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MMSE Weight for Single-carrier Overlap FDE

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***Abstract*—Overlap frequency-domain equalization (FDE) requires no cyclic prefix (CP) insertion. In this paper, a new minimum mean square error (MMSE) weight is derived for the overlap FDE. In the previous paper, the MMSE weight for overlap FDE is derived by approximating the IBI as a white Gaussian variable. However, the IBI is an independent colored noise. We derive a new MMSE weight by taking into account this fact. We evaluate the bit error rate (BER) performance by computer simulation, and show that overlap FDE using the newly derived MMSE weight reduces the residual inter-symbol interference (ISI) more and hence, improves the BER performance.**

Keywords; Frequency-selective fading channel, single-carrier transmission, overlap FDE, MMSE weight

I. INTRODUCTION

The broadband wireless channel is composed of many propagation paths with different time delays and hence, is characterized by a severely frequency-selective channel. In the single-carrier transmission, it produces the strong inter-symbol interference (ISI) and degrades the transmission performance [1], [2]. The frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can achieve a good bit error rate (BER) performance by exploiting the channel frequency-selectivity [3]-[5]. In the conventional FDE, the insertion of cyclic prefix (CP) is required to avoid the inter-block interference (IBI). Unfortunately, this results in the reduction of the transmission throughput. Furthermore, when the maximum time delay of the channel exceeds the CP length, the BER performance degrades due to the IBI.

In [6]-[10], overlap FDE which requires no CP insertion was presented and the approximate MMSE weight was derived by approximating the IBI as a white noise. However, the IBI is a colored noise. In this paper, we derive a new MMSE weight by taking into account this fact. The BER performance is evaluated by computer simulation to show that overlap FDE using the derived MMSE weight reduces the residual inter-symbol interference (ISI) more and hence, improves the BER performance.

The remainder of this paper is organized as follows. Section II describes overlap FDE. Section III presents the system model of the SC transmission using overlap FDE. In Sect. IV, we derive a new MMSE-FDE weight. The computer simulation results are shown in Sect V. Finally, Sect. VI offers some conclusions.

II. OVERLAP FDE

The residual IBI after MMSE-FDE is a circular convolution of the MMSE-FDE filter impulse response and the IBI in a received signal. Figure 1 shows the example of the impulse response of the MMSE-FDE filter for a fast Fourier transform (FFT) block size of N_c=256 and an L=16-path Rayleigh fading channel with uniform power delay profile. As seen from Fig. 1, the impulse response of the MMSE-FDE filter concentrates at a vicinity of time t=0. Therefore, the residual IBI is localized only near both edges of the equalized N_c-symbol block. The overlap FDE is based on this observation.

The received symbol sequence is divided into a sequence of M-symbol blocks (M<N_c). Then, N_c-point FFT is applied to an N_c-symbol block centering the M-symbol block of interest. After MMSE-FDE, the central M-symbol block in the equalized N_c-symbol block is picked up to suppress the residual IBI. The FFT intervals for consecutive M-symbol blocks are overlapped as shown in Fig. 2.

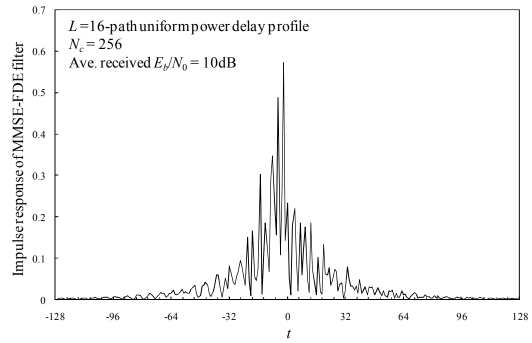


Fig. 1 Impulse response of MMSE-FDE filter.

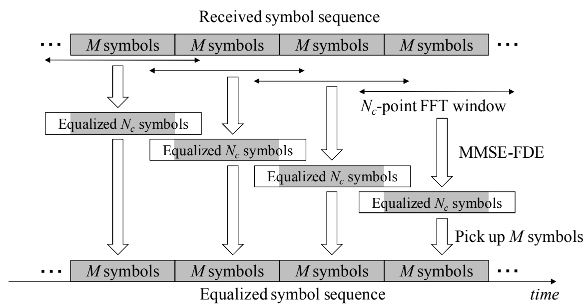


Fig. 2 Block signal processing of overlap FDE.

III. SYSTEM MODEL

A. Signal representation

Figure 3 illustrates the transmitter and receiver structure for the SC transmission using overlap FDE. In this paper, we use a symbol-spaced discrete-time representation. At the transmitter, the information bit sequence is data-modulated. Then the data-modulated symbol sequence $\{s(t)\}$ is transmitted without CP insertion. The transmitted symbol sequence $\{s(t)\}$ is received via a frequency-selective fading channel. In this paper, we assume a sample-spaced L-path frequency-selective block fading channel. The channel impulse response is expressed as

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l)\,, \qquad (1)$$

where h_l is the lth complex-valued channel gain with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$ ($E[.]$ denotes the ensemble average operation), and τ_l is the delay time of the lth path.

The received N_c-symbol block $\{r(t);t=0\sim N_c-1\}$ can be expressed as

$$r(t)=\sqrt{\frac{2E_s}{T_s}}\sum_{l=0}^{L-1}h_l s((t-\tau_l)\bmod N_c)+\nu(t)+\eta(t)\,, \qquad (2)$$

where E_s and T_s are respectively the symbol energy and the symbol duration and $\nu(t)$ and $\eta(t)$ are respectively the IBI component and the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$ with N_0 being the single-sided power spectrum density. $\nu(t)$ is given as

$$\begin{aligned}\nu(t)=\sqrt{\frac{2E_s}{T_s}}\sum_{l=0}^{L-1}&h_l\{s(t-\tau_l)-s((t-\tau_l)\bmod N_c)\}\\&\times\{u(t)-u(t-\tau_l)\}\end{aligned}\,, \qquad (3)$$

where $u(t)$ is the unit step function.

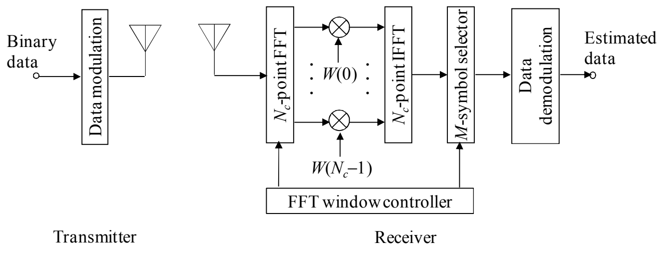


Fig. 3 System model of SC transmission using overlap FDE.

B. Overlap FDE

The received N_c-symbol block $\{r(t);t=0\sim N_c-1\}$ is transformed by an N_c-point FFT into the frequency-domain signal $\{R(k);k=0\sim N_c-1\}$ as

$$\begin{aligned}R(k)&=\sum_{t=0}^{N_c-1}r(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\&=\sqrt{\frac{2E_s}{T_s}}H(k)S(k)+\mathrm{N}(k)+\Pi(k)\end{aligned}\,, \qquad (4)$$

where $H(k)$ is the channel gain at the kth frequency and $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are respectively the signal component, the IBI component, and the noise component, given as

$$\begin{cases}H(k)=\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right)\\S(k)=\sum_{t=0}^{N_c-1}s(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\\mathrm{N}(k)=\sum_{t=0}^{N_c-1}\nu(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\\\Pi(k)=\sum_{t=0}^{N_c-1}\eta(t)\exp\left(-j2\pi k\frac{t}{N_c}\right)\end{cases}. \qquad (5)$$

FDE is done as

$$\begin{aligned}\hat{R}(k)&=R(k)W(k)\\&=\sqrt{\frac{2E_s}{T_s}}\hat{H}(k)S(k)+\hat{\mathrm{N}}(k)+\hat{\Pi}(k)\end{aligned}\,, \qquad (6)$$

where $R(k)$ is the MMSE-FDE weight, and $\hat{H}(k)$, $\hat{\mathrm{N}}(k)$, and $\hat{\Pi}(k)$ are respectively the equivalent channel gain, the IBI component, and the noise component after the MMSE-FDE, given as

$$\begin{cases}\hat{H}(k)=H(k)W(k)\\\hat{\mathrm{N}}(k)=\mathrm{N}(k)W(k)\\\hat{\Pi}(k)=\Pi(k)W(k)\end{cases}. \qquad (7)$$

The frequency-domain signal $\{\hat{R}(k);k=0\sim N_c-1\}$ is transformed by an N_c-point inverse FFT (IFFT) back to the time-domain signal $\{\hat{r}(t);t=0\sim N_c-1\}$ as

$$\begin{aligned}\hat{r}(k)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(j2\pi t\frac{k}{N_c}\right)\\&=\sqrt{\frac{2E_s}{T_s}}\left\{\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right\}s(t)+\mu(t)+\hat{\nu}(t)+\hat{\eta}(t)\end{aligned}\,, \qquad (9)$$

where $\mu(t)$, $\hat{\nu}(t)$, and $\hat{\eta}(t)$ are respectively the residual inter-symbol interference (ISI), the residual IBI, and the noise component, given as

$$\begin{cases} \mu(t) = \dfrac{1}{N_c}\sqrt{\dfrac{2E_s}{T_s}}\displaystyle\sum_{k=0}^{N_c-1}\hat{H}(k)\sum_{\substack{t'=0\\ \neq t}}^{N_c-1}s(t')\exp\left(j2\pi k\frac{t-t'}{N_c}\right) \\ \hat{\nu}(t) = \dfrac{1}{N_c}\displaystyle\sum_{k=0}^{N_c-1}\hat{\mathrm{N}}(k)\exp\left(j2\pi t\frac{k}{N_c}\right) \\ \hat{\eta}(t) = \dfrac{1}{N_c}\displaystyle\sum_{k=0}^{N_c-1}\hat{\Pi}(k)\exp\left(j2\pi t\frac{k}{N_c}\right) \end{cases} . \quad (10)$$

To suppress the residual IBI, only the central M-symbol block is picked up from the equalized N_c-symbol block. In next section, we discuss about the MMSE-FDE weight $W(k)$.

IV. Deriving MMSE-FDE Weight

A. Conventional MMSE weight

We define the equalization error at the kth frequency as

$$e(k) = \hat{R}(k) - \sqrt{\frac{2E_s}{T_s}}S(k) . \quad (11)$$

The MMSE-FDE weight $W(k)$ minimizes the mean square error (MSE) $E[|e(k)|^2]$. By solving $\partial E[|e(k)|^2]/\partial W(k) = 0$, we can obtain the MMSE-FDE weight. In the conventional overlap FDE, the IBI is treated as an independent white noise, and we obtain

$$W(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma_{IBI}^2 + \sigma_n^2} , \quad (12)$$

where σ_{IBI}^2 and σ_n^2 are the normalized IBI power and noise power, respectively. They are given as

$$\begin{cases} \sigma_{IBI}^2 = \dfrac{E[|\mathrm{N}(k)|^2]}{2N_cE_s/T_s} \\ \sigma_n^2 = \dfrac{E[|\Pi(k)|^2]}{2N_cE_s/T_s} = \left(\dfrac{E_s}{N_0}\right)^{-1} \end{cases} . \quad (13)$$

σ_{IBI}^2 of Eq. (13) can be approximately given as [8]

$$\sigma_{IBI}^2 = \frac{2}{N_c}\sum_{l=0}^{L-1}|h_l|^2\tau_l . \quad (14)$$

B. New MMSE weight

The IBI component $\mathrm{N}(k)$ of Eq. (5) can be rewritten as

$$\mathrm{N}(k) = \sqrt{\frac{2E_s}{T_s}}\sum_{k'=0}^{N_c-1}\Psi(k,k')\{S_{-1}(k') - S(k')\} , \quad (15)$$

where $S_{-1}(k)$ is the signal component of the previous block. $\Psi(k,k')$ is given as

$$\Psi(k,k') = \frac{1}{N_c}\sum_{l=0}^{L-1}\sum_{t=0}^{\tau_l-1}h_l\exp\left\{-j2\pi\frac{kt - k'(t-\tau_l)}{N_c}\right\} . \quad (16)$$

Using the above equation, an exact expression for the IBI power $\sigma_{IBI}^2(k)$ at the kth frequency can be given as

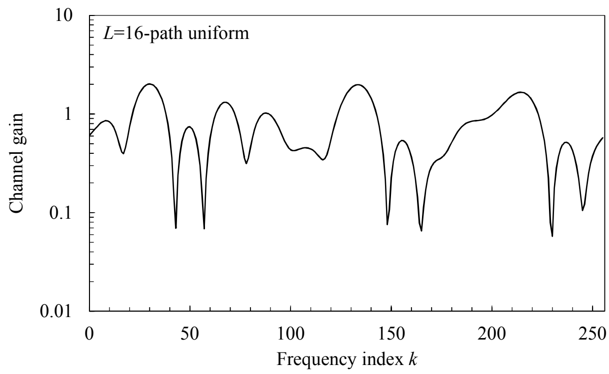
$$\sigma_{IBI}^2(k) = 2\sum_{k'=0}^{N_c-1}|\Psi(k,k')|^2 . \quad (17)$$

It is understood from Eq. (17) that the IBI power is a function of the frequency index k (i.e., the IBI is a colored noise).

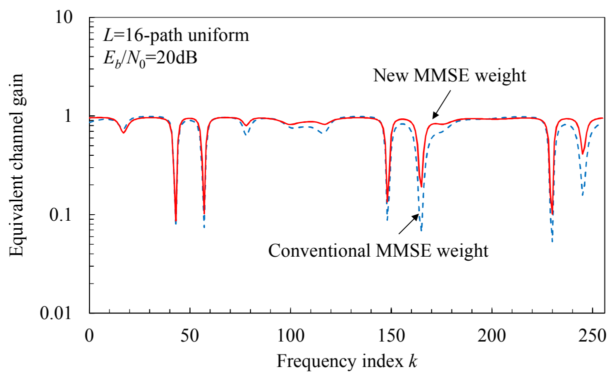
Using Eq. (15) and (16), we obtain a new MMSE-FDE weight as

$$W(k) = \frac{H^*(k) - \Psi^*(k,k)}{|H(k) - \Psi(k,k)|^2 - |\Psi(k,k)|^2 + \sigma_{IBI}^2(k) + \sigma_n^2} . \quad (18)$$

Figure 4 plots one-shot observations of the original channel gain $H(k)$ and the equivalent channel gain $\hat{H}(k) = H(k)W(k)$. It can be seen from Fig. 4(b) that the new MMSE weight provides smaller variations in the equivalent channel gain after FDE than the conventional MMSE weight and therefore, can reduce the residual ISI compared to the conventional MMSE weight.



(a) Original channel gain



(b) Equivalent channel gain

Fig. 4 One-shot observations of original channel gain and equivalent channel gain.

I. Computer Simulation

Table 2 summarizes the simulation condition. We assume QPSK data-modulation. The channel is assumed to be a frequency-selective block Rayleigh fading channel having a symbol-spaced L=16-path uniform power delay profile. The ideal channel estimation is assumed.

Table 2 Simulation condition.

Data modulation	QPSK	
FFT/IFFT block length	N_c=256	
Channel model	Frequency-selective block Rayleigh fading	
	Power delay profile	L=16-path uniform
	Delay time	τ_l=l
Channel estimation	Ideal	

Figure 5 plots the distribution of the average residual interference (ISI+IBI) power normalized by the signal power P=E_s/T_s as a function of symbol index t. As seen from Fig. 5, the residual interference power becomes larger near both ends of N_c-symbol block due to the residual IBI and becomes smaller in the central part of N_c-symbol block, and the new MMSE weight can reduce the interference power compared to the conventional MMSE weight. In the center region in an N_c-symbol block, the residual ISI is much stronger than the residual IBI and therefore, the new MMSE weight can improve the signal-to-interference plus noise power ratio (SINR) compared to the conventional MMSE weight.

Figure 6 compares the BER performances of overlap FDE using the new and conventional MMSE-FDE weights with M as a parameter. The average received bit energy-to-noise power spectrum density ratio is defined as E_b/N_0=0.5E_s/N_0 It can be seen from Fig. 6 that the new MMSE-FDE weight can achieve a better BER performance than the conventional one.

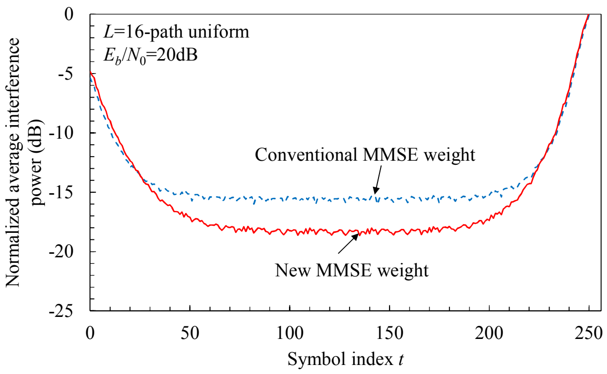


Fig. 5 Residual interference power distribution within the equalized N_c-symbol block.

Figure 7 shows the BER performance comparison among MMSE-FDE with CP and overlap FDE. In MMSE-FDE with CP, the GI length is assumed to be N_g=32 symbols (longer than the channel impulse response length) and the E_b/N_0 loss due to the GI insertion is taken into account. M=128 is used for overlap FDE. It can be understood from Fig. 7 that in a high E_b/N_0 region, the BER performance of overlap FDE using the conventional MMSE weight is worse than that of MMSE-FDE with CP. On the other hand, overlap FDE using the new MMSE weight can achieve almost the same BER performance as MMSE-FDE with CP.

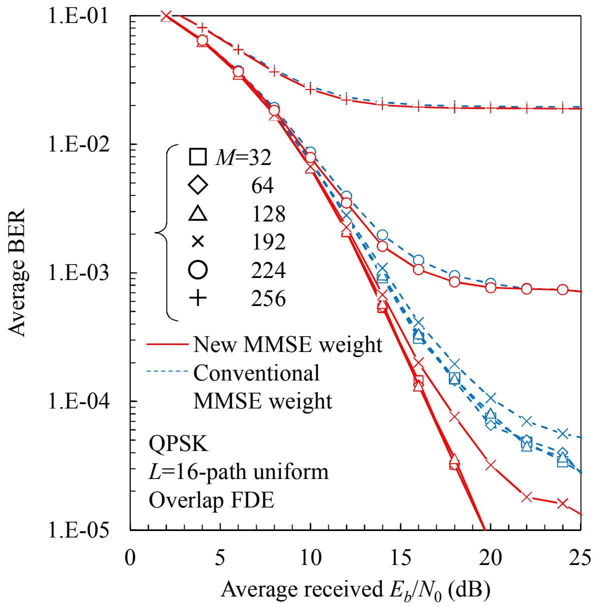


Fig. 6 Average BER performance of overlap FDE.

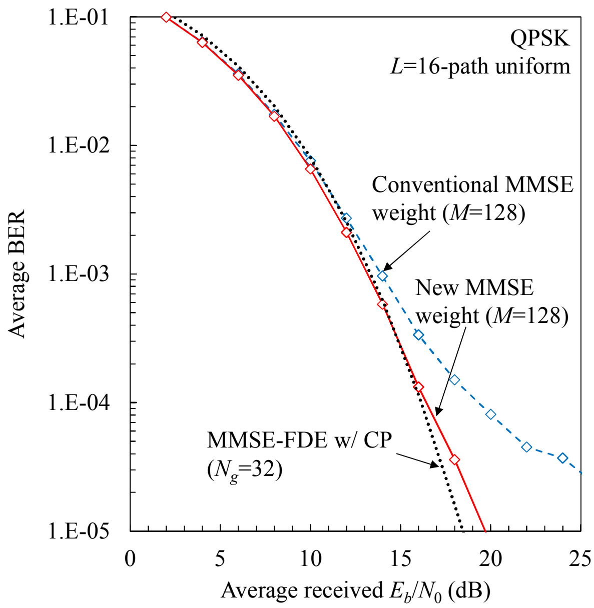


Fig. 7 Performance comparison of overlap FDE and MMSE-FDE with CP.

II. CONCLUSION

In this paper, we discussed about the MMSE-FDE weight for the SC transmission using overlap FDE. The conventional MMSE weight, which is derived based on the assumption that the IBI is a white noise, limits the performance improvement. We showed that the new MMSE weight, which is derived by taking into account the fact that the IBI is a colored noise, reduces the residual ISI more and hence, improves the BER performance.

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