 8/9, respectively.



(a) $R$=3/4



(b) $R$=8/9

Figure 2. Throughput performance.

Figure 3 shows how block size $N_m$ impacts the achievable throughputs of the recursive QRM-MLBD and MMSED. As the block size increases, the achievable throughput performance of MMSED improves because larger frequency diversity gain is obtained. On the other hand, the throughput performance of the recursive QRM-MLBD is almost the same despite of the number of block size. This is because when QRM-MLBD is used, the probability of removing the correct path at early stages increases as the block size increases. Consequently, the performance improvement of QRM-MLBD is larger when smaller block size is used. Below, we show the computational complexity comparison between the recursive QRM-MLBD and MMSED.

Figure 4 compares the computational complexities of the recursive QRM-MLBD and MMSED in the case of $N_t$=$N_r$=4, $N_c$=64, and 16QAM. The complexity here is defined as the number of complex multiply operations required in the signal detection. In the initial transmission ($p$=1), the recursive QRM-MLBD with $M$=16(64) requires about 3.2(9.7), 35(105), and 445(890) times higher computational complexity than MMSED when $N_m$=4, 16 and 64, respectively. As the block size increases, the recursive QRM-MLBD requires more complexity than MMSED. This is because QR decomposition of the equivalent channel matrix, which is a concatenation of the space and frequency-domain channel and DFT, is required. Path selection using M-algorithm also require high computational complexity. Therefore, further complexity reduction is necessary. This is left as an interesting future research topic. In the case of path selection using M-algorithm, the complexity can be reduced by using training sequence [13] and quadrant detection scheme [14].

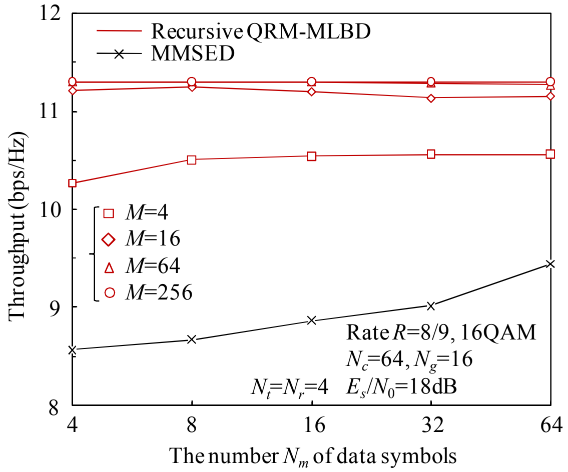


Figure 3. Influence of block size, $N_c$.

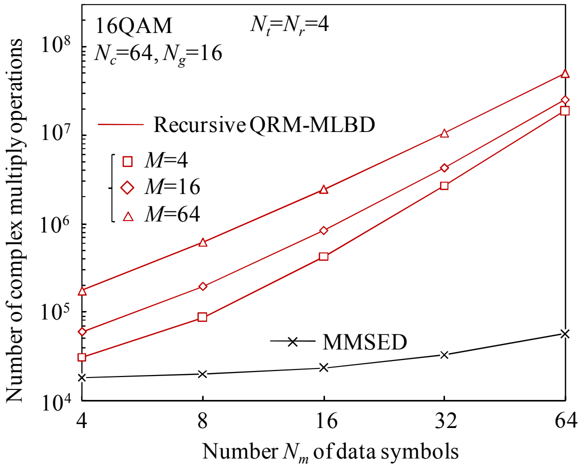


Figure 4. Complexity comparison.

## V. COMPUTER SIMULATION

In this paper, we proposed a block signal detection called MLBD using recursive QR packet combining and M-algorithm (recursive QRM-MLBD) suitable for SC-MIMO, which incorporates the packet combining into the QR decomposition. We also compared the achievable throughput performance of the recursive QRM-MLBD to that of the MMSED. The recursive QR packet combining requires less computational complexity than the expanded QR packet combining while achieving optimal packet combining gain. We showed that the recursive QRM-MLBD provides significantly higher throughput than the QRM-MLBD using the bit-level LLR packet combining with less computational complexity in a low $E_s/N_0$ region where retransmissions always likely occur. We also showed that the recursive QRM-MLBD achieves better throughput performance compared to the MMSED at the cost of increased computational complexity.

## REFERENCES

[1] G. J. Foschini and M. J. Gans, “On limit of wireless communications ini a fading environment when using multiple antennas,” Wireless Personal Commun., Vol. 6, No. 3, pp. 311-335, 1998.

[2] J. G. Proakis and M. Salehi, Digital communications, 5th ed., McGraw-Hill, 2008.

[3] K. Higuchi, H. Kawai, N. Maeda, H. Taoka, and M, Sawahashi, “Experiments on real-time 1-Gb/s packet transmission using MLD-based signal detection in MIMO-OFDM broadband radio access,” IEEE Journal on Selected Areas in Commun., Vol. 24, No. 6, pp. 1141-1153, June 2006.

[4] A. Nakajima, D. Garg, and F. Adachi, “Throughput of turbo coded hybrid ARQ using single-carrier MIMO multiplexing,” in Proc. IEEE 61st Vehicular Technology Conference (VTC2005-Spring), Vol. 1, pp. 610-614, 30 May-1 June 2005.

[5] N. Benjamin, L. Chan-Tong, and D. Falconer, “Trubo frequency domain equalization for single-carrier broadband wireless systems,” IEEE Trans. on Wireless Commun., Vol. 6, No. 2, pp. 759-767, Feb. 2007.

[6] A. van Zelst, R. van Nee, and G. A. Awater, “Space division multiplexing (SDM) for OFDM systems,” in Proc. IEEE 51st Vehicular Technology Conference (VTC2000-Spring), Vol. 2, pp. 1070-1074, May 2000.

[7] K. Nagatomi, K. Higuchi, and H. Kawai, “Complexity reduced MLD based on QR decomposition in OFDM MIMO multiplexing with frequency domain spreading and code multiplexing,” in Proc. IEEE Wireless Communications and Networking Conference (WCNC 2009), pp. 1-6, Apr. 2009.

[8] D. Chase, “Code combining – A maximum-likelihood decoding approach for combining an arbitrary number of noisy packets,” IEEE Trans. Commun., Vol. 33, No. 5, pp. 385-393, May 1985.

[9] E. W. Jang, J. Lee, H. Lou, and J. M. Cioffi, “Optimaol combining schemes for MIMO systems with hybrid ARQ,” in Proc. IEEE International Symposium on Information Theory, Jun. 2007.

[10] J. Lee, D. Toumpakaris, E. W. Jang, and H. L. Lou, “DFE-based receiver implementation for MIMO systems employing hybrid ARQ,” in Proc. GLOBECOM’08, 30 Nov. - 4 Dec. 2008.

[11] H. G. Myung, J. Lim, and D. J. Goodman, “Single-carrier FDMA for uplink wireless transmission,” in Proc. IEEE Vehicular Technology Magazine, Vol. 1, No. 3, pp. 30-38, Sep. 2006.

[12] W. Shin, H. Kim, M Son, and H. Park, “An improved LLR computation for QRM-MLD in coded MIMO systems,” in Proc. IEEE 66th Vehicular Technology Conference (VTC2007-Fall), pp.447-451, 30 Sept.-3 Oct. 2007.

[13] T. Tamamoto, K. Taleda, and F. Adachi, “Frequency-domain block signal detection with QRM-MLD for training sequence-aided single-carrier transmission,” EURASIP Journal on Advances in Signal Processing, Vol. 2011, Article ID 575706, 12 pages, 2010. doi: 10.1155/2011/575706.

[14] K. Temma, T. Yamamoto, and F. Adachi, "Computationally efficient 2-step QRM-MLD for single-carrier transmissions," in Proc. The IEEE International Conference on Communication Systems (IEEE ICCS 2010), Nov. 2010.