Joint MMSE-FDE & Spectrum Combining for Single-carrier Transmission with Antenna Diversity in the Presence of Timing Offset

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***Abstract*—Frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion is a powerful equalization technique for the broadband single-carrier (SC) transmission. However, the presence of timing offset produces the inter-symbol interference (ISI) and degrades the bit error rate (BER) performance. As the roll-off factor of the transmit filter increases, the performance degrades more. Recently, we proposed joint MMSE-FDE & spectrum combining which can achieve the frequency diversity gain while suppressing the negative impact of timing offset for the SC transmission. In this paper, we extend the joint MMSE-FDE & spectrum combining to include the antenna diversity reception.**

Keywords; Frequency-domain equalization, Nyquist filter, oversampling, timing offset, antenna diversity reception, single-carrier transmission

I. INTRODUCTION

The broadband wireless channel is composed of many propagation paths with different time delays and the strong frequency-selective fading channel is produced [1]-[3]. Therefore, the bit error rate (BER) performance of the broadband single-carrier (SC) transmission degrades due to the strong inter-symbol interference (ISI). The use of the frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can improve significantly the BER performance [4]-[6]. This is only true in the case of no timing offset between a transmitter and a receiver.

In many spectrum-efficient wireless communication systems, a square-root Nyquist filter is used at the transmitters to limit the signal bandwidth and the same filter at the receivers. However, the presence of timing offset between a transmitter and a receiver produces the ISI and degrades the BER performance as the roll-off factor of Nyquist filter increases as shown in Fig. 1. The reason for this performance degradation is that, when the received signal is sampled by the symbol rate, the received signal spectrum is distorted since adjacent frequency-shifted spectra are given different phase rotations and overlapped if the roll-off factor of Nyquist filter is larger than 0.

To solve the above problem, we proposed joint MMSE-FDE & spectrum combining [7]. The overlapping of spectra phase-rotated due to the timing offset can be avoided by 2-times oversampling. Therefore, when MMSE-FDE is applied to the oversampled received signal, the spectrum distortion due to the channel frequency-selectivity and the phase rotation due to the timing offset can be simultaneously compensated. MMSE-FDE and the spectrum combining are jointly performed to restore the ISI-free spectrum over the desired frequency range. The proposed MMSE-FDE can achieve a better BER performance as the filter roll-off factor increases. In our previous paper, we assumed the single antenna reception. The combination of MMSE-FDE and antenna diversity reception significantly improves the BER performance of the SC transmission [6]. In this paper, we extend the joint MMSE-FDE & spectrum combining to include the antenna diversity reception.

The remainder of this paper is organized as follows. Section II presents the system model of the proposed MMSE-FDE. The computer simulation results are discussed in Sect. III. Section IV offers some conclusions.

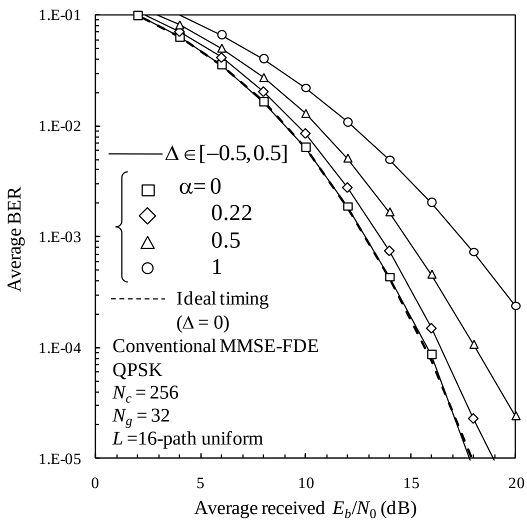

Fig. 1 BER performance of the conventional MMSE-FDE in the presence of timing offset.

II. JOINT MMSE-FDE & SPECTRUM COMBINING WITH ANTENNA DIVERSITY

In Fig. 2, the receiver structure of the SC transmission using the proposed joint MMSE-FDE & spectrum combining is illustrated. First, the received signal of each receive antenna is oversampled at a faster rate than the symbol rate to avoid the spectrum overlapping. When the square-root raised cosine filter is used as transmit filter, the spectrum overlapping can be avoided by using double oversampling (the received signal sampled at the rate $2/T_s$), as shown in Fig. 3. Then, MMSE-FDE is applied over the frequency range of $-N_c \le k < N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity. Finally, the spectrum combining (or the frequency-domain down sampling) and antenna diversity combining are performed to recover the desired signal spectrum over the frequency range of $-N_c/2 \le k < N_c/2$.

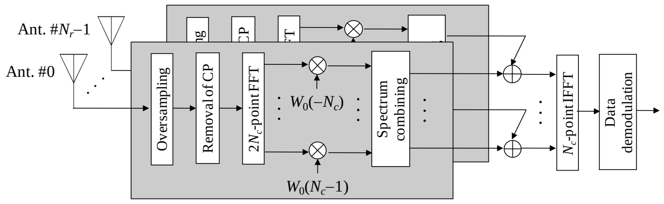

Fig. 2 Receiver structure of SC transmission using the proposed joint MMSE-FDE & spectrum combining.

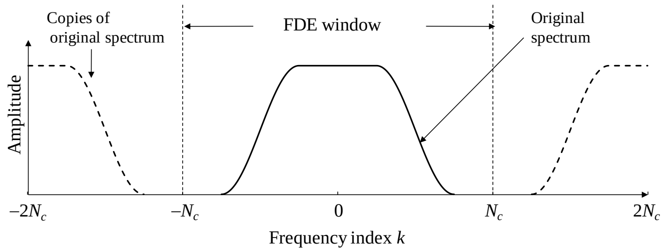

Fig. 3 Signal spectrum after double oversampling.

A. Signal representation

At the transmitter, the data-modulated symbol sequence is divided into a sequence of N_c-symbol blocks, where N_c is the size of fast Fourier transform (FFT). An N_g-symbol cyclic prefix (CP) is inserted into the guard interval (GI) of each symbol block. The GI-inserted symbol block is transmitted after passing through the square-root Nyquist transmit filter to limit the signal bandwidth.

The transmitted symbol block is received at the receiver via a frequency-selective fading channel. The $n(=0\sim N_r-1)$th antenna received signal oversampled at the rate $2/T_s$ can be expressed as

$$r_n(i) = \sqrt{\frac{2E_s}{T_s}} \sum_{l=0}^{L-1} \sum_{i'=-\infty}^{\infty} h_{n,l}\, s(i' \bmod N_c)\, \varphi\left(\frac{i}{2} + \Delta_n - \tau_l - i'\right) + \nu(i) + \eta(i), \tag{1}$$

where E_s is the symbol energy, $h_{n,l}$ and τ_l are respectively the complex-valued channel gain with $\sum_{l=0}^{L-1} E[|h_{n,l}|^2] = 1$ and delay time of l-th path, $\{s(m); m=0\sim N_c-1\}$ is the transmitted symbol block, $\nu(i)$ and $\eta(i)$ are respectively the inter-block interference (IBI) and the filter output of the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$ with N_0 being the single-sided power spectrum density, $\varphi(t)$ is the transmit filter impulse response, and Δ_n is the timing offset. In this paper, we assume that the root raised cosine filter with the roll-off factor α is used as the transmit filter.

After the removal of $2N_g$-sample CP, $2N_c$-point FFT is applied to transform the oversampled signal block $\{r_n(i); i=0\sim 2N_c-1\}$ of the nth receive antenna into the frequency-domain signal $\{R_n(k); k=-N_c\sim N_c-1\}$. The kth frequency component $R_n(k)$ can be expressed as

$$\begin{aligned} R_n(k) &= \frac{1}{\sqrt{2N_c}} \sum_{i=0}^{2N_c-1} r_n(i) \exp\left(-j2\pi k \frac{i}{2N_c}\right) \\ &= \sqrt{\frac{2E_s}{T_s}} \tilde{H}_n(k,\Delta_n) S(k) + \mathrm{N}_n(k) + \Pi_n(k) \end{aligned}, \tag{3}$$

where $\tilde{H}_n(k,\Delta_n)$, $S(k)$, $\mathrm{N}_n(k)$, and $\Pi_n(k)$ are the overall (transmit/receive filter + channel) transfer function, the signal component, the IBI component, and the noise component, respectively. $\tilde{H}_n(k,\Delta_n)$ and $S(k)$ are respectively given as

$$S(k) = \frac{1}{\sqrt{N_c}} \sum_{i=0}^{N_c-1} s(i) \exp\left(-j2\pi k \frac{i}{N_c}\right) \tag{4}$$

$$\begin{aligned} \tilde{H}_n(k,\Delta_n) = \sqrt{2} \sum_{p=-\infty}^{\infty} H_n(k-2pN_c)\Phi(k-2pN_c) \\ \times \exp\left\{ j2\pi(k-2pN_c)\frac{\Delta_n}{N_c} \right\} \end{aligned}, \tag{5}$$

where $H_n(k)$ is the channel gain at the kth frequency given as

$$H_n(k) = \sum_{l=0}^{L-1} h_{n,l} \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right). \tag{6}$$

$\Phi(k)$ is the transfer function of the transmit filter given as

$$\Phi(k) = \begin{cases} 1, & 0 \le \left|\frac{k}{N_c}\right| \le \frac{1-\alpha}{2} \\ \cos\frac{\pi}{2\alpha}\left(\left|\frac{k}{N_c}\right| - \frac{1-\alpha}{2}\right), & \frac{1-\alpha}{2} \le \left|\frac{k}{N_c}\right| \le \frac{1+\alpha}{2} \\ 0, & \text{elsewhere} \end{cases}. \tag{7}$$

$\tilde{H}_n(k,\Delta_n)$ can be estimated by using pilot-assisted channel estimation [8]-[10]. From Eq. (5), it can be understood that the copies of the original spectrum are phase-rotated due to the timing offset and frequency-shifted by an integer multiple of $2/T_s$.

B. Joint MMSE-FDE & spectrum combining

One-tap MMSE-FDE is performed over the frequency range of $-N_c \le k < N_c$ to simultaneously compensate for both the phase rotation due to the timing offset and the spectrum distortion due to the channel frequency-selectivity as

$$\begin{aligned}\hat{R}_n(k) &= R_n(k)W_n(k) \\ &= \sqrt{\frac{2E_s}{T_s}}\hat{H}_n(k,\Delta)S(k) + \hat{\mathrm{N}}_n(k) + \hat{\Pi}_n(k)\end{aligned}, \tag{8}$$

where $W_n(k)$ is the MMSE-FDE weight.

After MMSE-FDE, the spectrum combining and antenna diversity combining is performed to restore the ISI-free condition over the desired frequency range $-N_c/2 \le k < N_c/2$ as shown in Fig. 4. The frequency-domain signal after the spectrum combining and antenna diversity combining is given by

$$\begin{aligned}\breve{R}(k) &= \sum_{n=0}^{N_r-1}\sum_{q=-1}^{1}\hat{R}_n(k-qN_c) \\ &= \sqrt{\frac{2E_s}{T_s}}\breve{H}(k)S(k) + \breve{\mathrm{N}}(k) + \breve{\Pi}(k)\end{aligned}, \tag{9}$$

where

$$\begin{cases}\breve{H}(k) = \sum_{n=0}^{N_r-1}\sum_{q=-1}^{1}\hat{H}_n(k-qN_c,\Delta_n) \\ \breve{\mathrm{N}}(k) = \sum_{n=0}^{N_r-1}\sum_{q=-1}^{1}\hat{\mathrm{N}}_n(k-qN_c) \\ \breve{\Pi}(k) = \sum_{n=0}^{N_r-1}\sum_{q=-1}^{1}\hat{\Pi}_n(k-qN_c)\end{cases}. \tag{10}$$

The frequency-domain signal $\{\breve{R}(k); k = -N_c/2 \sim N_c/2-1\}$ after MMSE-FDE and spectrum combining is transformed by N_c-point IFFT into the time-domain signal block for succeeding data demodulation. Below, we derive the MMSE-FDE weight.

We define the equalization error $e(k)$ after the spectrum combining at the kth frequency as

$$\begin{aligned}e(k) &= \breve{R}(k) - \sqrt{\frac{2E_s}{T_s}}S(k) \\ &= \sum_{n=0}^{N_r-1}\sum_{q=-1}^{1}\hat{R}_n(k-qN_c) - \sqrt{\frac{2E_s}{T_s}}S(k)\end{aligned}, \tag{11}$$

where $-N_c/2 \le k < N_c/2$. The MMSE weight $\{W_n(k); k=-N_c \sim N_c-1\}$ for joint FDE & spectrum combining which minimizes the MSE $E[|e(k)|^2]$ can be derived as

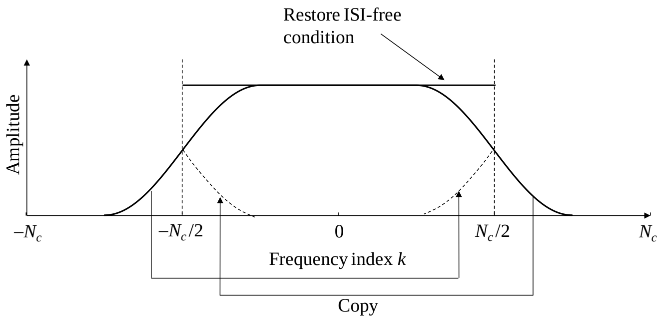

Fig. 4 Spectrum combining.

$$W_n(k) = \frac{\tilde{H}_n^*(k,\Delta_n)}{\sum_{n'=0}^{N_r-1}\sum_{q=-1}^{1}\frac{\Lambda_n^{-1}(\Delta_n)}{\Lambda_{n'}^{-1}(\Delta_{n'})}\left|\tilde{H}_{n'}(k-qN_c,\Delta_{n'})\right|^2 + \Lambda_n^{-1}(\Delta_n)}, \tag{12}$$

where $\Lambda_n(\Delta_n)$ denotes the signal-to-IBI plus noise power ratio (SINR) given by

$$\Lambda_n(\Delta_n) = \frac{(2E_s/T_s)}{E[|\mathrm{N}_n(k)|^2 + |\Pi_n(k)|^2]}. \tag{13}$$

Since $\tilde{H}_n(k,\Delta_n)$ includes the transfer function of the transmit filter, the MMSE-FDE weight also takes a role of the receive filter matched to the transmit filter (this is the reason why no receive filter is necessary in the receiver structure of Fig. 2).

III. COMPUTER SIMULATION

A. Simulation condition

The computer simulation condition is summarized in Table I. We assume QPSK data-modulation, a signal block length of N_c=256 symbols, and a CP length of N_g=32 symbols. The propagation channel is assumed to be L=16-path frequency-selective block Rayleigh fading channel having uniform power delay profile. The receiver has N_r(=2, 4) antennas. The timing offset Δ_n normalized by the symbol duration T_s is assumed to be uniformly distributed over [–0.5, 0.5] for n=0~N_r–1. The ideal channel estimation is also assumed.

TABLE I. SIMULATION CONDITION

Data modulation	QPSK	
Block length	N_c=256	
CP length	N_g=32	
Channel model	Frequency-selective block Rayleigh fading	
	Power delay profile	L=16-path uniform
Transmit filter	Root raised cosine filter	
	Roll-off factor	α=0~1
Receiver	No. of receive antennas	N_r=2, 4
	Timing offset	$\Delta_n \in$[–0.5, 0.5] (n=0~N_r–1)
	Channel estimation	Ideal

B. Impact of timing offset on the conventional MMSE-FDE

Figure 5 plots the average BER performance of the conventional MMSE-FDE as a function of the average received bit energy-to-noise power spectrum density ratio $E_b/N_0(=0.5(E_s/N_0)(1+N_g/N_c))$ per receive antenna. For comparison, the BER performance for N_r=1 and Δ_0=0 (no timing offset case) is also plotted.

As shown in Fig. 5, as N_r increases, the BER performance significantly improves compared to the case of N_r=1. However, when the timing offset is present, the BER performance degrades as α increases. This is because, as α increases, the overlapping interval of adjacent spectra which are phase-rotated due to the timing offset becomes wider, thereby enhancing the spectrum distortion. When α=0, the conventional MMSE-FDE in the presence of timing offset achieve almost the same performance as in the no timing offset case. This is because phase-rotated adjacent spectra do not overlap and therefore, no spectrum distortion is produced.

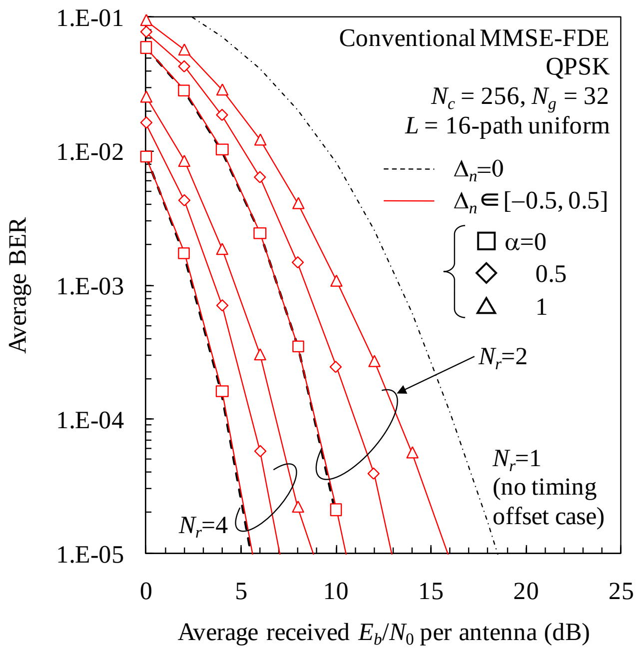

Fig. 4 Impact of timing offset on the conventional MMSE-FDE.

C. BER performance of the proposed joint MMSE-FDE & spectrum combining

Figure 6 plots the BER performance of the proposed joint MMSE-FDE & spectrum combining with α and N_r as parameters. For comparison, the BER performances for N_r=1 are also plotted.

The proposed joint MMSE-FDE & spectrum combining can achieve almost the same performance as in the no timing offset case irrespective of N_r and α. Furthermore, the performance of joint MMSE-FDE & spectrum combining improves as N_r and α increase. The reason for this is that, as the signal bandwidth becomes wider, the proposed joint MMSE-FDE & spectrum combining with antenna diversity can achieve increased frequency diversity gain due to joint equalization and frequency and antenna diversity combining.

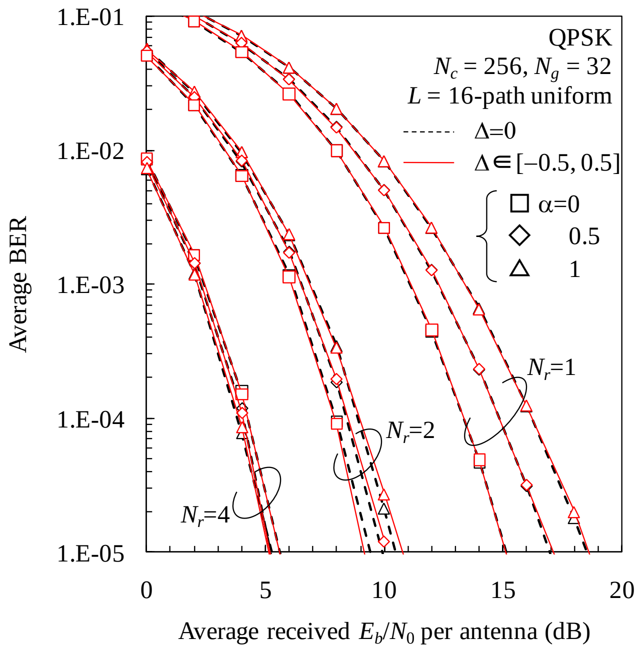

Fig. 4 BER performance of joint MMSE-FDE & spectrum combining.

IV. CONCLUSION

In this paper, we extended the joint MMSE-FDE & spectrum combining to include the antenna diversity reception. The joint MMSE-FDE & spectrum combining can provide almost the same performance as in the no timing offset case and much better performance due to larger frequency diversity gain as the filter roll-off factor and the number of receive antennas increase.

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