

# Multi-user Downlink Transmit Beamforming for the Broadband Single-Carrier Distributed Antenna Network

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**Abstract**—This paper studies the multi-user downlink transmit beamforming algorithms for the broadband single-carrier distributed antenna network (SC-DAN). The distributed antennas will be used as antenna arrays by using transmit beamforming. In this work, the beamforming weight is designed in order to avoid the interference cancellation from the mobile terminals. Two methods are adopted. One is to use the maximum ratio transmission (MRT) beamforming weight to maximize the desired signal's power at each mobile terminal. The other one is to use the null steering beamforming weight to minimize the interference power at each mobile terminal. The beamforming algorithms using the two methods are described and their performances are compared by computer simulations.

**Keywords**—distributed antenna network; downlink transmission; transmit beamforming

## I. INTRODUCTION

Broadband data services of greater than 1 Giga bps (in particular, downlink applications) are demanded in the future wireless communication networks. To realize such a high data rate, there are two major problems. On one hand, due to the multi-path fading, the wireless channel is characterized by severe frequency selectivity [1]. As a result, it is necessary to suppress the inter-symbol interference (ISI) at the receiver. The ISI can be suppressed by time domain equalization techniques such as maximum likelihood sequence estimation (MLSE) [2]. However, when the data rate increases, the number of resolvable paths increases as well and the complexity of MLSE grows exponentially as a result. Compared to the time domain equalization techniques, frequency domain equalization (FDE) [3], which is not a function of the channel frequency selectivity, has much less complexity.

On the other hand, an unacceptably high transmit power will be required to realize high data rate if the cell coverage is kept unchanged. Distributed antenna network (DAN) [4] was proposed as a solution to increase the cell coverage while to maintain low transmit power. In the DAN, a number of antennas are distributed in each cell and the distributed antennas are connected with a signal processing center (which is similar to the BS in conventional cellular system). The mobile terminal can communicate with the nearby located antennas even when it is located at near the cell edge. Therefore, the transmit power in the DAN can be kept low while the coverage of the cell can be greatly extended.

The signal processing ability of mobile terminals is limited by its small size and low power consumption. Fortunately, powerful signal processing can be allowed at the DAN signal processing center. In our previous study, we have proposed single-carrier frequency domain adaptive antenna array (SC-FDAAA) for the uplink (mobile terminal-to-BS) transmission in a cellular system. In this paper, the downlink transmission in SC-DAN is considered. Distributed transmit beamforming can be used to realize multi-user transmission while alleviating sophisticated interference cancellation techniques at mobile terminals. Transmit beamforming has been studied in [5] for point-to-point multiple-input multiple-output (MIMO) system. And it was applied to multi-user downlink transmission in [6] for the conventional cellular systems. However, multi-user transmit beamforming in the DAN has not been addressed yet.

In this study, we will consider multiple users and each user can be equipped with either single or multiple antennas. Two beamforming algorithms will be studied. One is designed to maximize the desired receive signal power at each user. The other is designed to minimize the interference between the users. In this paper, no transmit power control (TPC) is assumed so that we can focus on the transmit beamforming itself.

The rest of the paper is organized as follows. The system model of DAN is described in Section II. Two transmit beamforming algorithms for the DAN are proposed in Section III. And simulation results for the achievable bit error rates (BER) are shown in Section IV. Finally, the paper is concluded by Section V.

## II. SYSTEM MODEL

The DAN structure is shown in Fig. 1. The distributed antennas can be separately located or clustered, and they are connected to the central processor by optical fiber cables. It is assumed that there are  $N_t$  active transmit antennas and  $U$  users with each user being equipped with  $N_r$  antennas. SC transmission with FDE is a block transmission.

In this paper, we consider a cyclic-prefix inserted single-carrier (CP-SC) block transmission of  $N_c$  data symbols and assume a block fading channel between each user and each antenna (i.e., the channel remains unchanged during the

transmission period of an  $N_c$ -symbol block). It is assumed that the CP length is longer than the maximum time delay of the channel. The CP insertion makes the received symbol block to be a circular convolution of the transmitted symbol block and the channel impulse response as well as to avoid inter block interference (IBI). In the following, we omit the insertion and removal of the CP for the simplicity purpose. The symbol-spaced discrete time representation of the signal is used.

Assuming an  $L$ -path channel, the impulse response of the channel between the  $r^{\text{th}}$  antenna of the  $u^{\text{th}}$  user and the  $m^{\text{th}}$  transmit antenna can be expressed as

$$h_{r,u,m}(\tau) = \sum_{l=0}^{L-1} h_{r,u,m,l} \delta(\tau - \tau_l), \quad (1)$$

where  $h_{r,u,m,l}$  and  $\tau_l$  are the path gain and time delay normalized by the symbol duration of the  $l^{\text{th}}$  path, respectively.  $h_{r,u,m,l}$  follows the complex Gaussian distribution and satisfies  $\sum_{l=0}^{L-1} E\{|h_{r,u,m,l}|^2\} = 1$ , where  $E\{\cdot\}$  represents the expectation. It is assumed that the time delay of each path is an integer multiple of symbol duration and  $\tau_l = l$ .

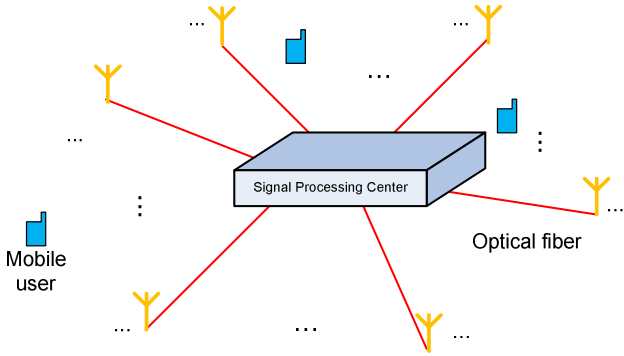


Figure 1 DAN structure.

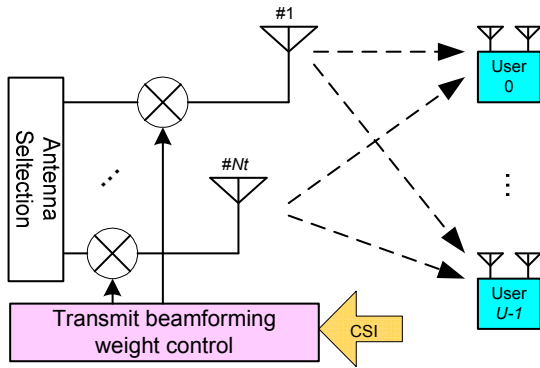


Figure 2 Downlink transmission using transmit beamforming.

The system model for downlink transmission using transmit beamforming is shown as in Fig. 2. In the DAN, there exist a large number of transmit antennas and antenna selection is necessary before transmit beamforming is performed. After antenna selection, each transmit signal will be weighted by the transmit beamforming weight. Both antenna selection and beamforming weight control are carried out by the signal

processing center (SPC). It is assumed that the signal processing center has perfect knowledge of channel state information (CSI). The baseband equivalent received signal block  $\{r_{r,u}(t); t=0 \sim N_c-1\}$  of  $N_c$  symbols at the  $r^{\text{th}}$  antenna of the  $u^{\text{th}}$  user can be expressed as

$$r_{r,u}(t) = \sum_{l=0}^{L-1} \mathbf{h}_{r,u,l}^T \sum_{i=0}^{U-1} \boldsymbol{\omega}_i(t) \sqrt{2P_i} s_i(t-l) + n_{r,u}(t), \quad (2)$$

where  $s_i(t)$  and  $P_i$  are the transmit signal and transmit signal power for the  $u^{\text{th}}$  user ( $u=0 \sim U-1$ ), respectively (as stated in the Introduction Section, no TPC is assumed in this study and unit transmit power for each user is used);  $\mathbf{h}_{r,u,l} = [h_{r,u,0,l} \xi_{r,u,0}^{-\alpha} \quad h_{r,u,1,l} \xi_{r,u,1}^{-\alpha} \quad \cdots \quad h_{r,u,N_c-1,l} \xi_{r,u,N_c-1}^{-\alpha}]^T$  and symbol  $(\cdot)^T$  represents the transpose operation;  $\boldsymbol{\omega}_i(t) = [\omega_{i,0}(t) \quad \omega_{i,1}(t) \quad \cdots \quad \omega_{i,N_c-1}(t)]^T$  is the transmit beamforming weight. In fact, the weight control is performed in frequency domain by the SPC, and  $\boldsymbol{\omega}_i(t)$  is the time domain transfer function of the frequency domain transmit beamforming weight;  $\xi_{r,u,m}$  is the distance between the  $r^{\text{th}}$  antenna of the  $u^{\text{th}}$  user and the  $m^{\text{th}}$  transmit antenna;  $\alpha$  represents the path loss exponent in dB. To simplify the analysis in this paper, no shadowing loss is assumed.  $n_{r,u}(t)$  is the additive white Gaussian noise (AWGN).

Frequency-domain representation of (2) is given by

$$\mathbf{R}_{r,u}(k) = \mathbf{H}_{r,u}^T(k) \sum_{i=0}^{U-1} \mathbf{W}_i(k) S_i(k) + \mathbf{N}_{r,u}(k), \quad (3)$$

where

$$\begin{cases} S_i(k) = \frac{1}{\sqrt{N_c}} \sum_{t=0}^{N_c-1} s_i(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ \mathbf{H}_{r,u} = [\mathbf{H}_{r,u,0} \quad \mathbf{H}_{r,u,1} \quad \cdots \quad \mathbf{H}_{r,u,N_c-1}] \\ \mathbf{W}_i(k) = \frac{1}{\sqrt{N_c}} \sum_{t=0}^{N_c-1} \boldsymbol{\omega}_i(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ \mathbf{N}_{r,u}(k) = \frac{1}{\sqrt{N_c}} \sum_{t=0}^{N_c-1} n_{r,u}(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \end{cases}$$

$\mathbf{H}_{r,u,m}(k) = \xi_{r,u,m}^{-\alpha} \sum_{l=0}^{L-1} \sum_{\tau=0}^{N_c-1} h_{r,u,m,l} \exp\left(-j2\pi k \frac{t}{N_c}\right)$ , and  $\mathbf{W}_i(k)$  is the frequency-domain transmit beamforming weight.

The received signal vector at the  $u^{\text{th}}$  user,  $\mathbf{R}_u(k)$ , is then expressed in matrix form as

$$\begin{aligned} \mathbf{R}_u(k) &= [\mathbf{R}_{0,u}(k), \mathbf{R}_{1,u}(k), \cdots, \mathbf{R}_{N_c-1,u}(k)]^T, \\ &= \mathbf{H}_u(k) \mathbf{W}(k) \mathbf{S}(k) + \mathbf{N}_u(k) \end{aligned} \quad (5)$$

where  $\mathbf{H}_u(k) = [\mathbf{H}_{0,u}(k) \quad \mathbf{H}_{1,u}(k) \quad \cdots \quad \mathbf{H}_{N_c-1,u}(k)]^T$  ;  
 $\mathbf{W}(k) = [\mathbf{W}_0(k) \quad \mathbf{W}_1(k) \quad \cdots \quad \mathbf{W}_{U-1}(k)]$  ;

$$\mathbf{S}(k) = [S_0(k) \ S_1(k) \ \cdots \ S_{U-1}(k)] \quad \text{and}$$

$$\mathbf{N}_u(k) = [N_{0,u}(k) \ \mathbf{N}_{1,u}(k) \ \cdots \ \mathbf{N}_{N_r-1,u}(k)]^T.$$

### III. TRANSMIT BEAMFORMING ALGORITHMS

In this section, multi-user downlink beamforming algorithms are proposed for the SC-DAN described in Section II. Two cases are considered; the first case is when each user has single receive antenna ( $N_r = 1$ ) and the other case is when each user has multiple receive antennas ( $N_r \geq 2$ ). Two design criteria are considered. One is to use the maximum ratio transmit (MRT) beamforming weight to maximize the received desired signal's power at each mobile terminal. The other one is to use the null steering beamforming weight to minimize the received interference power at each mobile terminal.

#### A. Transmit beamforming when $N_r = 1$

When  $N_r = 1$ , the dimension of the received signal vector at the  $u^{\text{th}}$  user,  $\mathbf{R}_u(k)$  in (5), reduces to one. As a result, the received signals of all the users can be written in matrix format as

$$\mathbf{R}(k) = \mathbf{H}(k)\mathbf{W}(k)\mathbf{S}(k) + \mathbf{N}(k) \quad (6)$$

where  $\mathbf{R}(k) = [\mathbf{R}_0(k) \ \mathbf{R}_1(k) \ \cdots \ \mathbf{R}_{U-1}(k)]^T$ ,  
 $\mathbf{H}(k) = [\mathbf{H}_0(k) \ \mathbf{H}_1(k) \ \cdots \ \mathbf{H}_{U-1}(k)]^T$  and  
 $\mathbf{N}(k) = [\mathbf{N}_1(k) \ \mathbf{N}_2(k) \ \cdots \ \mathbf{N}_{U-1}(k)]^T$ .

#### A1. MRT Beamforming

MRT beamforming weight vector is given by [1]

$$\mathbf{W}_u(k) = \mathbf{H}_u^*(k), \quad (7)$$

where symbol  $(\cdot)^*$  denotes transpose conjugate operation. MRT beamforming weight matrix is then equal to

$$\mathbf{W}(k) = \mathbf{H}^*(k). \quad (8)$$

By using MRT beamforming weight, the received signal vector of all the users can be rewritten by substituting (8) into (6) as

$$\mathbf{R}(k) = \mathbf{H}(k)\mathbf{H}^*(k)\mathbf{S}(k) + \mathbf{N}(k). \quad (9)$$

The received signal at the  $u^{\text{th}}$  user is then given by

$$\mathbf{R}_u(k) = \mathbf{H}_u(k)\mathbf{H}_u^*(k)\mathbf{S}_u(k) + \sum_{k=0, k \neq u}^{U-1} \mathbf{H}_k(k)\mathbf{H}_u^*(k)\mathbf{S}_k(k) + \mathbf{N}_u(k). \quad (10)$$

The first term in the right hand side of Eq. (10) is the desired signal component with maximized receive power. The second term and the third term are respectively multi-user interference and noise and the sum of them can be regarded as a complex valued Gaussian variable. If the cross correlation between the channel vectors is small, the interference term will be approximate zero. Otherwise, the received signal will be interfered by multi-user interference. Therefore, the performance of MRT beamforming will be affected by the cross correlation between the channel vectors.

The time domain signal block estimate is obtained by  $N_c$ -point IFFT for data decision, given by

$$\hat{d}_u(t) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} \mathbf{R}_u(k) \exp\left(j2\pi k \frac{t}{N_c}\right). \quad (11)$$

#### A2. Null steering Beamforming

The beamforming weight for null steering beamforming is given by [7]

$$\mathbf{W}(k) = \mathbf{H}^*(k) \left( \mathbf{H}(k)\mathbf{H}^*(k) \right)^{-1}. \quad (12)$$

The received signal vector by using null steering beamforming weight is obtained by substituting (12) into (6), given as

$$\begin{aligned} \mathbf{R}(k) &= \mathbf{H}(k)\mathbf{H}^*(k) \left( \mathbf{H}(k)\mathbf{H}^*(k) \right)^{-1} \mathbf{S}(k) + \mathbf{N}(k) \\ &= \mathbf{S}(k) + \mathbf{N}(k) \end{aligned} \quad (13)$$

Similarly, time domain signal block estimate is obtained by  $N_c$ -point IFFT for data decision. It is obvious that by using null steering beamforming, the received signal for each user is affected only by noise and will not be affected by the cross correlation between the channel vectors.

#### B. Transmit beamforming when $N_r \geq 2$

According to (5), the received signal vector at the  $u^{\text{th}}$  terminal can be rewritten as

$$\begin{aligned} \mathbf{R}_u(k) &= \mathbf{H}_u(k)\mathbf{W}_u(k)\mathbf{S}_u(k) \\ &+ \mathbf{H}_u(k) \sum_{i=0, i \neq u}^{U-1} \mathbf{W}_i(k)\mathbf{S}_i(k) + \mathbf{N}_u(k). \end{aligned} \quad (14)$$

When multiple antennas are available at each terminal, the received signals at multiple receive antennas can be combined by using the combining weight vector  $\boldsymbol{\gamma}_u = [\gamma_{u,0} \ \gamma_{u,1} \ \cdots \ \gamma_{u,N_r-1}]$  and the received signal after combining is given by

$$\begin{aligned} r'_u(k) &= \boldsymbol{\gamma}_u \mathbf{H}_u(k)\mathbf{W}_u(k)\mathbf{S}_u(k) \\ &+ \boldsymbol{\gamma}_u \mathbf{H}_u(k) \sum_{i=0, i \neq u}^{U-1} \mathbf{W}_i(k)\mathbf{S}_i(k) + \boldsymbol{\gamma}_u \mathbf{N}_u(k). \end{aligned} \quad (15)$$

#### B1. MRT Beamforming

In this study, we follow [4] to use the maximum ratio combining (MRC) weight vector at the receiver. The combining weight vector is given by

$$\boldsymbol{\gamma}_u = \left( \mathbf{H}_u(k)\mathbf{W}_u(k) \right)^* = \mathbf{W}_u^*(k)\mathbf{H}_u^*(k). \quad (16)$$

The MRC combined signal at the  $u^{\text{th}}$  terminal is then obtained by substituting (16) into (15) as

$$\begin{aligned} r'_u(k) &= \mathbf{W}_u^*(k)\mathbf{H}_u^*(k)\mathbf{H}_u(k)\mathbf{W}_u(k)\mathbf{S}_u(k) \\ &+ \mathbf{W}_u^*(k)\mathbf{H}_u^*(k) \left[ \mathbf{H}_u(k) \sum_{k=0, k \neq u}^{U-1} \mathbf{W}_k(k)\mathbf{S}_k(k) + \mathbf{N}_u(k) \right], \end{aligned} \quad (17)$$

$= \mathbf{W}_u^*(k)\mathbf{G}_u(k)\mathbf{W}_u(k)\mathbf{S}_u(k) + \mathbf{I}_u$   
where  $\mathbf{G}_u(k) = \mathbf{H}_u^*(k)\mathbf{H}_u(k)$ . Note that the objective of MRT beamforming is to maximize the received power of the desired signal  $S_u(k)$ . According to (17), MRT beamforming

weight should be the eigenvector of  $\mathbf{G}_u(k)$  which corresponds to the maximum eigenvalue as

$$\mathbf{W}_u(k) = \mathbf{e}_{u,\max}(k), \quad (18)$$

where  $\mathbf{e}_{u,\max}(k)$  represents the eigenvector of  $\mathbf{G}_u(k)$  paired with the maximum eigenvalue.

### B2. Null steering beamforming

Recall that the objective of null steering beamforming is to suppress the multi-user interference. And null steering beamforming weight vector should satisfy the following equation

$$\mathbf{W}_u^*(k) \sum_{i=0, i \neq u}^{U-1} \mathbf{G}_u(k) \mathbf{W}_i(k) = \mathbf{0}. \quad (19)$$

When  $N_i > N_r(U-1)$ , there will be  $N_i - N_r(U-1)$  solutions for Eq. (19). To get the maximum receive power of the desired signal, the null steering beamforming weight should be the solution which maximizes the value of  $\mathbf{W}_u^*(k) \mathbf{G}_u(k) \mathbf{W}_u(k)$ . Similarly, when  $N_r \geq 2$ , time domain signal block estimate is obtained by

$$\hat{d}_u(t) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} r'_u(k) \exp\left(j2\pi k \frac{t}{N_c}\right). \quad (20)$$

### C. Transmit Antenna Selection

In the DAN, since there exist a large number of distributed transmit antennas, an efficient transmit antenna selection algorithm is desired. According to the design criterion of the transmit beamforming algorithms described above, the following points should be taken into consideration when transmit antenna selection is addressed.

- C1. According to the design criterion of MRT beamforming, the selected transmit antennas should have small cross correlations between their channel vectors.
- C2. In order to realize the null steering beamforming, the minimum number of selected transmit antennas should be larger than  $N_r(U-1)$ .

Following C1 and C2, Antenna selection procedure is shown in Fig. 3.

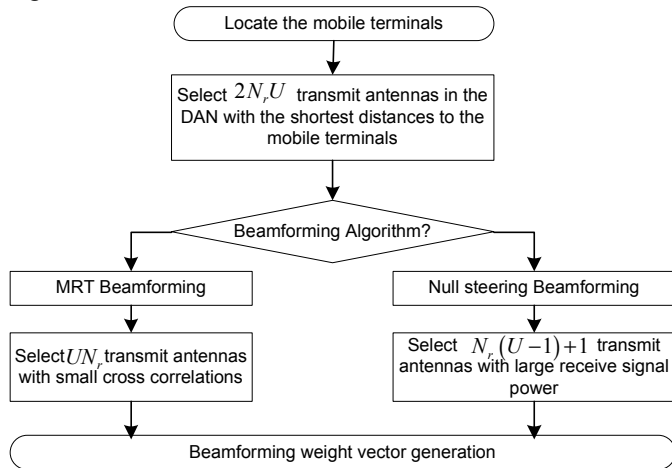


Figure 3 Transmit antenna selection.

## IV. SIMULATION RESULTS

In this section, the performance of transmit beamforming algorithms for the SC downlink transmission will be investigated by simulations. It is assumed that the distributed transmit antennas as well as mobile terminals are distributed randomly within the DAN cell. The simulation parameters are listed in Tab. 1. As stated in Section II, the shadowing loss is not considered for simplicity.

TABLE I. SIMULATION PARAMETERS

Modulation		QPSK
Frequency selective Channel	Fading Model	Block Rayleigh fading
	Number of paths	$L = 16$
	Power delay profile	Uniform
	Path loss	$\alpha = 3.5$
	Channel estimation	Perfect
Number of users $U$		2
Number of antennas at mobile terminal $N_r$		1, 2
Total number of transmit antennas $N$		8, 12, 16, 20
Number of selected antennas	$N_r = 1$ , MRT	2
	$N_r = 1$ , Null steering	2
	$N_r = 2$ , MRT	4
	$N_r = 2$ , Null steering	3
Transmit antenna distribution		Uniform
User distribution		Random
FFT (IFFT) points		$N_c = 256$

At first,  $N_r = 1$  case is considered. The average bit error rate (BER) performance as a function of transmit  $E_0/N_0$  where  $E_0$  is the desired signal power and  $N_0$  is the noise power. The average BER performance of MRT beamforming and null steering beamforming are shown in Fig. 4a and Fig. 4b, respectively. From the results in Fig. 4, the effectiveness of the antenna selection can be observed. When the total number of transmit antennas,  $N$ , increases from 8 to 20, the average BER performance improves significantly. The effect of antenna selection is two folds. On one hand, when more transmit antennas are available, the average distance between the selected transmit antennas and the mobile terminal becomes shorter, and the average receive power increases correspondingly. On the other hand, the cross correlation between the channel vectors reduces due to the larger collection of choices. By comparing the results in Fig. 4a and 4b, it is further observed that the performance of the null steering beamforming is better than that of the MRT beamforming. The reason can be found from (10) and (13). The received signal using MRT beamforming suffers from multi-user interference while the received signal using null steering beamforming does not have this problem.

Next,  $N_r = 2$  case is considered. The average BER performance of MRT beamforming and null steering beamforming are shown in Fig. 5a and Fig. 5b, respectively. It is observed that both the MRT beamforming algorithm and null steering beamforming algorithm work effectively when multiple receive antennas are available. In addition, the antenna

selection is also effective. Similar to  $N_r = 1$  case, null steering beamforming provides a better performance than MRT beamforming. Note that in both  $N_r = 1$  and  $N_r \geq 2$  cases, the calculation of null steering beamforming weight costs much higher computational complexity than the MRT beamforming weight calculation. Therefore, the performance gain is achieved at the cost of higher computational complexity.

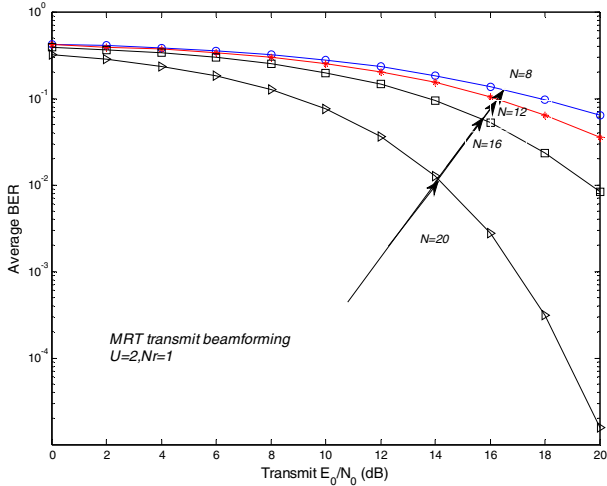


Figure 4a Average BER performance of MRT beamforming,  $N_r = 1$ .

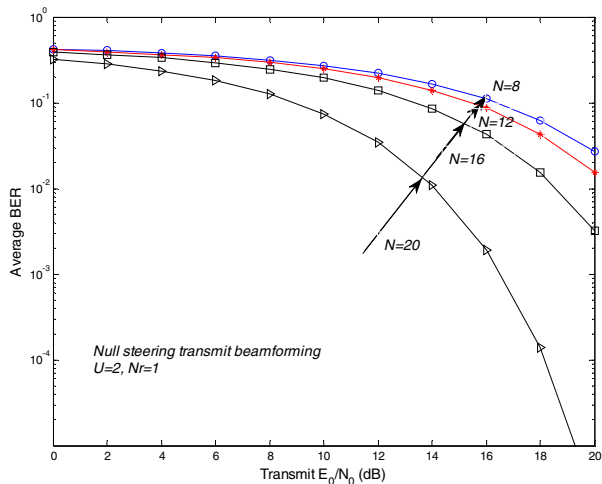


Figure 4b Average BER performance of Null steering beamforming,  $N_r = 1$ .

### V. CONCLUSIONS

In this paper, we have studied the transmit beamforming for downlink transmission in SC-DAN. Two beamforming algorithms, namely, MRT beamforming and null steering beamforming, together with antenna selection algorithm were discussed. The average BER performance of both beamforming algorithms were evaluated and compared by simulations. It has been shown that both MRT beamforming and null steering beamforming are effective when the total number of transmit antennas is large enough. In addition, it was shown that the performance of null steering beamforming is better than that of the MRT beamforming. However, the performance gain is achieved at the cost of higher

computational complexity.

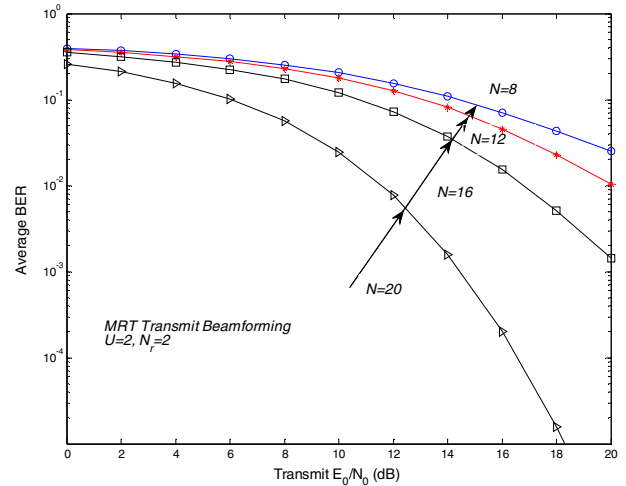


Figure 5a Average BER performance of MRT beamforming,  $N_r = 2$ .

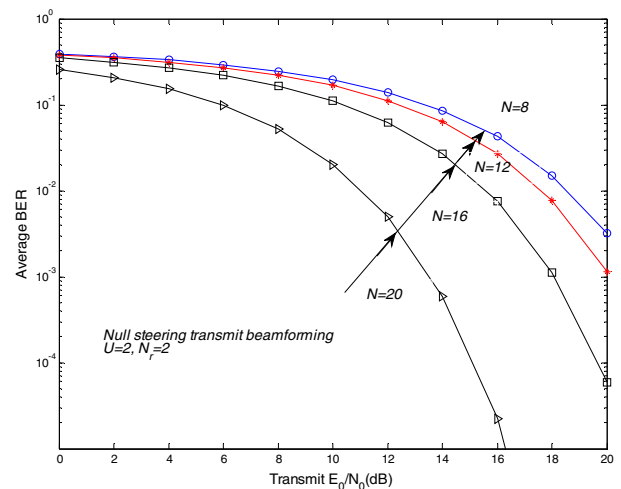


Figure 5b Average BER performance of Null steering beamforming,  $N_r = 2$ .

### References

- [1] J. G. Proakis, Digital Communications, fourth edition, New York: McGraw Hill, 2001.
- [2] R. Price and P. E. Green, "A Communication Technique for Multipath Channels," Proc. IRE, vol. 46, pp. 555-570, March 1958.
- [3] B.K. Ng and D. Falconer, "A Novel Frequency Domain Equalization Method for Single-carrier Wireless Transmissions over Doubly-selective Fading Channels," Proc. IEEE Globecom 2004, pp. 237 -241, Texas, USA, Nov. 2004.
- [4] F. Adachi, K. Takeda, T. Obara, T. Yamamoto and H. Matsuda, "Recent advances in single-carrier frequency-domain equalization and distributed antenna network," Proc. IEEE ICICS 2009, pp.1-5, Macau, China, 7-10 Dec. 2009.
- [5] T. Nishimura, T. Ohgane, Y. Ogawa, Y. Doi, and J. Kitakado, "Downlink Beamforming Performance for an SDMA Terminal with Joint Detection," Proc. IEEE VTC Fall 2001, vol.3, pp. 1538 - 1542, New Jersey, USA, Sept. 2001.
- [6] H. Matsuda, K. Takeda and F. Adachi, "Downlink Transmit Diversity for Broadband Single-Carrier Distributed Antenna Network," Proc. IEEE VTC Spring 2010, pp. 1-5, Taipei, May 2010.
- [7] L.C. Godara, "Application of Antenna Arrays to Mobile Communications. II. Beam-forming and Direction-of-arrival Considerations," Proc. of the IEEE, vol. 85, no. 8, pp. 1195 - 1245, Dec. 1994.