Joint Overlap FDE & Spectrum Combining for Single-carrier Transmission without Cyclic Prefix Insertion

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Abstract— **In the square-root Nyquist transmit-filtered single-carrier (SC) transmission, increasing the roll-off factor can reduce the peak-to-average power ratio (PAPR) of the transmit signal. Furthermore, since the signal bandwidth becomes wider, larger frequency diversity gain can be achieved. In our previous work, we proposed joint minimum mean square error frequency-domain equalization (MMSE-FDE) & spectrum combining for the cyclic prefix (CP) inserted SC block transmission. However, the CP insertion reduces the spectrum efficiency. In this paper, we propose a joint overlap FDE & spectrum combining for the Nyquist filtered SC transmission without CP insertion. The BER performance of the proposed method is evaluated by computer simulation. We show that the proposed method can achieve almost the same performance as the conventional MMSE-FDE with CP insertion and improve the BER performance by increasing the roll-off factor.**

Keywords; Overlap FDE, transmit filter, oversampling, single-carrier transmission

I. INTRODUCTION

The broadband wireless channel composed of many propagation paths with different time delays produces strong frequency-selective fading channel and accordingly degrades the bit error rate (BER) performance of the broadband single-carrier (SC) transmission due to the strong inter-symbol interference (ISI) [1]-[3]. The use of the minimum mean square error (MMSE) based frequency-domain equalization (FDE) can improve significantly the BER performance [4]-[6].

In most of SC communication systems, a square-root Nyquist filter is used at the transmitter to limit the signal bandwidth and is used as the matched filter at the receiver. Furthermore, increasing the roll-off factor α of the transmit filter can reduce the peak-to-average power ratio (PAPR) of the transmit signal. In our previous work, we proposed the joint MMSE-FDE & spectrum combining for the cyclic prefix (CP) inserted transmit-filtered SC block transmission [7]. The proposed scheme can obtain larger frequency diversity gain and achieve better BER performance by increasing the value of α since the signal bandwidth gets wider. The CP is inserted to avoid the inter-block interference (IBI) and to make the received signal to be a circular convolution of the transmit signal block and the channel impulse response. However, the CP insertion reduces the spectrum efficiency. Recently, overlap FDE which requires no CP insertion was proposed [8], [9]. Based on the fact that the residual IBI after MMSE-FDE is localized only near both edges of the equalized signal block, overlap FDE can sufficiently suppress the residual IBI and achieve higher throughput than the FDE with CP insertion at the cost of increased complexity [10].

In this paper, we propose the joint overlap FDE & spectrum combining for the square-root Nyquist filtered SC block transmissions. In the joint overlap FDE & spectrum combining, the received signal is 2-times oversampled and then, MMSE-FDE and spectrum combining are jointly used to restore the near original spectrum with reduced ISI. After MMSE-FDE and spectrum combining, only the central part of the equalized signal block is picked up to suppress the residual IBI. The BER performance achievable with the proposed method is evaluated by computer simulation. We show that the proposed scheme can achieve almost the same performance as the conventional MMSE-FDE with CP insertion and improve the BER performance by increasing the roll-off factor α.

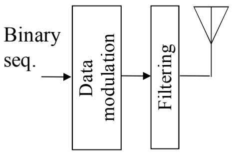
The remainder of this paper is organized as follows. Section II presents the transmission system model using the proposed joint overlap FDE & spectrum combining. The computer simulation results are discussed in Sect. III. Section IV offers some conclusions.

II. JOINT OVERLAP FDE & SPECTRUM COMBINING

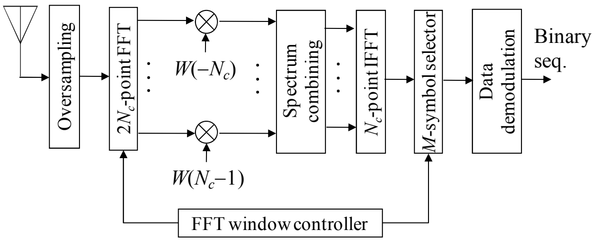
Figure 1 illustrates the transmitter and receiver structures of the square-root Nyquist filtered SC transmission using the proposed joint overlap FDE & spectrum combining. At the transmitter, the data-modulated symbol sequence is transmitted after passing through the square-root Nyquist transmit filter to limit the signal bandwidth. At the receiver, the received signal is oversampled at a faster rate than the symbol rate to avoid the spectrum overlapping. When the square-root raised cosine filter is used as transmit filter, the spectrum overlapping can be avoided by using double oversampling (the received signal sampled at the rate $2/T_s$). Then, joint overlap FDE & spectrum combining is applied to the oversampled signal sequence.

In FDE, the received signal is transformed by fast Fourier transform (FFT) into the frequency-domain signal. However, when the CP is not inserted, the IBI appears at the beginning of the FFT window. The residual IBI after MMSE-FDE is a circular convolution of the IBI and the FDE filter impulse response. Figure 2 shows the impulse response of the MMSE-FDE filter for an L=16-path Rayleigh fading channel with uniform power delay profile. As seen from Fig. 2, the MMSE-

FDE filter impulse response concentrates around time t=0. Therefore, the residual IBI is localized only near the both edges of N_c-symbol block after MMSE-FDE. The proposed joint overlap FDE & spectrum combining is based on this observation. Figure 3 shows the block signal processing of the joint overlap FDE & spectrum combining. The oversampled received signal sequence is divided into a sequence of $2M$-sample blocks. Then, the received $2N_c$-sample block centering $2M$-sample block of interest ($M \le N_c$) is transformed into the frequency-domain signal by applying $2N_c$-point FFT. After applying MMSE-FDE and spectrum combining [7], the equalized and down-sampled N_c-symbol block can be obtained. The central M symbols are picked up from the equalized N_c symbols to suppress the residual IBI.



(a) Transmitter



(b) Receiver

Fig. 1 Transmitter and receiver structure of SC transmission using the proposed joint overlap FDE & spectrum combining.

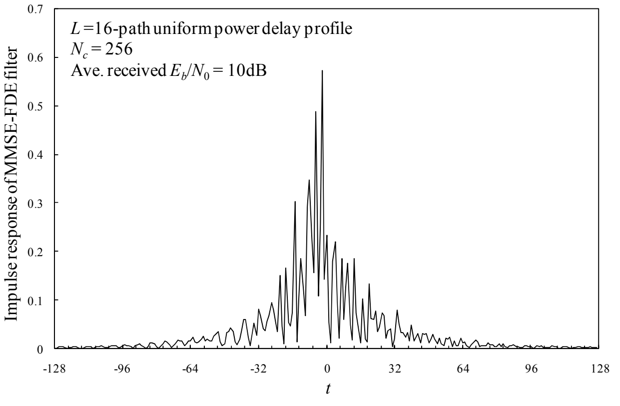


Fig. 2 Impulse response of MMSE-FDE filter.

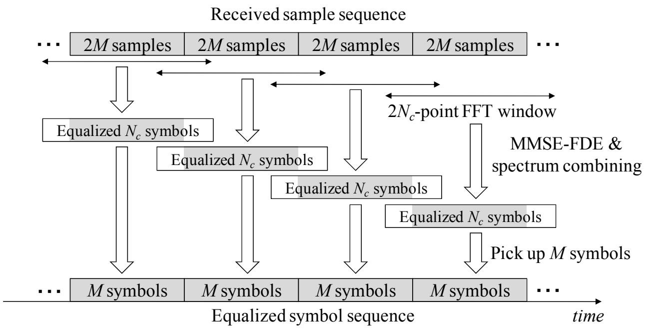


Fig. 3 Joint overlap FDE & spectrum combining

A. *Receive signal representation*

The received signal block $\{r(i);\ i=0\sim 2N_c-1\}$ oversampled at the rate $2/T_s$, where T_s is the symbol duration, can be expressed as

$$r(i)=\sqrt{2P}\sum_{l'=-\infty}^{\infty}\tilde{h}_{l'}\tilde{s}((i-l')\bmod 2N_c)+\nu(i)+\eta(i)\,, \qquad (1)$$

where P is the transmit power. $\{\tilde{s}(i); i=0\sim 2N_c-1\}$ is defined as

$$\tilde{s}(i)=\begin{cases} s(n), & i=2n \\ 0, & else \end{cases}, \qquad (2)$$

where $\{s(n);n=0\sim N_c-1\}$ is the transmitted symbol block. $\nu(i)$ and $\eta(i)$ denote respectively the inter-block interference (IBI) and the filter output of the additive white Gaussian noise (AWGN) with zero mean and variance $2N_0/T_s$, where N_0 is the single-sided power spectrum density. $\tilde{h}_{l'}$ is the overall channel (transmit filter + channel) gain given as

$$\tilde{h}_{l'}=\sum_{l=0}^{L-1}h_l\varphi\left(\frac{l'}{2}-\tau_l\right), \qquad (3)$$

where h_l and τ_l are respectively the complex-valued channel gain with $\sum_{l=0}^{L-1}E[|h_{n,l}|^2]=1$ and delay time of l-th path, $\{s(m);m=0\sim N_c-1\}$ is the transmitted symbol block, and $\varphi(t)$ is the transmit filter impulse response. In this paper, we assume the square-root raised cosine filter with the roll-off factor α as the transmit filter, given as [3]

$$\varphi(t)=\frac{\sin\{\pi t(1-\alpha)\}+4\alpha t\cos\{\pi t(1+\alpha)\}}{\pi t\{1-(4\alpha t)^2\}}. \qquad (4)$$

$2N_c$-point FFT is applied to transform the oversampled signal block $\{r(i);i=0\sim 2N_c-1\}$ into the frequency-domain signal $\{R_n(k);k=-N_c\sim N_c-1\}$. The kth frequency component $R_n(k)$ can be expressed as

$$\begin{aligned} R(k)&=\frac{1}{\sqrt{2N_c}}\sum_{i=0}^{2N_c-1}r(i)\exp\left(-j2\pi k\frac{i}{2N_c}\right), \\ &=\sqrt{2P}\tilde{H}(k)S(k)+\mathrm{N}(k)+\Pi(k) \end{aligned} \qquad (5)$$

where $\tilde{H}(k)$, $S(k)$, $\mathrm{N}(k)$, and $\Pi(k)$ are the overall (transmit filter + channel) transfer function, the signal component, the IBI component, and the noise component, respectively. $\tilde{H}(k)$ and $S(k)$ are respectively given as

$$\tilde{H}(k)=\sum_{l'=-\infty}^{\infty}\tilde{h}_{l'}\exp\left(-j2\pi k\frac{l'}{2N_c}\right)=H(k)\Phi(k), \tag{6}$$

and

$$S(k)=\frac{1}{\sqrt{N_c}}\sum_{i=0}^{N_c-1}s(i)\exp\left(-j2\pi k\frac{i}{N_c}\right). \tag{7}$$

$H(k)$ is the channel gain at the kth frequency given as

$$H(k)=\sum_{l=0}^{L-1}h_{n,l}\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right). \tag{8}$$

$\Phi(k)$ is the transfer function of the transmit filter given as

$$\Phi(k)=\begin{cases}1, & 0\le\left|\frac{k}{N_c}\right|\le\frac{1-\alpha}{2}\\ \cos\frac{\pi}{2\alpha}\left(\left|\frac{k}{N_c}\right|-\frac{1-\alpha}{2}\right), & \frac{1-\alpha}{2}\le\left|\frac{k}{N_c}\right|\le\frac{1+\alpha}{2}\\ 0, & \text{elsewhere}\end{cases}. \tag{9}$$

B. Joint MMSE-FDE & spectrum combining

One-tap MMSE-FDE is performed over the frequency range of $-N_c\le k<N_c$ as

$$\begin{aligned}\hat{R}_n(k)&=R_n(k)W_n(k)\\&=\sqrt{2P}\hat{H}(k)S(k)+\hat{\mathrm{N}}(k)+\hat{\Pi}(k)\end{aligned}, \tag{10}$$

where $W_n(k)$ is the MMSE-FDE weight.

After MMSE-FDE, the spectrum combining is performed to restore the ISI-free condition over the desired frequency range $-N_c/2\le k<N_c/2$ as shown in Fig. 4. The frequency-domain signal after the spectrum combining is given by

$$\begin{aligned}\breve{R}(k)&=\sum_{q=-1}^{1}\hat{R}(k-qN_c)\\&=\sqrt{2P}\breve{H}(k)S(k)+\breve{\mathrm{N}}(k)+\breve{\Pi}(k)\end{aligned}, \tag{11}$$

where

$$\begin{cases}\breve{H}(k)=\sum_{q=-1}^{1}\hat{H}(k-qN_c,\Delta_n)\\ \breve{\mathrm{N}}(k)=\sum_{q=-1}^{1}\hat{\mathrm{N}}(k-qN_c)\\ \breve{\Pi}(k)=\sum_{q=-1}^{1}\hat{\Pi}(k-qN_c)\end{cases}. \tag{12}$$

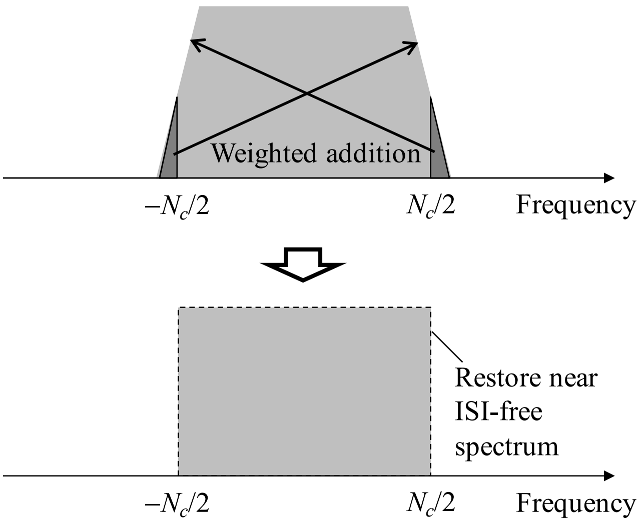


Fig. 4 Spectrum combining.

The frequency-domain signal $\{\breve{R}(k);k=-N_c/2\sim N_c/2-1\}$ after MMSE-FDE and spectrum combining is transformed by N_c-point inverse FFT (IFFT) into the down-sampled time-domain N_c-symbol block $\{\breve{r}(i);i=0\sim N_c-1\}$ given as

$$\breve{r}(i)=\frac{1}{\sqrt{N_c}}\sum_{k=-N_c/2}^{N_c/2-1}\breve{R}(k)\exp\left(j2\pi i\frac{k}{N_c}\right). \tag{13}$$

Finally, the central M symbols are picked up from the equalized N_c symbols of Eq. (13) to suppress the residual IBI. In next subsection, we derive the MMSE-FDE weight.

C. MMSE-FDE weight

We define the equalization error $e(k)$ after the spectrum combining at the kth frequency as

$$\begin{aligned}e(k)&=\breve{R}(k)-\sqrt{2P}S(k)\\&=\sum_{q=-1}^{1}\hat{R}(k-qN_c)-\sqrt{2P}S(k)\end{aligned}, \tag{14}$$

where $-N_c/2\le k<N_c/2$. The MMSE weight $\{W(k-qN_c);q=-1,0,1;k=-N_c\sim N_c-1\}$ for joint FDE & spectrum combining which minimizes the MSE $E[|e(k)|^2]$ can be derived as

$$W(k-qN_c)=\frac{\tilde{H}^*(k-qN_c)}{\sum_{q'=-1}^{1}\left|\tilde{H}(k-q'N_c)\right|^2+\sigma^2/P}, \tag{15}$$

where σ^2 denotes the sum of the IBI and noise powers given by

$$\sigma^2=\frac{1}{2}\{E[|\mathrm{N}(k)|^2]+E[|\Pi(k)|^2\}. \tag{16}$$

Since $\tilde{H}(k)$ includes the transfer function of the transmit filter, the MMSE-FDE weight also takes a role of the receive filter matched to the transmit filter (this is the reason why no receive filter is necessary in the receiver structure of Fig. 2).

III. COMPUTER SIMULATION

A. Simulation condition

The computer simulation condition is summarized in Table I. We assume QPSK data-modulation, the FFT size of $2N_c$=512 samples, and the IFFT size of N_c=256. The propagation channel is assumed to be a frequency-selective block Rayleigh fading channel having $T_s/2$-spaced L=16-path uniform power delay profile. The ideal channel estimation is also assumed.

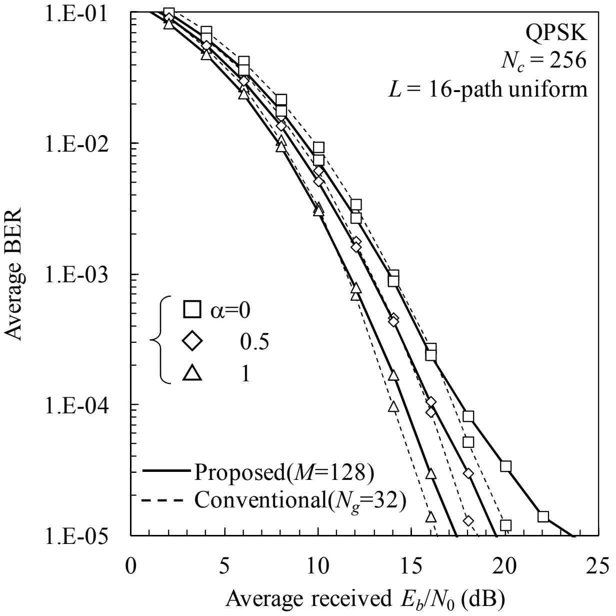
TABLE I. SIMULATION CONDITION

Data modulation	QPSK	
FFT window length	$2N_c$=512	
IFFT window length	N_c=256	
Channel model	Frequency-selective block Rayleigh fading	
	Power delay profile	L=16-path uniform
	Delay time	$\tau_l=l/2$
Nyquist filter	Square-root raised cosine filter	
	Roll-off factor	α=0~1
Channel estimation	Ideal	

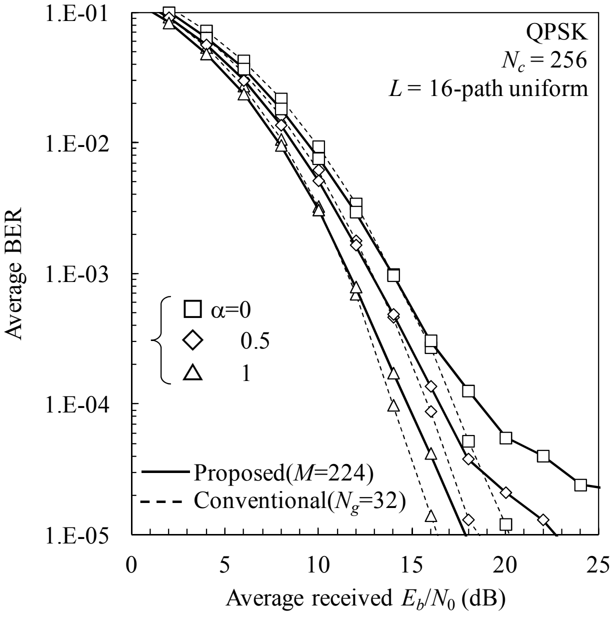
Figure 5 plots the average BER performance of the proposed joint overlap FDE & spectrum combining as a function of the average received bit energy-to-noise power spectrum density ratio E_b/N_0(=0.5(E_s/N_0)) with the roll-off factor α as a parameter. Also plotted is the BER performance of the conventional joint MMSE-FDE & spectrum combining with CP insertion for comparison (we assume the CP length is N_g=32 symbols and the E_b/N_0 loss due to the CP insertion is taken into account).

Figure 5(a) plots the BER performance when M=128. It can be seen from Fig. 5(a) that increasing α improves the BER performances. The reason for this is that, as α increases, the signal bandwidth becomes wider and the proposed joint MMSE-FDE & spectrum combining can achieve increased frequency diversity gain. The required E_b/N_0 to achieve BER=10^{-3} can be reduced by about 1(2)dB for α=0.5(1) compared to α=0. When M=128, the proposed scheme can achieve almost the same BER performance as the joint MMSE-FDE & spectrum combining with CP insertion. In low E_b/N_0 region, the proposed scheme can achieve slightly better performance than the joint MMSE-FDE & spectrum combining with CP insertion since there is no E_b/N_0 loss in the proposed scheme. In high E_b/N_0 region, the BER performance of the proposed scheme degrades from that of the conventional MMSE-FDE with CP due to the residual IBI. However, as α increases, the impact of the residual IBI can be reduced due to larger frequency diversity gain and the performance of the proposed scheme can approach that of the conventional MMSE-FDE with CP insertion.

Figure 5(b) plots the BER performance when M=224. When M is larger, the performance degradation of the proposed scheme becomes larger due to the residual IBI. However, as α increases, the performance degradation can be smaller compared to α=0 even if larger M is used.



(a) M=128



(b) M=224

Fig. 4 BER performance of the joint overlap FDE & spectrum combining.

IV. CONCLUSION

In this paper, we proposed the joint overlap FDE & spectrum combining for the transmit-filtered SC transmission without CP insertion. The proposed scheme can suppress the residual IBI by picking up only the central part from the equalized symbol block and provide almost the same performance as the conventional MMSE-FDE with CP insertion. In addition, as the filter roll-off factor increases, better BER performance can be achieved since larger frequency diversity gain is obtained.

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