Adaptive Single-Carrier Transmission Using QRM-ML Block Signal Detection

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***Abstract*—In this paper, we propose an adaptive single-carrier (SC) transmission suitable for near maximum likelihood (ML) block signal detection using QR decomposition and M-algorithm (QRM-MLBD). QRM-MLBD can significantly improve the bit error rate (BER) performance of SC block transmission. However, in order to achieve a close-to-ML performance, the use of a fairly large number *M* of surviving paths in the M-algorithm is required and hence its computational complexity is still high. In this paper, to reduce the required number of surviving paths, we introduce an unequal bit loading in an SC transmission block. In QRM-MLBD, the signal-to-noise power ratio (SNR) degrades for symbols to be detected at early stages in the M-algorithm. By noting this fact, we propose to load less number of bits (i.e., lower modulation level) on symbols close to the end of block. We show that the introduction of an unequal bit loading to SC block transmission using QRM-MLBD can achieve almost the same BER performance as the conventional QRM-MLBD while significantly reducing the required number *M* of surviving path in the M-algorithm.**

Keywords-component; Single-carrier, bit loading, MLD, QR decomposition, M-algorithm

I. INTRODUCTION

The broadband wireless channel is characterized by a severe frequency-selective channel and therefore, the severe inter-symbol interference (ISI) limits the transmission performance [1]. Recently, a near maximum likelihood (ML) block signal detection using QR decomposition and M-algorithm (called the QRM-MLBD) was proposed [2, 3] for single-carrier (SC) block transmission. It was shown [3] that QRM-MLBD can significantly improve the bit error rate (BER) performance of SC block transmission, compared to the frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion [4, 5]. However, in order to achieve the sufficiently improved BER performance, the use of a fairly large number M of surviving paths in the M-algorithm is required and hence the computational complexity is still very high.

In QRM-MLBD, the concatenation of the discrete Fourier transform (DFT) and the propagation channel is regarded as an equivalent channel, to which the QR decomposition is applied. The available signal power at each stage in the M-algorithm depends on the elements of an upper triangular matrix **R**, obtained by QR decomposition of the equivalent channel matrix. However, the magnitude of an element in close-to-the-rightmost columns of **R** may drop with higher probability [6]. Therefore, the achievable signal-to-noise power ratio (SNR) gets lower at early stages in the M-algorithm and hence the probability of removing the correct path at early stages may increase when smaller M is used.

In this paper, in order to reduce the required number M of surviving paths for achieving the sufficiently improved BER performance, we propose an adaptive SC block transmission for QRM-MLBD. The probability of removing the correct path depends on the SNR of each stage. In QRM-MLBD, if small M is used, the achievable SNR tends to degrade for symbols to be detected at early stages in the M-algorithm. By noting this fact, we propose an unequal bit loading in which less number of bits are loaded for symbols close to the end of block.

The remainder of this paper is organized as follows. In Sect. II, the system model of SC block transmission using QRM-MLBD is presented. In Sect. III, the proposed adaptive SC transmission scheme is described. In Sect. IV, we will show some simulation results. We will show that the proposed adaptive SC block transmission can reduce the required number M of surviving paths (thereby can reduce the computational complexity) while achieving almost the same performance as the conventional SC transmission. Sect. V offers some concluding remarks.

II. SC BLOCK TRANSMISSION USING QRM-MLBD

A. Signal Transmission Model

The system model of SC block transmission using QRM-MLBD is illustrated in Fig. 1. Throughout the paper, the symbol-spaced discrete time representation is used. We consider block data transmission of N_c symbols. The data symbol block is expressed using the vector form as $\mathbf{d}=[d(0),\ldots d(n),...,d(N_c-1)]^T$. The last N_g symbols of each block are copied as a cyclic prefix (CP) and inserted into the guard interval (GI) placed at the beginning of each block and a CP-inserted data block of N_c+N_g symbols is transmitted.

The signal block is transmitted over a frequency-selective fading channel. We assume a symbol-spaced frequency-selective fading channel composed of L propagation paths with different time delays. The GI-removed received signal block $\mathbf{y}=[y(0),...,y(t),\ldots,y(N_c-1)]^T$ can be expressed using the vector form as

$$\mathbf{y}=\sqrt{2E_s/T_s}\,\mathbf{hd}+\mathbf{n}\,, \tag{1}$$

where E_s and T_s are respectively the symbol energy and the symbol duration. $\mathbf{n}=[n(0),\ldots,n(t),\ldots,n(N_c-1)]^T$ is the noise vector. The tth element $n(t)$ of $\mathbf{n}$ is the zero-mean additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ with N_0 being the one-sided noise power spectrum density. $\mathbf{h}$ is the $N_c\times N_c$ channel impulse response matrix given as

$$\mathbf{h}=\begin{bmatrix} h_0 & & & & h_{L-1} & \cdots & h_1 \\ h_1 & h_0 & & & & \ddots & \vdots \\ \vdots & h_1 & h_0 & \mathbf{0} & & & h_{L-1} \\ h_{L-1} & \vdots & h_1 & \ddots & & & \\ & h_{L-1} & \vdots & & h_0 & & \\ & & h_{L-1} & & h_1 & \ddots & \\ \mathbf{0} & & & \ddots & \vdots & & h_0 \end{bmatrix}, \quad (2)$$

where h_l is the complex-valued path gain with $E[\sum_{l=0}^{L-1}|h_l|^2]=1$. The lth path time delay is assumed to be l symbols. The received signal block after GI removal is transformed by N_c-point DFT into the frequency-domain signal. The frequency-domain received signal vector $\mathbf{Y}=[Y(0),\ldots,Y(k),\ldots,Y(N_c-1)]^T$ is expressed as

$$\mathbf{Y}=\mathbf{F}\mathbf{y}=\sqrt{2E_s/T_s}\,\mathbf{F}\mathbf{h}\mathbf{d}+\mathbf{F}\mathbf{n}, \quad (3)$$

where $\mathbf{F}$ is the DFT matrix of size $N_c \times N_c$ given by

$$\mathbf{F}=\frac{1}{\sqrt{N_c}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{1\times 1}{N_c}} & \cdots & e^{-j2\pi\frac{1\times(N_c-1)}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{(N_c-1)\times 1}{N_c}} & \cdots & e^{-j2\pi\frac{(N_c-1)\times(N_c-1)}{N_c}} \end{bmatrix}. \quad (4)$$

Since the channel impulse response matrix $\mathbf{h}$ is a circulant matrix, the eigenvalue decomposition using $\mathbf{F}$ can be applied [7]. We have

$$\mathbf{F}\mathbf{h}\mathbf{F}^H = diag[H(0),\ldots,H(N_c-1)] \equiv \mathbf{H}, \quad (5)$$

where $H(k)=\sum_{l=0}^{L-1}h_l\exp(-j2\pi k\tau_l/N_c)$, k=0~N_c–1, and $(.)^H$ is the Hermitian transpose operation. Using Eq. (5), Eq. (3) can be rewritten as

$$\mathbf{Y}=\sqrt{2E_s/T_s}\,\mathbf{H}\mathbf{F}\mathbf{d}+\mathbf{N}=\sqrt{2E_s/T_s}\,\overline{\mathbf{H}}\mathbf{d}+\mathbf{N}, \quad (6)$$

where $\overline{\mathbf{H}}=\mathbf{H}\mathbf{F}$ and $\mathbf{N}=[N(0),..,N(k),\ldots,N(N_c-1)]^T$ are respectively the equivalent channel matrix and the frequency-domain noise vector.

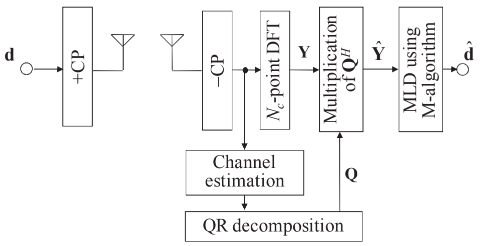


Figure 1. System model of SC block transmission using QRM-MLBD.

B. QRM-MLBD

QRM-MLBD can be applied to the SC block transmission by treating a concatenation of the frequency-domain channel and DFT as the equivalent channel. The QR decomposition is applied to the equivalent channel matrix $\overline{\mathbf{H}}=\mathbf{H}\mathbf{F}$ to obtain $\overline{\mathbf{H}}=\mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is an $N_c \times N_c$ unitary matrix and $\mathbf{R}$ is an $N_c \times N_c$ upper triangular matrix. The transformed frequency-domain received signal $\hat{\mathbf{Y}}=[\hat{Y}(0),\ldots,\hat{Y}(k),\ldots,\hat{Y}(N_c-1)]^T$ is obtained as

$$\hat{\mathbf{Y}}=\mathbf{Q}^H\mathbf{Y}=\sqrt{\frac{2E_s}{T_s}}\mathbf{R}\mathbf{d}+\mathbf{Q}^H\mathbf{N}$$
$$=\sqrt{\frac{2E_s}{T_s}}\begin{bmatrix} R_{0,0} & R_{0,1} & \cdots & R_{0,N_c-1} \\ & R_{1,1} & \cdots & R_{1,N_c-1} \\ & & \ddots & \vdots \\ \mathbf{0} & & & R_{N_c-1,N_c-1} \end{bmatrix}\begin{bmatrix} d(0) \\ d(1) \\ \vdots \\ d(N_c-1) \end{bmatrix}+\mathbf{Q}^H\mathbf{N}. \quad (7)$$

From Eq. (7), the ML solution can be obtained by searching for the best path having the minimum Euclidean distance in the tree diagram composed of N_c stages. In each stage, the best M paths are selected as surviving paths by comparing the path metrics based on the squared Euclidean distance for all surviving paths and are passed to the next stage. The path metric e_i, at the ith stage (i=0,1,...,N_c–1) is calculated as

$$e_i=\sum_{n=0}^{i}\left|\hat{Y}(N_c-1-n)-\sqrt{\frac{2E_s}{T_s}}\sum_{j=0}^{n}R_{N_c-1-n,N_c-1-j}\,\overline{d}(N_c-1-j)\right|^2, \quad (8)$$

where $\overline{d}(n)$ is the symbol-candidate for $d(n)$. The most possible transmitted symbol sequence is determined by tracing back the path with the smallest path metric at the last stage (i=N_c–1).

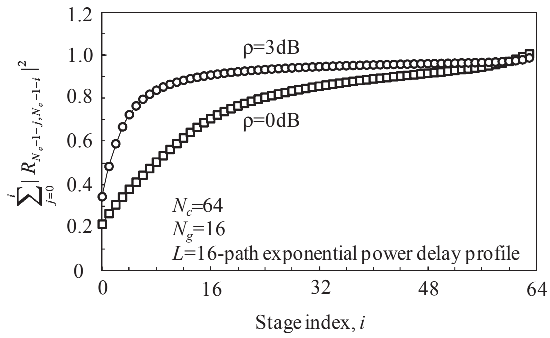


Figure 2. Received signal power available at each stage of the M-algorithm.

In QRM-MLBD, the received signal power associated with the symbol $d(N_c-1-j)$ at the ith stage, i=0,1,…,N_c–1 and j=0,1,…,i, is the sum of the squared magnitude $|R_{N_c-1-i,N_c-1-j}|^2$ of the (N_c–1–i, N_c–1–j)th elements in $\mathbf{R}$ from i=0 to n. Therefore, the received signal power available at early stages likely becomes low since only the elements of matrix $\mathbf{R}$ closer to the lower right positions are available. Furthermore, in the case of SC block transmission, the magnitude of elements of matrix $\mathbf{R}$ closer to the lower right positions may drop with higher probability because the channel impulse response matrix is circulant [6]. As a result, the received signal power available at early stages significantly drops. Figure 2 plots the normalized received signal power available at each stage of the M-algorithm for N_c=64-symbol block transmission in an L=16-path frequency-selective channel with exponential power delay profile having the decay factor ρdB. It can be seen from Fig. 2 that the received signal power available at early stages significantly drops and the number of the stages at which the received signal power significantly drops is larger as the channel frequency-selectivity gets stronger. Since the available signal power is lower at early stages, the probability of removing the correct path at early stages may increase when smaller M is used. Therefore, a fairly large M must be used to achieve the sufficiently improved BER performance. However, the use of larger M increases the computational complexity.

III. Adaptive SC Block Transmission

In this paper, in order to reduce the required number M of surviving paths for achieving the sufficiently improved BER performance, we introduce an unequal bit loading [8, 9] to the SC block transmission. It is desirable that less number of bits are loaded (i.e., lower modulation level is used) on the symbols to be detected at early stages in the M-algorithm. If lower modulation level is used, all possible paths likely can survive at early stages even when small M is used and hence, the probability of removing the correct path at early stages can be reduced. The number of loaded bits is determined block-by-block based on the received signal power at each stage of the M-algorithm. The adaptive bit loading to determine the modulation level is based on the Chernoff upper bound [1].

A. *Adaptive Bit Loading Scheme*

A total number of bits per SC transmission block is denoted by B. The bit loading is represented by $\{c(0),\ldots,c(n),\ldots,c(N_c-1)\}$, where $c(n)$ represents the number of loaded bits for the nth symbol. The adaptive bit loading is done as follows.

Step 1) Apply the QR decomposition to the equivalent channel matrix at the receiver.

Step 2) Select the bit loading $\{c(0),\ldots,c(n),\ldots,c(N_c-1)\}$ that minimizes the BER averaged over block.

Step 3) Feedback the best combination to the transmitter.

Assuming Gray code mapping, the conditional BER $P_{b,n}$ associated with the nth symbol for the given set of modulation level, SNR, and channel is given as [1]

$$P_{b,n}=\alpha(n)\mathrm{erfc}\left(\sqrt{\frac{\gamma(n)}{\beta(n)}}\right), \tag{9}$$

where erfc(.) denotes the complementary error function and $\gamma(n)$ represents the achievable SNR associated with the nth symbol which is introduced in Sect III-B. Table I shows the value of $\alpha(n)$ and $\beta(n)$ for QPSK, 16QAM and 64QAM. In this paper, the Chernoff upper bound of the BER is used. Eq. (9) can be upper-bounded as [1]

$$P_{b,n}\le 2\alpha(n)\exp\left(-\frac{\gamma(n)}{\beta(n)}\right). \tag{10}$$

The upper bounded BER averaged over block is given as

$$\overline{P}_b\le\frac{1}{B}\sum_{n=0}^{N_c-1}2c(n)\alpha(n)\exp\left(-\frac{\gamma(n)}{\beta(n)}\right). \tag{11}$$

The best bit loading is found under the following condition.

$$\begin{cases}\sum_{n=0}^{N_c-1}c(n)=B\\ c(0)\ge\ldots\ge c(n)\ldots\ge c(N_c-1)\end{cases}. \tag{12}$$

First equation means that adaptive bit loading is done under the total number of bits constraint. The second equation means that less number of bits (including no symbol transmission represented by $c(n)=0$) are loaded on symbols to be detected at early stages in the M-algorithm (symbols near the end of block) since the signal power tends to drop at early stages in the M-algorithm as shown in Fig. 2. Note that from the second equation in Eq. (12), what needs to feedback to the transmitter is only the modulation boundary in a block. For example, in the case of $\{c(n)=6;\ n=0\sim4,\ c(n)=4;\ n=5\sim60,\ c(n)=2;\ n=61,\ c(n)=0;\ n=62\sim63\}$, it is sufficient to feedback only n=5, 61, and 62 to the transmitter.

TABLE I. $\alpha(n)$ AND $\beta(n)$

Data modulation	$\alpha(n)$	$\beta(n)$
QPSK	1/2	4
16QAM	3/8	20
64QAM	7/24	84

B. *SNR of Each Symbol After QRM-MLBD*

Since an exact analysis of achievable SNR associated with the nth symbol after QRM-MLBD is quite difficult if not impossible, we use an approximate SNR expression based on the signal power of the M-algorithm given by

$$\gamma(n)=\frac{2E_s}{N_0}\sum_{i=0}^{I_n}|R_{n-i,n}|^2, \tag{13}$$

where the value of I_n is chosen from [0, 1,…,n]. If the received signal power of the nth symbol depends only on the diagonal element $R_{n,n}$, I_n should be 0 (i.e., $\gamma(n)=(2E_s/N_0)|R_{n,n}|^2$). When $M<X$ for X-QAM, the path selection containing the nth symbol greatly depends on the nth diagonal element of $\mathbf{R}$. Therefore, $I_n=0$ may be chosen. On the other hand, when $M\ge X$, the received signal power of the nth symbol depends not only on the diagonal element $R_{n,n}$ but also on other elements, i.e., the (n–1) to 0th elements in the nth column. Therefore, $I_n>0$ is chosen. It is expected that as M is larger, the value of I_n becomes larger and the optimum I_n does exist, but it depends on the channel condition, data modulation, and the number M of surviving paths. In this paper, we found, by preliminary computer simulation, the best I_n for each M that minimizes the BER performance.

IV. Computer Simulation

We evaluate, by computer simulation, the achievable BER performance of adaptive SC block transmission using QRM-MLBD and compare it with the conventional SC block transmission using QRM-MLBD. The simulation condition is summarized in Table II. The block size is N_c=64 and CP length is N_g=16. The channel is assumed to be a frequency-selective block Rayleigh fading channel having symbol-spaced L=16-path exponential power delay profile having the decay factor ρdB. Ideal channel estimation is assumed.

In the case of the conventional SC block transmission, 16QAM is used for all symbols (the total number of transmitted bits per SC block is $4N_c$). In the case of the adaptive SC block transmission, the total number of bits per block is set to $B=4N_c$ and the number of the loaded bits for each symbol is selected from 4 candidates: 0 (no symbol transmission), 2 (QPSK), 4 (16QAM), and 6 (64QAM) bits, according to the channel condition. Perfect feedback of the modulation boundary is assumed.

TABLE II. COMPUTER SIMULATION CONDITION

Transmitter	Data modulation	16QAM for the conventional SC block transmission
	Block size	N_c=64
	CP lengths	N_g=16
Channel	Fading type	Frequency-selective block Rayleigh
	Power delay profile	L=16 path exponential power delay profile
	Decay factor	ρ=0, 3dB
Receiver	Channel estimation	Ideal

A. Optimum I_n

Figure 3 plots the average BER of adaptive SC block transmission using QRM-MLBD as a function of I_n for M=1, 4, 16, and 64. The average received bit energy-to-noise power spectrum density ratio E_b/N_0 is set to 16dB. The decay factor ρ=0 (corresponding to the uniform power delay profile) is assumed. When M=1 and 4, I_n=0 provides the minimum BER. This is because the path selection containing the nth symbol greatly depends on the nth diagonal element of **R**. On the other hand, when M=16 and 64, as I_n increases, the BER decreases. This is because the received signal power of the nth symbol depends not only on the diagonal element but also on other elements. However, I_n is set to too large, the BER increases because the optimum bit loading cannot be performed by overestimating the SNR. When the SNRs of the symbols near the end of block are overestimated, larger number of bits is loaded for these symbols and then, the probability of removing the correct path at early stages increases. We found that I_n=7 and 9 minimize the BER when M=16 and 64, respectively. In the following simulation, we use the best I_n for each M that minimizes the BER.

B. Average BER Performance

The BER performance of adaptive SC block transmission using QRM-MLBD is plotted in Fig. 4 as a function of average received E_b/N_0 for M=1, 4, 16, and 64. For comparison, the BER performance of the conventional SC block transmission is also plotted. It can be seen from Fig. 4 that when small M is used, the achievable BER performance of the conventional block transmission degrades. On the other hand, adaptive SC block transmission can achieve better BER performance even if small M is used. When ρ=0dB, adaptive SC block transmission with M=16(64) achieves almost the same BER performance of the conventional SC block transmission using M=64(256). When ρ=3dB, adaptive SC block transmission with M=16 achieves almost the same BER performance of the conventional SC block transmission using M=64.

Figure 4 shows the probability of the number of loaded bits for the nth symbol when E_b/N_0=16dB. The decay factor ρ=0 is assumed. It can be seen from Fig. 4 that less number of bits is likely to be loaded for symbols to be detected at early stages in the M-algorithm (symbols near the end of block). When less number of bits is loaded for the symbols to be detected at early stages, all paths likely survive at early stages even when small M is used. Therefore, the BER performance does not degrade even if small M is used. The probability of less number of bits is loaded for the symbol to be detected at early stages in the M-algorithm is higher for M=1 and 4 (I_n=0) than that for M=16 (I_n=7). This is because the estimated SNR of those symbols for M=1 and 4 is smaller than that for M=16.

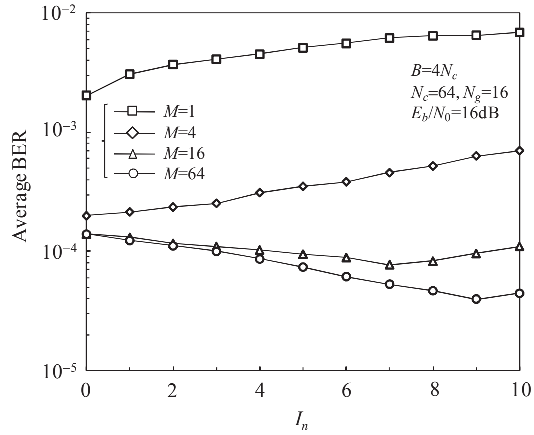
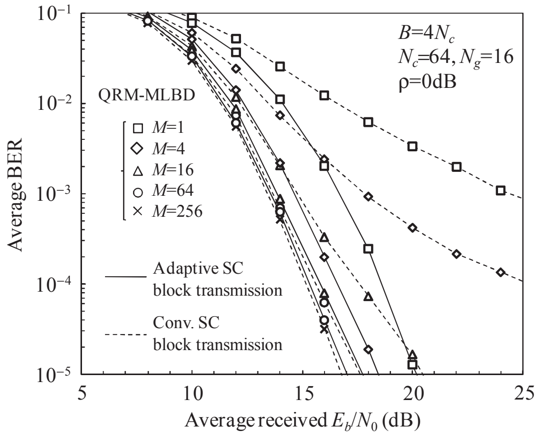
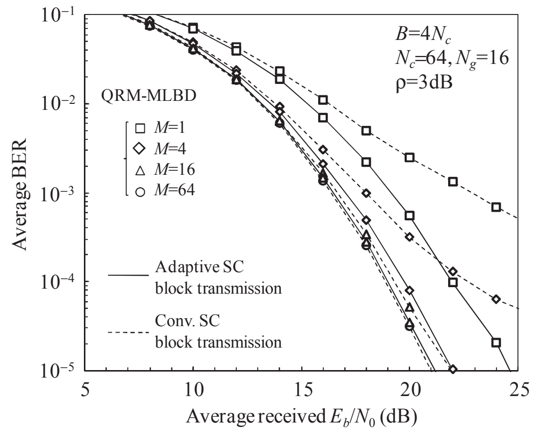


Figure 3. Impact of I_n on average BER.



(a) ρ=0dB



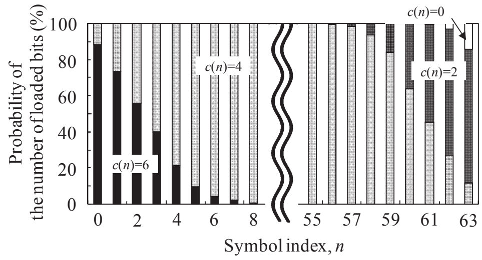
(b) ρ=3dB

Figure 4. Average BER performance.

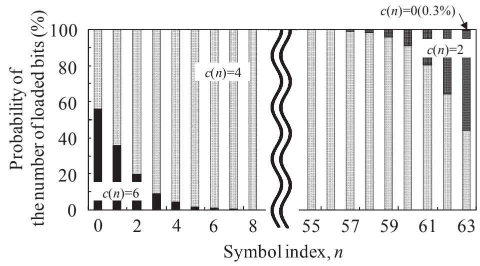
Finally, we discuss the effect the number of symbols for which bits are loaded adaptively. So far, the best combination of the number of loaded bits which minimizes the BER averaged over block is found among the all possible combinations which satisfy the condition expressed by Eq. (12). However, as the block size increases, it requires a prohibitively large number of searching. On the other hand, it can be understood from Fig. 5 that the symbols for which adaptive bit loading is applied can be localized only near the beginning and end of blocks and that 16QAM is always used for the symbols in the middle of the block. Therefore, it is not necessary to apply the adaptive bit loading for all symbols in a block. Figure 6 shows BER dependency on the number N of symbols for which adaptive bit loading is applied. The bits are loaded adaptively for N symbols near the beginning and end of block. When ρ=0dB, the adaptive bit loading can be applied for at most N=16 symbols and this achieves almost the same BER as N=64 case (adaptive bit loading is applied for all symbol). When ρ=3dB, the number of symbols for which adaptive bit loading is applied can be further reduced. This is because the number of the stages at which the received signal power drops becomes less as the channel frequency-selectivity gets weaker as shown in Fig. 2.

V. CONCLUSION

In this paper, in order to reduce the required number M of surviving paths in QRM-MLBD for achieving the sufficiently improved BER performance, we proposed an adaptive SC block transmission using QRM-MLBD. In the proposed scheme, less number of bits is loaded for the symbols to be detected at early stages in the M-algorithm, and therefore, all paths likely survive at early stages even when small M is used. As a result, the probability of removing the correct path at early stages can be reduced and the BER performance does not degrade even if small M is used. We showed that adaptive SC block transmission using QRM-MLBD can achieve almost the same BER performance as the conventional SC block transmission using QRM-MLBD while reducing the number M of surviving path to 1/4.

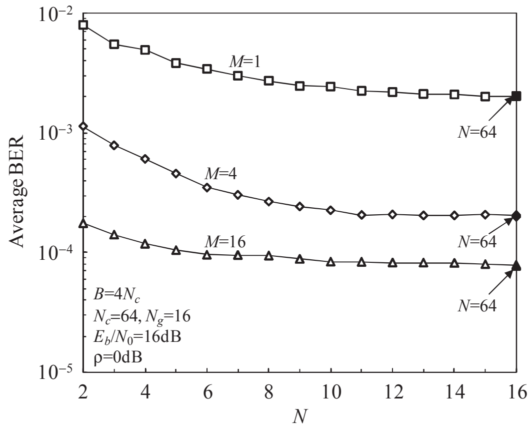


(a) M=1 and 4 (I_n=0)

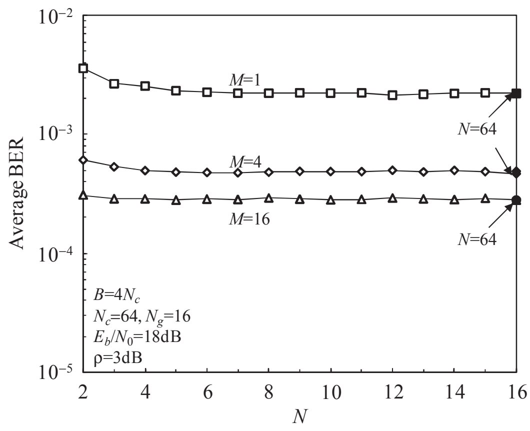


(b) M=16 (I_n=7)

Figure 5. Probability of the number of loaded bits.



(a) ρ=0dB



(b) ρ=3dB

Figure 6. BER dependency on the number of symbols for which adaptive bit loading is applied.

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