Performance Evaluation of Low-PAPR Transmit Filter for Single-Carrier Transmission

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***Abstract*—Single-carrier transmission is a promising transmission technique due to its low peak-to-average power ratio (PAPR) property compared to multicarrier transmission, and using frequency-domain equalization (FDE) at the receiving side also mitigate the adverse effect of channel frequency-selectivity. An introduction of transmit filter can improve the system performance in terms of either PAPR or error probability. In this paper, we focus on the transmit filter aiming to reduce PAPR by employing the minimization of variance of instantaneous transmit power. Filter roll-off factor is also considered in order to provide excess-bandwidth transmission. With a combination of an FDE and spectrum combining at the receiver, excess-bandwidth transmission inherits additional frequency diversity gain, and therefore improves error probability. Performance evaluation of the proposed filtering algorithm is done by computer simulation, while the PAPR and bit-error rate (BER) performance of proposed algorithm are compared with conventional square-root Nyquist filter.**

***Index Terms*—Single-carrier transmission, transmit filter, peak-to-average power ratio (PAPR), frequency-domain equalization (FDE).**

I. Introduction

High-speed and high-quality are the main requirements for the next generation mobile network [1]. However, the existence of multipath propagation and time delay leads to frequency-selective fading channel, consequently, decreases system performance in terms of error probability [2]. Orthogonal frequency division multiplexing (OFDM) is a promising technique that is robust to fading, but its high peak-to-average power ratio (PAPR) property is the main drawback [3]. On the other hand, single-carrier (SC) transmission [4] provides lower PAPR. Even though SC transmission itself suffers from inter-symbol interference (ISI), frequency-domain equalization (FDE) is serviceable to improve error probability [5].

SC transmission is typically equipped with transmit filter for limiting the bandwidth. One of generally-used transmit filters is square-root raised cosine filter [6]. However, system performance also changes when roll-off factor changes since the roll-off factor controls the excess bandwidth. In aspect of PAPR, a certain value of roll-off factor gives very low PAPR [7]. In aspect of bit-error rate (BER), the excess bandwidth can inherit additional frequency diversity gain as long as the original spectrum can be recovered, hence BER performance improves [8]. One thing that should be mentioned is that spectrum efficiency decreases when the bandwidth increases. Besides roll-off factor, any modification on transmit filter such as filter shape also alters system performance. In this paper, we aim to improve PAPR performance rather than BER performance.

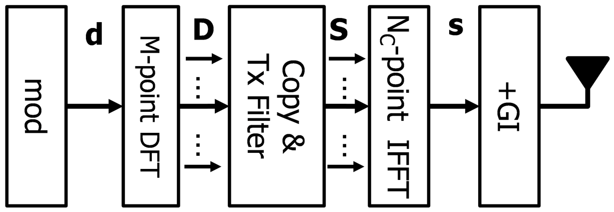
There exist literatures which proposed PAPR reduction algorithms for both OFDM and SC schemes. Such algorithms based on transmit filter and precoder are preferable because of low complexity and no changes on transceiver. Some literatures propose filter shapes by either improving over the conventional filters or deriving from Nyquist prototype such as [9], [10], [11], and [12]. Slimane [13] has shown PAPR performance of various conventional filters for OFDM. However, most of proposed filters cannot guarantee the lowest PAPR since they are not optimum, and this also indicates possibility to determine a filter which provides lower PAPR than existing filters. On the other hand, Falconer [14] suggests that the variance of instantaneous transmit power corresponds to PAPR, and also proposes a low-PAPR precoder based on minimization of variance of instantaneous transmit power for OFDM scheme by using gradient search. Even though our objective is to determine a filter instead of precoder, proposed algorithm in [14] is still usable with some modification. In addition, the roll-off factor is not applied in [14].

In this paper, we determine a new transmit filter for SC transmission by minimizing variance of instantaneous transmit power. PAPR performance of proposed filter better compares to square-root raised cosine filter, which can be claimed as conventional filter. Roll-off factor is also applied to achieve excess bandwidth. On the receiving side, an FDE based on the minimum mean-square error criterion (MMSE-FDE) is applied to combat ISI. Spectrum combining [15] is also introduced to recover the original spectrum and obtain additional frequency diversity gain from excess bandwidth.

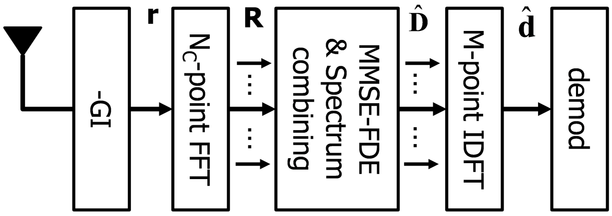
This paper is organized as follows. SC transmission system model is introduced in Section II. Low-PAPR filtering algorithm based on minimization of variance of instantaneous power is presented in section III. Simulation results in aspects of PAPR and BER are shown in Section IV. Finally, Section V concludes the paper.

II. Transmission System Model

Fig.1 illustrate SC system model considered in this paper, while transmission is indicated as block transmission of M symbols over available N_c subcarriers. On the other hand,



(a) Transmitter



(b) Receiver

Fig. 1. Nyquisted-Filtered SC system model

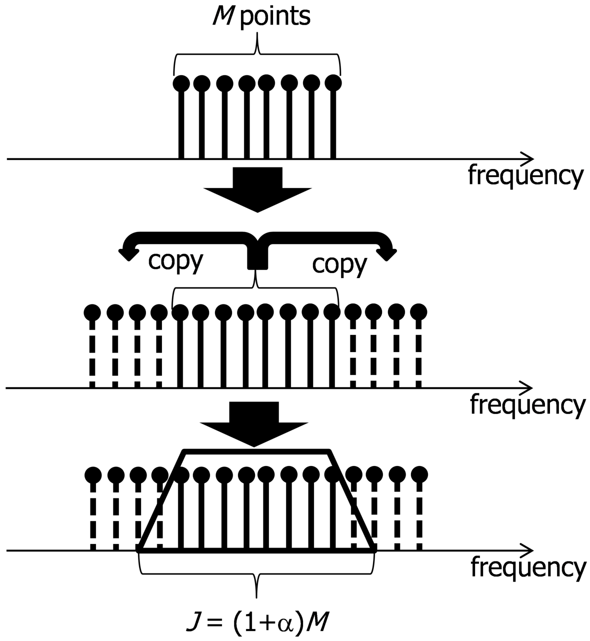


Fig. 2. Transmission algorithm

MMSE-FDE and spectrum combining are equipped at the receiver. MMSE-FDE reduces the effect from frequency-selective channel while spectrum combining inherits additional frequency diversity gain. In addition, transmission is conducted over frequency-selective fading channel, and hence, guard interval (GI) is required.

A. Transmitter

First of all, we have a block of M QPSK modulated symbols $\mathbf{d}$, where $\mathbf{d} = [d(0), d(1), \ldots, d(M-1)]^T$. The block $\mathbf{d}$ is transformed to frequency domain and then copied to entire N_c-point ($N_c = 2M$). Prior to this, a matrix $\mathbf{E}_M$ is introduced for the operation, which is a repetition of M-point discrete Fourier transform (DFT) matrix. That is

$$\mathbf{E}_M \equiv \begin{bmatrix} \mathbf{F}_M \\ \mathbf{F}_M \end{bmatrix}, \tag{1}$$

where $\mathbf{F}_M$ is M-point DFT matrix, which is

$$\mathbf{F}_M = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi(1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(1)(M-1)}{M}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{\frac{-j2\pi(M-1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(M-1)(M-1)}{M}} \end{bmatrix}. \tag{2}$$

In addition, we also determine frequency-domain signal vector $\mathbf{D} = \mathbf{E}_M \mathbf{d}$.

Transmit filter is generally used for limiting the signal bandwidth. Square-root raised cosine filter is an example of Nyquist filter, which can be referred as conventional filter. In this paper, transmit filter is introduced by a matrix $\mathbf{H}_T$. $\mathbf{H}_T$ is $N_c \times N_c$ diagonal matrix which the first J elements of diagonal contains filter coefficients $\left\{H_T\left(\frac{-J}{2}\right), \cdots, H_T\left(\frac{J}{2}-1\right)\right\}$. In addition, $J = (1+\alpha)M$ while α represents filter roll-off factor. Then, $\mathbf{H}_T$ can be illustrated as

$$\mathbf{H}_T = \begin{bmatrix} H_T\left(\frac{-J}{2}\right) & & & \mathbf{0} \\ & \ddots & & \\ & & H_T\left(\frac{J}{2}-1\right) & \\ \mathbf{0} & & & \mathbf{0} \end{bmatrix}. \tag{3}$$

In case of square-root raised cosine filter [6], each filter coefficients can be calculated by using (4).

Instead of conventional filter, we determine a new set of filter coefficients $\left\{H_T\left(\frac{-J}{2}\right), \cdots, H_T\left(\frac{J}{2}-1\right)\right\}$ for (3) which gives lower PAPR, and the method of determination will be discussed later.

After that, N_c-point inverse DFT (IDFT) matrix $\mathbf{F}_{N_c}^H$ is applied for transforming the filtered signal back to time domain. Before adding GI, transmit time-domain signal $\mathbf{s} = [s(0), \cdots, s(N_c-1)]^T$ after passing through all processes in (1) and (3) can be described as in (5), and transmission algorithm is illustrated as shown in Fig.2.

$$\mathbf{s} = \mathbf{F}_{N_c}^H \mathbf{H}_T \mathbf{D} = \mathbf{F}_{N_c}^H \mathbf{H}_T \mathbf{E}_M \mathbf{d}. \tag{5}$$

B. Receiver

The transmission is conducted under independent L-path block fading channel [2]. Regarding to this, the channel impulse response can be expressed as

$$h(\tau) = \Sigma_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \tag{6}$$

where h_l and τ_l are complex-valued path gain and time delay of lth-path, respectively. The received signal vector after removing guard interval $\mathbf{r} = [r(0), \cdots, r(N_c-1)]^T$ can also be expressed as

$$H_T(k) = \begin{cases} 1, & 0 \le |k| < \frac{1-\alpha}{2}M \\ \cos\left[\frac{\pi}{2\alpha M}\left(|k| - \frac{1-\alpha}{2}M\right)\right], & \frac{1-\alpha}{2}M \le |k| < \frac{1+\alpha}{2}M \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$\mathbf{r} = \sqrt{\frac{2E_s}{T_s}}\mathbf{hs} + \mathbf{n}, \tag{7}$$

where E_s and T_s represent symbol energy and symbol period, respectively. Transmit signal vector $\mathbf{s}$ is represented by (5). A vector $\mathbf{n}$ represents zero-mean Gaussian noise. In addition, $\mathbf{h}$ represents time-domain channel response matrix, which is circular matrix and can be expressed as follow.

$$\mathbf{h} = \begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ \mathbf{0} & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}. \tag{8}$$

The received signal vector $\mathbf{r}$ is transformed into frequency domain by N_c-point DFT matrix $\mathbf{F}_{N_c}$, obtaining frequency-domain received signal $\mathbf{R}$ which is

$$\begin{aligned} \mathbf{R} &= \sqrt{\tfrac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{hs} + \mathbf{F}_{N_c}\mathbf{n} \\ &= \sqrt{\tfrac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{h}\mathbf{F}_{N_c}^H\mathbf{H}_T\mathbf{D} + \mathbf{F}_{N_c}\mathbf{n} \\ &= \sqrt{\tfrac{2E_s}{T_s}}\mathbf{H}_C\mathbf{H}_T\mathbf{D} + \mathbf{N} \end{aligned} \tag{9}$$

Here, we define $\mathbf{H}_C = \mathbf{F}_{N_c}\mathbf{h}\mathbf{F}_{N_c}^H$, which is diagonal matrix determining frequency-domain channel gain with respect to each subcarrier. In this paper, we also employ MMSE-FDE, as a receive filter, with spectrum combining as same as in [8] and [15]. Hence, the frequency-domain desired signal at the receiver can be expressed as $\hat{\mathbf{D}} = \mathbf{WR}$. Time-domain desired signal vector $\hat{\mathbf{d}}$ is obtained consequently after transforming $\hat{\mathbf{D}}$ into time-domain by M-point IDFT matrix $\mathbf{F}_M^H$. Here, the operation matrix $\mathbf{W}$ for MMSE-FDE and spectrum combining is determined as in (10) and (11). We also define $\hat{\mathbf{H}} = diag\left\{\hat{H}(0), \ldots, \hat{H}(N_c-1)\right\} = \mathbf{H}_C\mathbf{H}_T$. In addition, spectrum combining is also illustrated as in Fig.3.

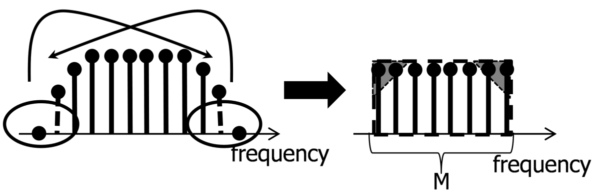


Fig. 3. Spectrum combining

$$\mathbf{W} = \begin{bmatrix} W(0) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & W\left(\frac{N_C}{2}-1\right) \\ W\left(\frac{N_C}{2}\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & W(N_C-1) \end{bmatrix}, \tag{10}$$

$$W(k) = \frac{\hat{H}^*(k)}{\Sigma_{g=0}^1\left|\hat{H}(k \bmod M + gM)\right|^2 + (E_S/N_0)^{-1}}. \tag{11}$$

III. Low-PAPR Filtering Algorithm

As previously mentioned, [14] suggested that variance of instantaneous transmit power relates to PAPR characteristic and also proposed an algorithm to find a suitable precoding matrix which minimizes the variance of instantaneous power for OFDM transmission. In this paper, [14] is used as a guideline to find an appropriate filter for SC transmission which can reduce PAPR. Filter roll-off factor is also approachable. The method of filter determination can be done by firstly determining the variance and precoding matrix, then optimizing the precoding matrix, and finally obtaining the filter coefficients from the optimum precoder.

A. Variance and Precoding Matrix Determination

We consider an SC transmission (same as indicated in Section II) consisting of M-length block of QPSK modulated symbols. For simplicity, we also define a matrix $\mathbf{X}$ as an overall operation matrix, which means $\mathbf{X} = \mathbf{F}_{N_c}^H\mathbf{H}_T\mathbf{E}_M$. Then the transmit signal vector $\mathbf{s}$ can be expressed as $\mathbf{s} = \mathbf{Xd}$, and each element in transmit block $\mathbf{s} = [s(0), \cdots, s(N_c-1)]^T$ is described as

$$s(n) = \sum_{m=0}^{M-1} x_{nm} d(m), \tag{12}$$

where x_{nm} represents an element in $\mathbf{X}$ at the nth-row and mth-column. Therefore, the instantaneous transmit power is $|s(n)|^2$, and the variance of instantaneous power σ^2 can be expressed by averaging over a block of N_c samples as

$$\begin{aligned} \sigma^2 &= \tfrac{1}{N_c}\textstyle\sum_{n=0}^{N_c-1} E\left[\left||s(n)|^2 - E\left[|s(n)|^2\right]\right|^2\right] \\ &= \tfrac{1}{N_c}\textstyle\sum_{n=0}^{N_c-1} E\left[|s(n)|^4\right] - P_{avg}^2 \end{aligned} \tag{13}$$

Here, P_{avg} represents average transmit power of $\mathbf{s}$. Substitute (12) into (13) yields the definition of variance of instantaneous transmit power in (14). Note that (14) is for M-PSK modulation, which gives $E\left[|d(m)|^4\right] = 1$. Variance of instantaneous power in (14) is similar to one of OFDM transmission since they both are derived from overall operation matrix $\mathbf{X}$. However, overall operation matrix of this paper and [14] are difference due to different transmission scheme. This implies filtering algorithm needs to be modified in order to be compatible with SC transmission.

On the other hand, a precoding matrix is also defined. Even though the main objective of this paper is to obtain filter coefficients, we determine a precoder in order to make the optimization simpler. A precoding matrix $\mathbf{P}$, whose dimension is $N_c \times M$, is defined as $\mathbf{P} = \mathbf{H}_T\mathbf{E}_M$. After the optimization, optimum filter coefficients matrix $\mathbf{H}_{T,opt}$ is obtained from $\mathbf{H}_{T,opt} = \mathbf{P}_{opt}\mathbf{E}_M^{+}$, where $\mathbf{A}^{+}$ represents Moore-Penrose pseudoinverse operation of $\mathbf{A}$. Note that the magnitude of each column vector of precoding matrix, $|\mathbf{p}_m|^2$, is one due to power constraint.

B. Minimization of Variance of Instantaneous Power

Regarding to the precoding matrix $\mathbf{P}$ and variance of instantaneous transmit power, as indicated in (14), the objective function of constrained minimization is expressed as

$$\arg\min_{\{\mathbf{p}_m\}} \ \sigma^2 \quad s.t.\ |\mathbf{p}_m|^2 = 1. \tag{15}$$

The objective functions in (14) and (15) are non-convex. Alternatively, we minimize the objective function numerically by employing gradient search with respect to the real and imaginary part of $\mathbf{p}_m$. The gradient of (14) with respect to $\mathbf{p}_m$ is

$$\nabla_{\mathbf{p}_m}\sigma^2 = \frac{4}{N_c}\sum_{n=0}^{N_c-1}\left[2\sum_{q=0}^{M-1}|x_{nq}|^2 - |x_{nm}|^2\right]x_{nm}\mathbf{e}_n, \tag{16}$$

where $\mathbf{e}_n = \frac{1}{\sqrt{N_c}}\left[1, e^{\frac{-j2\pi n(1)}{N_c}}, \cdots, e^{\frac{-j2\pi n(J-1)}{N_c}}\right]^T$. Gradient search is done iteratively for each column vector $\mathbf{p}_m, m = 0, \ldots, M-1$. At the t^{th} step of minimization, gradient search algorithm is done by

$$\begin{array}{c}\tilde{\mathbf{p}}_m[t+1] = \mathbf{p}_m[t] - \gamma\nabla_{\mathbf{p}_m[t]}\sigma^2 \\ \mathbf{p}_m[t+1] = \tilde{\mathbf{p}}_m[t+1] / |\tilde{\mathbf{p}}_m[t+1]|\end{array}, \tag{17}$$

where γ means step-size parameter.

One important thing for iterative gradient search algorithm is the starting point. DFT matrix is typically selected as a starting point, which refers to unmodified SC transmission. In this paper, a starting precoder is constructed by $\mathbf{P} = \mathbf{H}_T\mathbf{E}_M$, where $\mathbf{H}_T$ is constructed from square-root raised cosine filter. Iterative optimization beginning from square-root raised cosine filter provides less number of iterations compared to other starting points.

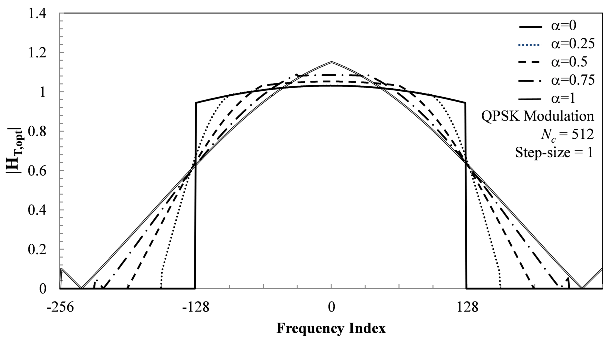


Fig. 4. Proposed filter shapes

IV. Simulation Results

Simulation parameters are summarized as follow. We assume block transmission consisting of $M = 256$ QPSK-modulated symbols. The number of available DFT points is $N_c = 2M = 512$. The transmission is conducted under 16-path block Rayleigh fading with uniform power-delay profile. Perfect channel estimation and zero timing offset are also assumed in this simulation model. Cyclic prefix length is assumed to be $N_g = 32$. Such particular values of roll-off factor ,i.e., α = 0, 0.25, 0.5, 0.75, and 1 are evaluated for both square-root raised cosine filter and proposed filter. As mentioned in the previous parts, we expect lower PAPR compared to conventional filtering algorithm, while the increasing of filter roll-off factor gives better error probability.

A. Filter Coefficients

Gradient search algorithm, as mentioned in (16) and (17), is done iteratively at the transmitting side, where its iteration starts from a precoding matrix determined by square-root raised cosine filter. In this paper, sets of filter coefficients $\left\{H_T(k); k = \frac{-J}{2} \sim \frac{J}{2} - 1\right\}$ are discovered after 100 iterations of gradient search, while step-size parameter γ is assumed to be 1. Filter coefficients are shown in Fig.4 with various roll-off factors. Note that the number of iterations and step-size parameter are chosen by trial and error, and different modulation schemes contribute different filter shapes, as referred in [14] and (14). We also would like to mention that MMSE-FDE is always required at the receiving side as a receive filter in order to meet Nyquist criterion.

B. PAPR

PAPR over a block of transmission is defined as

$$PAPR = \frac{\max\left\{|s(n)|^2\right\}_{n=0\sim N_c-1}}{E\left[|s(n)|^2\right]}. \tag{18}$$

We examine the complementary cumulative distribution function (CCDF) at oversampling factor of 2. Fig.5 shows the CCDF of PAPR of both conventional filter (i.e., square-root raised cosine filter) and proposed filter with particular values of rool-off factor α. It is obviously seen that PAPR

$$\begin{aligned}\sigma^2 &= \frac{1}{N_c}\left\{\sum_{n=0}^{N_c-1}\left(2\left[\sum_{m=0}^{M-1}|x_{nm}|^2\right]^2-\left(2-E\left[|d(m)|^4\right]\right)\left[\sum_{m=0}^{M-1}|x_{nm}|^4\right]\right)-P_{avg}^2\right\}\\ &= \frac{1}{N_c}\left\{\sum_{n=0}^{N_c-1}\left(2\left[\sum_{m=0}^{M-1}|x_{nm}|^2\right]^2-\left[\sum_{m=0}^{M-1}|x_{nm}|^4\right]\right)-P_{avg}^2\right\}\end{aligned} \tag{14}$$

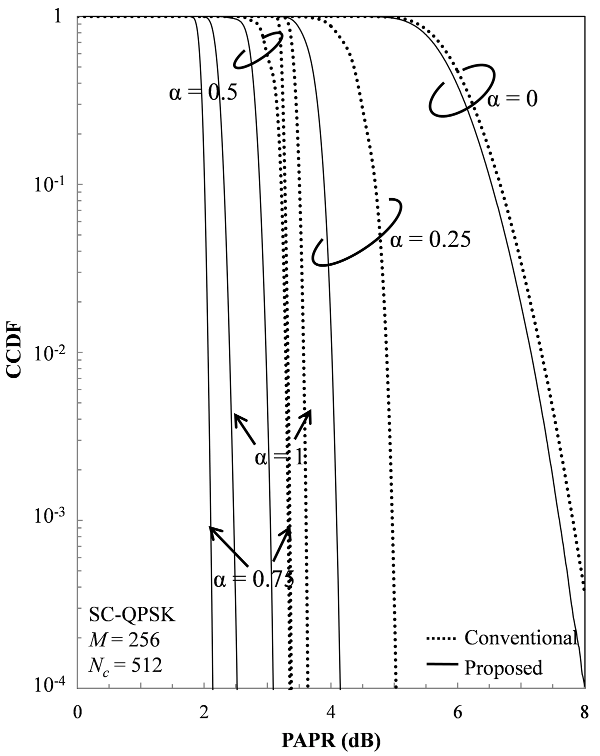


Fig. 5. CCDF of PAPR

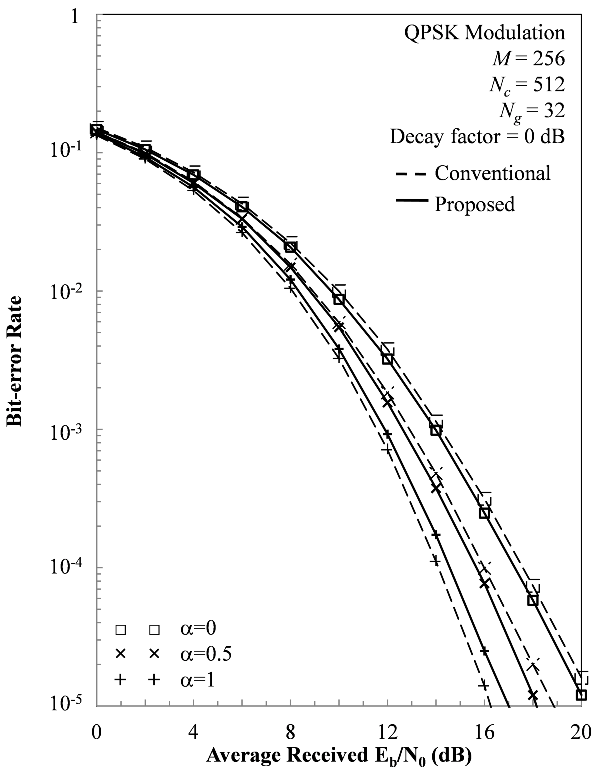


Fig. 6. BER performance of square-root raised cosine filter and proposed filter

of proposed algorithm is lower than conventional filter for all values of roll-off factor. As previously mentioned, the objective function is non-convex and hence provides many points of local minima. Even though gradient search cannot guarantee the global minimum, we can still reduce PAPR from the conventional filter.

At a place where probability of occurrence equals 0.1%, called $\text{PAPR}_{0.1\%}$, approximately 0.3 dB reduction is obtained when α=0, while 1.3 dB reduction is shown when α=0.75. Note that expanding the filter bandwidth until such a particular value can reduce PAPR, as seen that the lowest PAPR can be achieved when α is 0.5 for conventional filter, and 0.75 for proposed filter, respectively.

C. BER Performance

BER performance of both conventional filter and proposed filter with various values of roll-off factors are shown in Fig.6 as a function of average received energy-to-noise power spectrum density ratio $E_b/N_0 = (E_s/N_0)(1+N_g/N_c)/2$. With the aid of MMSE-FDE and spectrum combining, BER performance is better as roll-off factor α increases. As previously mentioned, using MMSE-FDE with spectrum combining can obtain additional frequency diversity gain which is inherited from excess-bandwidth transmission. It is also noticed that the BER performance of proposed filter is very similar compared to square-root raised cosine filter, which means that we can reduce PAPR at the transmitter while achieving the similar BER performance when the proposed filtering algorithm is applied.

V. Conclusion

In this paper, a novel low-PAPR filtering algorithm based on the minimization of variance of instantaneous power for SC transmission has been investigated. We also introduce the roll-off factor to the proposed filter and hence excess-bandwidth transmission is achievable. A combination of proposed filtering algorithm at the transmitter and MMSE-FDE with spectrum combining at the receiver provides lower PAPR while additional frequency diversity gain from excess bandwidth is still obtained. Simulation results confirmed that proposed filter gave better PAPR performance and BER characteristic is also similar to the square-root raised cosine filter, which implies that BER performance is better by increasing the roll-off factor.

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