

# Power Planning for Power limited Two-Relay Network

Wei Peng

Department of Electrical and Communication  
Tohoku University  
Sendai, Japan  
peng@mobile.ecei.tohoku.ac.jp

Fumiyuki Adachi

Department of Electrical and Communication  
Tohoku University  
Sendai, Japan  
adachi@ecei.tohoku.ac.jp

**Abstract**—In this paper, power limited relay network is considered. We take one example of the limited relay, i.e. energy harvesting (EH) relay to study the power planning problem. It is assumed that the EH process is stationary and periodical. A two-EH-relay network is considered for simplicity. Efficient power planning can maximize the sum capacity that can be achieved by the saved power of EH relays. Several power planning schemes are compared and the sum capacity performance of them are presented by numerical results.

**Keywords**- energy harvesting; relay network, power planning

## I. INTRODUCTION

Cooperative relay network has been widely studied as a promising technique to improve the communication reliability [1]-[3]. Some cooperation schemes use one or more relays to forward to the destination an amplified version of the signal they have received from the source [4], i.e., amplify and forward (AF), while other cooperation schemes use the relay(s) to decode the signal from the source and then forward the decoded signal to the destination, i.e., decode and forward (DF).

In order to forward the signal to the destination, a relay will consume its own energy. The energy of a relay usually comes from a battery rather than power supply cable because that using power supply cable is more costly and impractical [5]. Therefore, when the battery is drained out, the relay can't serve anymore even if it has very good channel condition. An alternate emerging solution is to use energy harvesting nodes as relays. These nodes harvest energy from the environment and save the energy in a battery or capacitor [6]. An EH node can harvest energy again and again when the saved energy is used up. Therefore, EH relays are attractive due to the fact that they can extend the relay's lifetime without replacing battery or relying on power supply cable. Since the EH nodes are always simple in its structure, they usually serve in AF mode and in this study, we focus on the AF EH relay network.

The EH capability is the main limitation for an EH relay. In general, EH relay follows the energy neutrality constraint, i.e., the energy used by a relay can't exceed the harvested energy [7]. Therefore, transmit power of the EH relay in each time slot should be carefully planned in order to optimize the sum capacity or for other objectives. In this paper, an EH relay network with two relay nodes is considered. Power planning is carried out to maximize the sum capacity and several power settings are used to make comparison. In

our future work, the power planning method is going to be extended to more complicated EH relay network.

The rest of the paper is organized as follows. System model of two-relay network with EH relays will be described in Section II. The power planning algorithm is described in Section III. Numerical results for different power settings are shown in Section IV. Finally, the paper will be concluded by Section V.

## II. SYSTEM MODEL

A cooperative relay network with two EH relays is shown in fig. 1. There is a source (S), a destination (D) and two EH relays (R1 and R2) in the considered system. The source and destination are conventional terminals which are not considered as EH nodes. And it is assumed that there is no direct link between S and D. The information to be sent from S to D is divided into two parts and the communication between S and D is completed by 4 time slots, as shown in fig. 1. In time slot #1, S transmits the 1<sup>st</sup> part of information to R1 and R2, R1 and R2 will then amplify and forward the information to D in time slot #2. Similarly, in time slot #3, S transmits the 2<sup>nd</sup> part of information to R1 and R2, and R1 and R2 will then amplify and forward the information to D in time slot #4. As stated in the introduction, EH relay follows the energy neutrality constraint and the energy used by the relay cannot exceed the harvested energy. Without loss of generality, it is assumed that the two EH relays have harvested and saved a unit of energy before the transmission, and they cannot harvest energy until they finish the transmission. If the transmit power  $P_{R_1,2} (P_{R_1,2} \leq 1)$  is used by R1 to forward the signal in time slot #2, then the remaining power  $P_{R_1,4} = 1 - P_{R_1,2}$  will be used to forward the 2<sup>nd</sup> part signal in time slot #4. Similarly, if  $P_{R_2,2} (P_{R_2,2} \leq 1)$  is used by R2 in time slot #2, then  $P_{R_2,4} = 1 - P_{R_2,2}$  will be used by R2 in time slot #4. It is also assumed that channels between S and R1, S and R2, R1 and D as well as R2 and D are all frequency flat and independent from each other while they keep unchanged during the 4 time slots.

The baseband received signal at R1 and R2 in the 1<sup>st</sup> time slot transmission is given by

$$y_{R_1,1} = \sqrt{P_t} \cdot h_{SR_1} \cdot I_1 + n_{R_1}, \quad (1)$$

$$y_{R_2,1} = \sqrt{P_t} \cdot h_{SR_2} \cdot I_1 + n_{R_2}, \quad (2)$$

where  $y_{R_1,1}$  and  $y_{R_2,1}$  represent the received signal at R1 and R2 respectively;  $\sqrt{P_t}$  is the transmit power of S;  $h_{SR_1}$  and  $h_{SR_2}$  are the channel gain between S and R1 and R2 respectively, they are following Rayleigh fading;  $I_1$  is the 1<sup>st</sup> part of information (M-PSK modulated) and  $n_{R_1}$  and  $n_{R_2}$  are the Gaussian noise at R1 and R2 respectively.

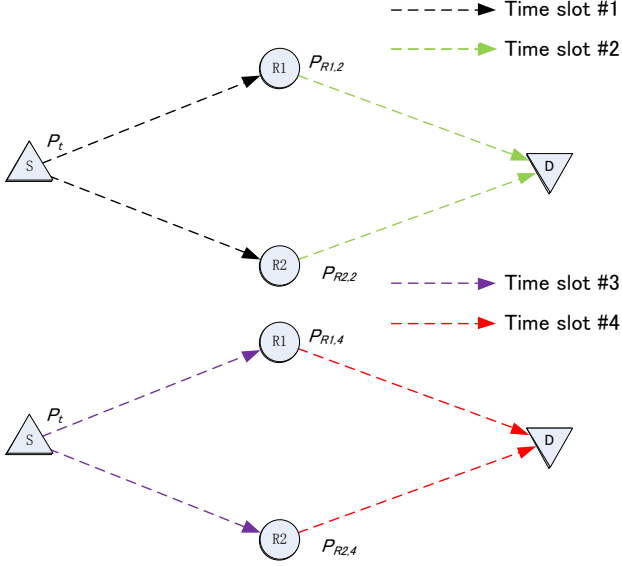


Figure 1 Cooperative relay network with two EH relays.

### III. CAPACITY ANALYSIS

Let  $\alpha_{1,2}$  denote the signal amplification by R1 in time slot #2.  $\alpha_{1,2}^2$  is given by [8]

$$\alpha_{1,2}^2 = \frac{P_{R_1,2}}{P_t |h_{SR_1}|^2 + N_R}, \quad (3)$$

where  $N_R$  is the noise power at R1. It is assumed that the noise power at both R1 and R2 are the same. Similarly, the amplification by R2 in time slot #2 is given by

$$\alpha_{2,2}^2 = \frac{P_{R_2,2}}{P_t |h_{SR_2}|^2 + N_R}. \quad (4)$$

Assuming that maximum ratio combining (MRC) is applied to the signals from the two relays, the signal to noise ratio (SNR) at D for the 1<sup>st</sup> part of information is then given by [9]

$$\gamma_{D,2} = \sum_{i=1}^2 \frac{\gamma_{SR_i,1} \gamma_{R_i,D,2}}{\gamma_{SR_i,1} + \gamma_{R_i,D,2} + 1}, \quad (5)$$

where  $\gamma_{SR_i,1} = \frac{P_t \cdot |h_{SR_i}|^2}{N_R}$  and  $\gamma_{R_i,D,2} = \frac{P_{R_i,2} \cdot |h_{R_i,D}|^2}{N_D}$ , here  $N_D$  denotes the noise power at D.

Following (1) ~ (5), the SNR for the 2<sup>nd</sup> part of information can be given by

$$\gamma_{D,4} = \sum_{i=1}^2 \frac{\gamma_{SR_i,3} \gamma_{R_i,D,4}}{\gamma_{SR_i,3} + \gamma_{R_i,D,4} + 1}, \quad (6)$$

where  $\gamma_{SR_i,3} = \frac{P_t \cdot |h_{SR_i}|^2}{N_R}$  and  $\gamma_{R_i,D,4} = \frac{(1 - P_{R_i,2}) \cdot |h_{R_i,D}|^2}{N_D}$ .

According to the information theory, the capacity of the 1<sup>st</sup> part information transmission is given by [9]

$$C_2 = \log_2(1 + \gamma_{D,2}), \quad (7)$$

and the capacity of the 2<sup>nd</sup> part information transmission is given by

$$C_4 = \log_2(1 + \gamma_{D,4}). \quad (8)$$

Therefore, the sum capacity of the transmission between S and D can be calculated by

$$\begin{aligned} C &= C_2 + C_4 \\ &= \log_2(1 + \gamma_{D,2}) + \log_2(1 + \gamma_{D,4}) \\ &= \log_2(1 + \gamma_{D,2} + \gamma_{D,4} + \gamma_{D,2} \cdot \gamma_{D,4}) \end{aligned} \quad (9)$$

A closed-form of the sum capacity can then be obtained by submitting (5) and (6) into (9), as

$$\begin{aligned} C &= \log_2 \left( 1 + \sum_{i=1}^2 \frac{P_t \cdot P_{R_i,2} |h_{SR_i}|^2 \cdot |h_{R_i,D}|^2}{N \cdot (P_t \cdot |h_{SR_i}|^2 + P_{R_i,2} \cdot |h_{R_i,D}|^2 + N)} \right. \\ &\quad + \sum_{i=1}^2 \frac{P_t \cdot (1 - P_{R_i,2}) |h_{SR_i}|^2 \cdot |h_{R_i,D}|^2}{N \cdot (P_t \cdot |h_{SR_i}|^2 + (1 - P_{R_i,2}) \cdot |h_{R_i,D}|^2 + N)} \\ &\quad \left. + \left( \sum_{i=1}^2 \frac{P_t \cdot P_{R_i,2} |h_{SR_i}|^2 \cdot |h_{R_i,D}|^2}{N \cdot (P_t \cdot |h_{SR_i}|^2 + P_{R_i,2} \cdot |h_{R_i,D}|^2 + N)} \right) \right. \\ &\quad \left. \cdot \left( \sum_{i=1}^2 \frac{P_t \cdot (1 - P_{R_i,2}) |h_{SR_i}|^2 \cdot |h_{R_i,D}|^2}{N \cdot (P_t \cdot |h_{SR_i}|^2 + (1 - P_{R_i,2}) \cdot |h_{R_i,D}|^2 + N)} \right) \right) \end{aligned} \quad (10)$$

In the above calculation,  $N_R = N_D = N$  is used to simplify the expression, i.e., it is assumed that the noise power is the same for the relay nodes and destination node. However, it is obvious that we can use different values for  $N_R$  and  $N_D$  for practical implementation.

Our objective is to optimize the sum capacity in (10), where  $P_t$ ,  $h_{SR_i}$ ,  $h_{R_i,D}$  and  $N$  can be treated as constants in each transmission. It is now obvious that the problem of

$$\max \left\{ C \mid P_t, h_{SR_i}, h_{R_i,D}, N \right\} \quad (11)$$

is equivalent to find out the optimal solution for  $P_{R_i,2}$  ( $i=1,2$ ).

It can be observed from (10) that neither  $P_{R_1,2}$  nor  $P_{R_2,2}$  can be calculated without knowing the value of the other. Therefore,  $P_{R_1,2}$  and  $P_{R_2,2}$  should be jointly optimized.

For a given  $P_{R_2,2}$ , (10) is rewritten as

$$C = f_1(P_{R_1,2}), \quad (12)$$

and the optimal  $P_{R_1,2}$  is the solution of the equation

$$\frac{\partial f_1(P_{R_1,2})}{\partial P_{R_1,2}} = 0, \quad (13)$$

where  $\partial f(x)/\partial x$  denotes the derivative operation. Similarly, for a given  $P_{R_1,2}$ , (10) is rewritten as

$$C = f_2(P_{R_2,2}), \quad (14)$$

and the optimal  $P_{R_2,2}$  is the solution for

$$\frac{\partial f_2(P_{R_2,2})}{\partial P_{R_2,2}} = 0. \quad (15)$$

If unit transmit power at S is used for simplicity, the optimal solution for  $P_{R_1,2}$  equals to the solution for equation

$$\frac{\partial f_1'(x)}{\partial x} = 0, \quad (16)$$

where

$$f_1'(x) = \frac{Na_1^2b_1 + N^2a_1b_1 + Na_1b_1c_2 + a_1^2b_1c_2 + \left( 2Na_1b_1^2 + a_1^2b_1^2 + a_1b_1^2c_1 + a_1b_1^2c_2 + a_1^2b_1c_1 + a_1^2b_1c_2 + Na_1b_1c_1 - Na_1b_1c_2 \right) x - \left( 2Na_1b_1^2 + a_1^2b_1^2 + a_1b_1^2c_1 + a_1b_1^2c_2 + a_1b_1^2c_2 \right) x^2}{N^2(a_1 + b_1x + N)[a_1 + b_1(1-x) + N]}. \quad (17)$$

$$\text{Here } a_1 = |h_{SR_1}|^2, \quad b_1 = |h_{R_1,D}|^2, \quad a_2 = |h_{SR_2}|^2, \quad b_2 = |h_{R_2,D}|^2, \\ c_1 = \frac{(1 - P_{R_2,2})a_2b_2}{a_2 + (1 - P_{R_2,2})b_2 + N} \text{ and } c_2 = \frac{P_{R_2,2}a_2b_2}{a_2 + P_{R_2,2}b_2 + N}.$$

And the optimal solution for  $P_{R_2,2}$  equals to the solution for equation

$$\frac{\partial f_2'(x)}{\partial x} = 0, \quad (18)$$

where

$$f_2'(x) = \frac{Na_2^2b_2 + N^2a_2b_2 + Na_2b_2d_2 + a_2^2b_2d_2 + \left( 2Na_2b_2^2 + a_2^2b_2^2 + a_2b_2^2d_1 + a_2b_2^2d_2 + a_2^2b_2d_1 + a_2^2b_2d_2 + Na_2b_2d_1 - Na_2b_2d_2 \right) x - \left( 2Na_2b_2^2 + a_2^2b_2^2 + a_2b_2^2d_1 + a_2b_2^2d_2 + a_2b_2^2d_2 \right) x^2}{N^2(a_2 + b_2x + N)[a_2 + b_2(1-x) + N]}, \\ d_1 = \frac{(1 - P_{R_2,1})a_1b_1}{a_1 + (1 - P_{R_2,1})b_1 + N} \text{ and } d_2 = \frac{P_{R_2,1}a_1b_1}{a_1 + P_{R_2,1}b_1 + N}.$$

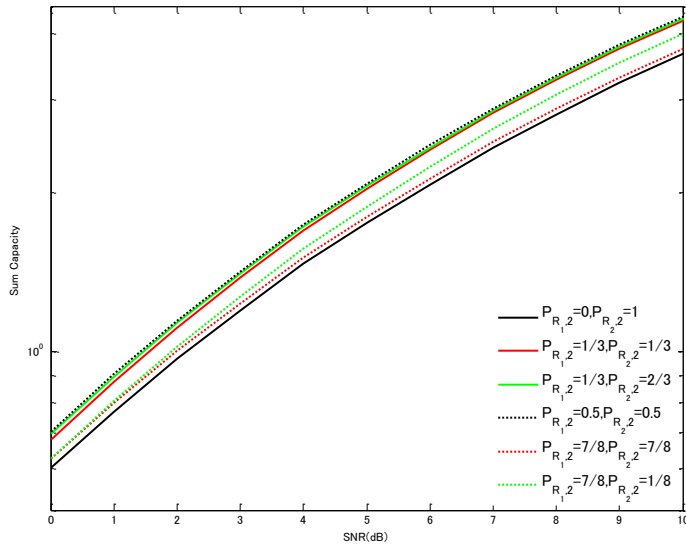
#### IV. NUMERICAL RESULTS

In this section, several power setting plans are considered. The sum capacity achieved by those power setting plans are calculated following (10). Instead of using the optimal solution for (17) and (19), some simple power settings are going to be used for simplicity. The optimal solution for (17) and (19) and extension to cooperative relay systems with more than two EH relays will be presented in our future work. The parameters used for numerical calculation are listed in Tab. I.

TABLE I. PARAMETERS

<i>Number of Relays</i>	2	
<i>Relay type</i>	EH relay	
<i>Saved power per relay</i>	1 unit	
<i>Transmit power at S</i>	$P_t = 1$ unit	
SNR $P_t/N$	0dB ~ 20dB	
<i>Channel</i>	Non-selective Rayleigh fading	
<i>Power setting</i>	Plan 1	$P_{R_1,2} = 0, P_{R_2,2} = 1$
	Plan 2	$P_{R_1,2} = 0.5, P_{R_2,2} = 0.5$
	Plan 3	$P_{R_1,2} = 1/3, P_{R_2,2} = 1/3$
	Plan 4	$P_{R_1,2} = 1/3, P_{R_2,2} = 2/3$
	Plan 5	$P_{R_1,2} = 7/8, P_{R_2,2} = 7/8$
	Plan 6	$P_{R_1,2} = 7/8, P_{R_2,2} = 1/8$

The numerical results of the sum capacity by using the power setting plans in Tab. 1 are shown in Fig. 2. Note that in power setting plan 1,  $P_{R_1,2} = 0$  and  $P_{R_2,2} = 1$ . It represents the case that only one relay is used in time slot #2 or #4 and when the relay is active, it will use up its saved energy. In this situation, SNR on the link between the active relay and D is maximized; however, no diversity gain can be obtained. While in plan 2,  $P_{R_1,2} = 0.5$  and  $P_{R_2,2} = 0.5$ . It corresponds to the situation while the saved energy of the relays will be evenly used in time slot #2 and time slot #4. It can be observed from fig. 2 that among the 6 power setting plans, plan 1 gives the lowest sum capacity while plan 2 achieves the largest sum capacity. Note that we have not considered the effect of path loss and the distribution of the relays in this study. In addition, simple power setting plans are used instead of using the optimal power planning in (17) and (19). From the results in fig. 2, it is intuitively implied that the maximal sum capacity might be achieved by evenly allocating the saved power of each relay when the effect of path loss is not considered. Theoretical explanation will be made in our future work, and the analysis in this paper will also be generalized to cooperative relay network with more than two EH relays.



## V. CONCLUSIONS

In this paper, we have studied power planning of cooperative power limited relay network with two EH relays. Theoretical analysis is done on the sum capacity and the optimal solution of the transmit power of the two relays have been given. Numerical results are carried out by assuming 6 simple power setting plans. Power planning for more complicated system, e.g., where the effect of path loss has to be considered or when the number of EH relays are larger than 2, will be presented in our future work.

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