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# Pilot-assisted Channel Estimation Without Feedback for Bi-directional Broadband ANC

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*Abstract*— **Broadband analog network coding (ANC) has been recently introduced to increase the network capacity by exploiting the broadcasting nature of the wireless channel. However, channel state information (CSI) knowledge is required for self-information removal and signal detection. Recently, a pilot-assisted channel estimation (PACE) scheme has been presented for broadband ANC, where feedback of the channel estimates from the relay to the users is required. In this work, we introduce a PACE scheme without feedback from the relay for broadband ANC using orthogonal frequency-division multiplexing (OFDM). In the first time slot the users transmit their respective pilots to the relay and in the second time slot the relay simply amplifies and forwards the received pilot signals to both users. Each user can then estimate all the CSI it needs for self-information removal and coherent signal detection, without requiring any feedback from the relay. The bit error rate (BER) performance of broadband ANC using the proposed PACE is evaluated by computer simulation. It was shown that the proposed PACE scheme causes only a slight BER performance degradation compared to the conventional PACE scheme while eliminating the feedback channel requirement.**

***Keywords- Broadband ANC, OFDM, channel estimation***

## I. INTRODUCTION

Increasing broadband services such as multicasting, video conferences, video on demand, etc. will demand very high capacity from the next generation wireless communication networks. In wired networks, network coding [1] was proposed to increase the network capacity. The same concept can be used in wireless networks to exploit the broadcasting nature of wireless transmission and further increase the network capacity [2]. It has been shown that for bi-directional wireless communication, network coding at the physical layer (PNC) can double the network capacity [3], [4]. Narrowband analog network coding (ANC) was introduced in [5] as a simplification of PNC where the signals from the two users are mixed in the wireless medium.

It was shown in [6] that broadband ANC with perfect knowledge of channel state information (CSI) has the ability to overcome the frequency-selectivity of the wireless channel. Self-information removal and coherent signal detection in the broadband ANC scheme require accurate channel estimation (CE). A straightforward approach would be allocating four time slots for pilot-assisted channel estimation to avoid interference among users' signals in the same time slot. However, this might greatly decrease the capacity benefit of ANC. In [7], a two-phase protocol for CE in two-way relay networks is proposed. The CSIs of the equivalent channels are first estimated through the least-square (LS) algorithm, then in order to overcome the interference of the two pilot signals, considering the channel orders known, the CSIs of the individual channels (between each user and the relay) are identified; finally the gains of the equivalent channels to be used for self-information removal and data detection are recomputed from the individual channels. This algorithm requires the users to have *a priori* knowledge of the channel order. Moreover, it is shown in [7] that small overestimation of the channel order severely degrades the performance of the CE scheme. Recently in [8], a two-phase carrier frequency offset (CFO) and channel estimation scheme that estimates the CSIs of the equivalent channels has been introduced. This scheme has a high computational complexity, as it is utilizing a nulling-based LS estimation and the performance degrades in comparison to the scheme in [7] due to residual interference between the two pilot signals. In [9], the authors propose to superimpose pilots at the relay and then apply a complex algorithm to estimate the CFO and the CSIs of the equivalent channels. However, the scheme requires a large number of pilot tones or a large number of iterations of the estimation algorithm. In [10], a low-complexity two-slot pilot-assisted channel estimation scheme that uses cyclically shifted pilot signals for the two users to avoid the pilot signals interference is introduced. The individual channels from the users to the relay are first estimated by the relay, then sent to the users through feedback. However, the scheme requires CSI feedback from the relay to the users, which is practically difficult to achieve and reduces the bandwidth efficiency.

In this paper we present a pilot-assisted CE scheme for broadband ANC using orthogonal frequency-division multiplexing (OFDM) that estimates the CSIs of the equivalent channels, but unlike the work in [10], it does not require feedback from the relay. In the first stage, both users transmit their pilot signals to the relay. In order to avoid interference in the first time slot [10], the pilot signal of one of the users is cyclically shifted [11] to allow the signals to be separated in delay time domain at the destination for channel estimation. Then, unlike the work in [10], the relay does not estimate the individual CSIs, but rather broadcasts the received pilot signals to both users. Finally, the CSIs of the equivalent channels are estimated by the users. We evaluate the performance of broadband ANC with the proposed CE scheme by computer simulation and compare it to the perfect CSI case and the

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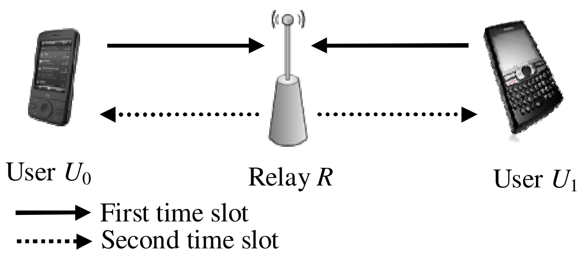


Figure 1 Network model.

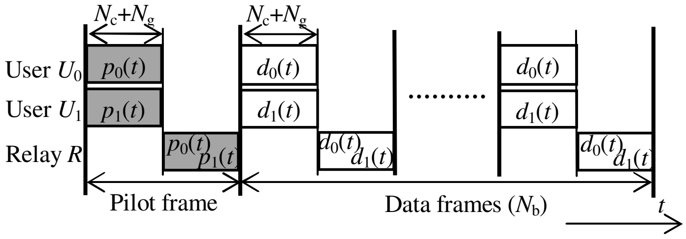


Figure 2 Transmission frame structure

CE scheme introduced in [10] (in this paper referred to as "conventional PACE scheme"). We show that the proposed PACE scheme causes only a slight BER performance degradation compared to that of the conventional PACE scheme, due to noise enhancement, while costly feedback from the relay is not required.

The rest of the paper is organized as follows. In Section II, we present the network model. The proposed CE scheme is presented in Section III. Section IV shows the results of the computer simulation and discussions. We summarize our findings in Section V.

## II. NETWORK MODEL

We consider a two-way relay network, with users $U_0$ and $U_1$ outside each other's coverage area, that communicate through the relay $R$, as shown in Fig.1. The communication between the users and the relay is done using time division duplex (TDD) in two slots: in the first time slot $U_0$ and $U_1$ transmit their signals to the relay and in the second time slot the relay broadcasts the received signal to the users through the amplify-and-forward (AF) protocol. In this paper, we assume that the channel between the users and the relay does not change during the two slots.

### A. First time slot

The information bit sequence is channel coded and the encoded sequence is mapped to a complex-valued finite constellation such as quadrature phase shift keying (QPSK) modulation. The data-modulated symbol sequence of the $j$th user $U_j$ is represented by $\{d_j(n);\ n=0\sim N_c-1\}$ for $j\in\{0,1\}$. An $N_c$-point inverse fast Fourier transform (IFFT) is applied to $\{d_j(n);\ n=0\sim N_c-1\}$ in order to generate the respective OFDM signals. An $N_g$-sample guard interval (GI) is inserted, and the signals from the users are transmitted through the frequency-selective fading channel. The GI length is assumed to be longer than the maximum time delay of the equivalent channels, in order to maintain subcarrier orthogonality.

The frequency-domain received signal $\{R_r(n);\ n=0\sim N_c-1\}$ at the relay can be expressed as

$$R_r(n)=\sum_{j=0}^{1}\sqrt{2P_t}\,d_j(n)H_j(n)+N_r(n), \tag{1}$$

where $P_t(=E_s/T_cN_c)$, $H_j(n)$, and $N_r(n)$ denote the transmit signal power of the users, the channel gain between user $U_j$ and the relay, and the additive white Gaussian noise (AWGN) at the $n$th frequency with single-sided power spectral density $N_0$, respectively. $E_s$ and $T_c$ denote the symbol energy and the sampling period of IFFT, respectively.

### B. Second time slot

The relay terminal normalizes the received signal shown in Eq. (1) by the factor $G=\sqrt{1/E[R_r(n)^2]}$ to make the received average signal power unity and then broadcasts it with transmit power $P_r$. The frequency-domain representation of the received signal $\{R_j(n);\ n=0\sim N_c-1\}$ at user $U_j$'s receiver can be written as

$$R_j(n)=\sqrt{2P_r}\,G\cdot R_r(n)H_j(n)+N_j(n), \tag{2}$$

where $N_j(n)$ denotes the AWGN at the user side with single-sided power spectral density $N_0$.

The $j$th user $U_j$ removes its self-information. The frequency-domain signal $\{\tilde{R}_j(n);\ n=0\sim N_c-1\}$ after self-information removal can be expressed as

$$\tilde{R}_j(n)=R_j(n)-d_j(n)H_j^{(0)}(n), \tag{3}$$

where $H_j^{(0)}(n)$ denotes the channel between user $U_j$ and $U_j$ via the relay, given by

$$H_j^{(0)}(n)=2\sqrt{P_rP_t}\,G\cdot H_j^2(n). \tag{4}$$

One-tap zero forcing frequency domain equalization (ZF-FDE) is then applied as

$$\hat{R}_j(n)=\tilde{R}_j(n)W_j(n), \tag{5}$$

where $W_j(n)$ is the equalization weight for the $n$th subcarrier, given by

$$W_j(n)=\frac{\{H_j^{(1)}(n)\}^*}{\left|H_j^{(1)}(n)\right|^2}, \tag{6}$$

where $H_j^{(1)}(n)=2\sqrt{P_rP_t}\,GH_0(n)H_1(n)$ is the channel gain of the channel between user $U_0$ and $U_1$ and $(\cdot)^*$ denotes the complex conjugate operation. Finally, the equalized signal is demodulated and the decision variables are decoded using Viterbi algorithm.

The self-information removal given by Eq. (3) and the ZF-FDE given by Eq. (6) require knowledge of the equivalent channel gains, $\{H_j^{(0)}(n);n=0=N_c-1\}$ and $\{H_j^{(1)}(n);n=0=N_c-1\}$. In practice, the equivalent channel gains must be estimated. Their estimates are denoted by

$\{\breve{H}_j^{(0)}(n); n=0=N_c-1\}$ and $\{\breve{H}_j^{(1)}(n); n=0=N_c-1\}$, respectively.

## III. CHANNEL ESTIMATION

Accurate channel estimation is crucial to broadband ANC transmission, due to the use of CSI in both self-information removal and FDE. The straightforward approach for CE for a TDD bi-directional relay network would be a four time slot scheme, to separate the pilot signals of different users. However, in a fast fading environment, where we need an increased pilot insertion rate in order to improve the tracking ability, this would significantly decrease the capacity benefit of the ANC scheme. High-complexity two-slot CE schemes [7-9] have been introduced for similar systems. A low-complexity two-slot CE scheme that uses cyclically shifted pilot signals to separate the signals from the users has been introduced for broadband ANC systems [10], but it requires feedback of the channel estimates from the relay to the users. Perfect feedback poses some difficult implementation problems, and accurate feedback can be very costly. We propose a low-complexity channel estimation scheme that addresses the above mentioned problems.

Our proposal is a pilot-assisted channel estimation scheme with time-domain multiplexed (TDM) pilots. The transmission frame structure of the users and the relay is shown in Fig. 2. Both the pilot and data frames are divided into two time slots, each of length $N_c + N_g$ samples. In the first time slot of the pilot stage, both users $U_0$ and $U_1$ transmit their respective pilot signals, $\{p_0(t); t=0\sim N_c-1\}$ and $\{p_1(t); t=0\sim N_c-1\}$, to the relay. Unlike the work in [10], where the relay estimates the channel and transmits it to the users, in our scheme during the second time slot the relay broadcasts the received superimposed pilot signals of the two users through an AF protocol. Consequently, the CE processing is done only at the users' side, and the estimates of the equivalent channels (one from user $U_j$ to user $U_j$ via the relay, and one between the two users $U_0$ and $U_1$ through the relay) are used in the following $N_b$ data frames for self-information removal and equalization. The two pilot stage time slots are described below.

### A. First pilot time slot

As shown in Fig. 2, during the first time slot, users $U_0$ and $U_1$ transmit their pilot signals to the relay through a frequency-selective fading channel. The received pilot signal $\{R_{r,p}(n); n=0\sim N_c-1\}$ at the relay can be expressed in frequency domain as

$$R_{r,p}(n) = \sum_{j=0}^{1} \sqrt{2P_t}\, P_j(n) H_j(n) + N_r(n), \tag{7}$$

where $P_j(n)$ is the frequency domain representation of the pilot signal from user $j$.

In order to avoid the problem of the two channel impulse responses overlapping, we adapt a technique introduced in [11] for OFDM systems with multiple transmit antennas. The pilot signal $\{p_1(t); t=0\sim N_c-1\}$ of user $U_1$ is cyclically shifted by $\theta$ samples relative to user $U_0$'s pilot signal $\{p_0(t); t=0\sim N_c-1\}$, so that $p_1(t)=p_0((t-\theta) \bmod N_c)$. Thus the pilot signal of user $U_1$ can be expressed in frequency domain as

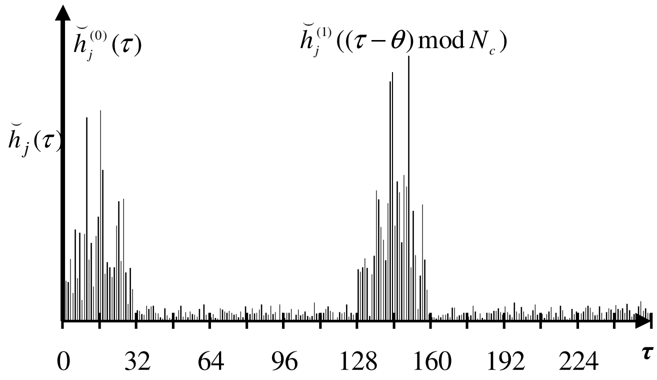


Figure 3 Estimated channel impulse response.

$$P_1(n) = P_0(n) \exp\left(-j2\pi\theta \frac{n}{N_c}\right) \tag{8}$$

for $n=0\sim N_c-1$, and the pilot signal received by the relay can be written as

$$R_{r,p}(n) = \sqrt{2P_t}\, P_0(n) \left\{ H_0(n) + H_1(n) \exp\left(-j2\pi\theta \frac{n}{N_c}\right) \right\} + N_r(n). \tag{9}$$

### B. Second pilot time slot

In the CE scheme we propose the relay handles the pilot frame in the exact same way it handles data frames, without performing any processing on the pilot signal it receives. It simply normalizes the received superimposed pilot signals by a factor of $G$ and broadcasts the mixed signal with power $P_r$.

After GI removal and $N_c$-point FFT, the frequency-domain received signal $\{R_{j,p}(n); n=0\sim N_c-1\}$ at user $U_j$'s receiver can be represented as

$$R_{j,p}(n) = \sqrt{2P_r}\, G \cdot R_{r,p}(n).H_j(n) + N_j(n). \tag{10}$$

This can be rewritten as

$$R_{j,p}(n) = P_0(n) \left\{ \begin{array}{l} H_j^{(0)}(n) \\ + H_{1j}^{(1)}(n) \exp\left(-j2\pi\theta \frac{n}{N_c}\right) \end{array} \right\} + \tilde{N}_j(n), \tag{11}$$

where $\tilde{N}_j(n) = \sqrt{2P_r}\, G N_r(n) H_j(n) + N_j(n)$ denotes the composite noise.

The reverse modulation is applied to $R_{j,p}(n)$ to remove the pilot as

$$\begin{aligned} \breve{H}_j(n) &= \frac{R_{j,p}(n)}{P_0(n)} \\ &= H_j^{(0)}(n) + H_j^{(1)}(n) \exp\left(-j2\pi\theta \frac{n}{N_c}\right) + \breve{N}_j(n), \end{aligned} \tag{12}$$

TABLE I. SIMULATION PARAMETERS

| | | |
|---|---|---|
| Transmitter $U_0$ ,$U_1$ | Data modulation | QPSK |
| | Block size | $N_c = 256$ |
| | GI | $N_g = 32$ |
| Channel | $L$ = 16-path block Rayleigh fading (uniform power delay profile) | |
| Relay $R$ | Protocol | Amplify-and-forward |
| Receiver $U_0$ ,$U_1$ | FDE | ZF |
| | Channel estimation | Pilot-assisted (Chu seq.) |

where $\breve{N}_j(n) = \tilde{N}_j(n) \big/ P_0(n)$. The inverse Fourier transform $\{\, \breve{h}_j(\tau)\,;\, \tau = 0\text{~}N_c\text{–}1\}$ of $\{\, \breve{H}_j(n)\,;\, n = 0\text{~}N_c\text{–}1\}$ is obtained, by taking an $N_c$-point IFFT, as

$$\breve{h}_j(\tau) = h_j^{(0)}(\tau) + h_j^{(1)}((\tau - \theta) \bmod N_c) + \breve{n}_j(\tau), \tag{13}$$

where $h_j^{(0)}(\tau)$ and $h_j^{(1)}(\tau)$ denote the desired impulse responses of the channel between user $U_j$ and $U_j$ via the relay and of the channel between $U_0$ and $U_1$ via the relay, respectively. The third term is the noise. Note that due to the fact that the pilot of user $U_1$ is cyclically shifted by $\theta$ samples, the impulse response of the channel from this user has a delay of $\theta$ samples, which separates it from the impulse response of the channel from user $U_0$ and allows us to estimate both channels at the same time.

An example of an actual estimate of the channel impulse response at the receiver for a multipath channel with the number of sample-spaced paths $L$=16, when $N_c$=256 subcarriers, $N_g$=32 samples and $\theta$=128 samples, is shown in Fig. 3. It can be clearly seen from the figure that the channel impulse responses from the two users are completely separated in the delay time domain. Note that the delay spread of the equivalent channels increases compared to the original channel impulse response, and the GI size $N_g$ and the cyclic shift of the pilot signal $\theta$ must be set accordingly, in order to avoid overlapping of the two channel impulse responses. Since the equivalent channels that are estimated can be expressed as the convolution of two sample-spaced $L$-path channels, the delay spread will become as high as double, and in order to avoid overlapping, we must have $2L \le N_g \le \theta \le N_c/2$ for the case of FFT sample-spaced time delays.

A delay time domain window is used to separate the two channel impulse responses estimates as described in [10], by taking

$$\breve{h}_j^{(0)}(\tau) = \begin{cases} \breve{h}_j(\tau), & \text{for } \tau = 0 \sim N_g - 1 \\ 0, & \text{elsewhere} \end{cases} \tag{14}$$

and

$$\breve{h}_j^{(1)}(\tau) = \begin{cases} \breve{h}_j(\tau + \theta), & \text{for } \tau = 0 \sim N_g - 1 \\ 0, & \text{elsewhere.} \end{cases} \tag{15}$$

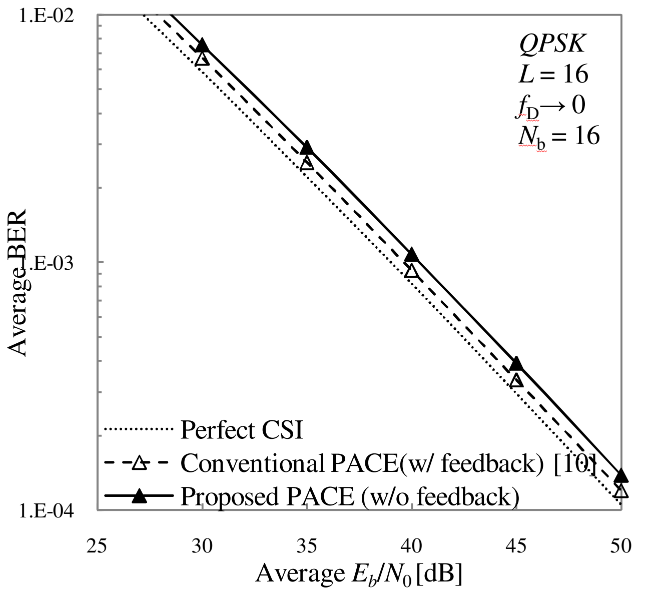


Figure 4 Uncoded BER performance.

Finally, an $N_c$-point FFT is applied to both channel impulse response estimates, $\{\breve{h}_j^{(0)}(\tau); \tau = 0 \sim N_g - 1\}$ and $\{\breve{h}_j^{(1)}(\tau); \tau = 0 \sim N_g - 1\}$, to obtain the estimates of the channel gains $\{\breve{H}_j^{(0)}(n); n = 0 = N_c - 1\}$ and $\{\breve{H}_j^{(1)}(n); n = 0 = N_c - 1\}$, respectively.

## IV. SIMULATION RESULTS

This section shows the computer simulation results for the bit-error rate (BER) performance of an ANC network such as the one described in Sect. II, using the CE scheme introduced in Sect. III. The parameters used in the simulation are summarized in Table I. We assume an OFDM system with ideal coherent quadrature phase-shift keying (QPSK) modulation and demodulation, $N_c$=256 subcarriers, and $N_g$=32 samples. For forward error control we apply rate-1/2 convolutional coding with the generator vectors $g_1$=(111) and $g_2$=(101) with constraint length 3. Hard decision Viterbi decoding is used. The propagation channel is FFT sample-spaced $L$=16-path block Rayleigh fading, having a uniform power delay profile. For the pilot signal we use a Chu-sequence given by $\{p_0(t)=\exp\{j\pi t^2/N_c\};\ t = 0 \sim N_c - 1\}$ [12]. We can see from the network model in Fig. 1 that the tracking ability against fading in the channel estimation is identical for the proposed and conventional PACE schemes. In order to compare the accuracy of the CE of the proposed scheme to that of the conventional scheme, we consider a quasi-static fading channel (i.e., the Doppler frequency $f_D \to 0$) for the computer simulation.

Fig. 4 illustrates the BER performance as a function of the total average signal energy per bit-to-AWGN power spectrum density ratio $E_b/N_0$ (=0.5·($P_t$+$P_r$)$T_c$/$N_0$·(1+$N_g$/$N_c$)·(1+1/$N_b$)) when channel coding is not employed. The power loss due to GI and pilot insertion is taken into consideration. We compare

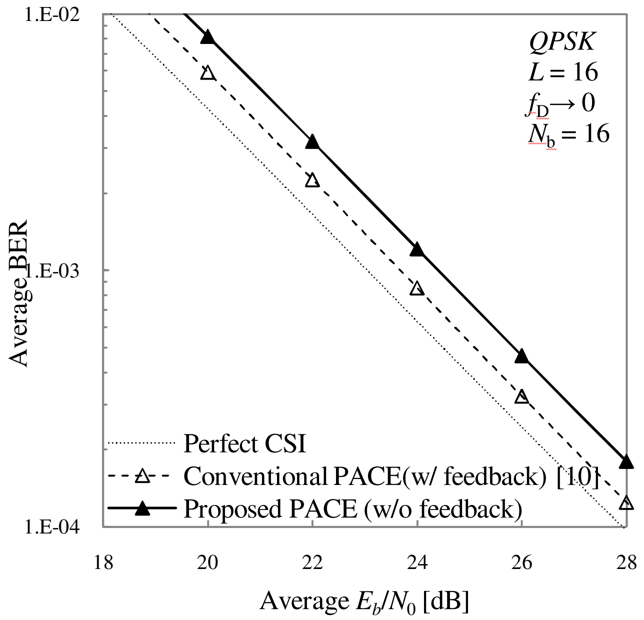


Figure 5 Coded BER performance.

the performance of the broadband ANC system with the proposed CE scheme, denoted by "Proposed PACE (w/o feedback)", to the case of perfect knowledge of CSI, denoted by "Perfect CSI" and the conventional CE scheme, introduced in [10], denoted by "Conventional PACE (w/ feedback)".

As can be seen from Fig. 4, the proposed PACE scheme achieves a satisfactory performance, while eliminating the need of a costly relay feedback. For example, for BER=$10^{-3}$ the $E_b/N_0$ degradation is 1.5dB in comparison to the perfect CSI case, and only 0.75dB in comparison to the CE scheme introduced in [10]. This degradation is due to the noise enhancement that occurs at the relay during the pilot stage. In the conventional scheme the channel is estimated both at the relay and at the users' side, and the pilot signals are not amplified and forwarded by the relay. In the proposed scheme the relay forwards the pilot signals along with the noise added at the relay, thus the noise component during the estimation process is enhanced. However, we can conclude that the noise enhancement does not severely affect the performance.

Fig. 5 shows the BER performance when convolutional channel coding is employed. As we can see from the figure, asides from the coding gain we achieve, the performance trend is unchanged, with the proposed scheme showing 1.5dB degradation in comparison to the perfect CSI knowledge case, and 0.75dB degradation in comparison to the conventional scheme, for BER=$10^{-3}$.

Although the proposed scheme shows a small degradation in the BER performance in comparison to the conventional CE scheme for both the uncoded and coded cases, we emphasize the fact that the BER performance does not reflect the benefit the proposed scheme induces by eliminating feedback. Depending on the feedback method employed with the conventional CE scheme, the throughput would possibly become lower than that of the proposed scheme, due to reduced bandwidth efficiency and the BER performance degradation caused by feedback errors. However, the design of a feedback method is out of the scope of this paper and it is left as an interesting future study.

## V. CONCLUSION

In this paper, we proposed a practical PACE scheme for broadband ANC systems that does not require feedback of CSI from the relay. The CE scheme is divided into two stages: first the users transmit their respective pilot signals to the relay, and then the relay amplifies and forwards the received combined signals back to the users, where the CSI required for self-information removal and coherent detection are estimated. Since each user can estimate all the CSI it needs for processing the data, there is no need for a costly feedback from the relay to the users.

The BER performance of an ANC system using the proposed CE scheme was evaluated by computer simulation, and compared to the case of perfect knowledge of CSI and the conventional CE scheme that requires feedback from the relay. Our results show that the proposed scheme shows similar performance to the conventional CE scheme, with only slight performance degradation due to noise enhancement at the relay during the pilot stage of the transmission, but achieves that without the need of feedback from the relay.

While the tracking ability of the proposed CE scheme is identical to that of the conventional scheme, the tracking problem has not been discussed in this paper. Also, although the BER performance of the proposed scheme shows slight degradation, depending on the feedback method employed by the conventional scheme, the throughput performance characteristics could change. These are left as interesting future work.


## ACKNOWLEDGMENT

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