Uplink Capacity of Cellular System Using Frequency Domain Adaptive Antenna Array

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***Abstract*—Space division multiple access (SDMA) using adaptive antenna array (AAA) allows multiple users to simultaneously access using the same bandwidth and hence is spectrum efficient. Single-carrier (SC) transmission has been widely adopted by uplink communication due to its lower peak-to-average-power-ratio (PAPR) than multi-carrier (MC) transmission. However, broadband SC channel is frequency-selective and the inter-symbol interference (ISI) is produced in addition to the multiuser interference. Frequency domain AAA (FDAAA) is a combination of AAA and frequency-domain equalization (FDE). In this paper, the uplink capacity is investigated for the cellular system with FDAAA. The cellular systems using fixed frequency reuse factor (FRF) and using hybrid FRF are considered. Their achievable uplink capacities are evaluated by computer simulation.**

Keywords: *Single-carrier, Uplink, FDAAA, multiple users, Hybrid FRF*

I. INTRODUCTION

The available bandwidth is limited while broadband services are demanded. To improve the utilization of the limited bandwidth, cellular systems reuse the same carrier frequency in spatially separated cells. As a result, the cellular capacity in bps/Hz/BS is limited by the co-channel interference (CCI) from co-cell cells [1]. To reduce the CCI, the frequency reuse factor (FRF) is introduced to adjust the distance between co-channel cells. The co-channel cell distance increases for higher FRF and accordingly, CCI becomes less significant. However, the spectrum efficiency degrades because a wider bandwidth is required.

To further improve the spectrum efficiency of the cellular systems, space division multiple access (SDMA) using adaptive antenna array (AAA) can be used. It allows multiple users to simultaneously access using the same bandwidth. The single-carrier (SC) transmission has an advantage of lower peak-to-average power ratio (PAPR) over the multi-carrier (MC) transmission. However, broadband channels are severely frequency selective due to the presence of multiple paths having different time delays [2]. In such a frequency-selective channel, the inter symbol interference (ISI) is produced and degrades the SC transmission performance significantly. The frequency-domain adaptive antenna array (AAA), which is a combination of frequency-domain equalization (FDE) and adaptive antenna array (AAA), can effectively suppress the

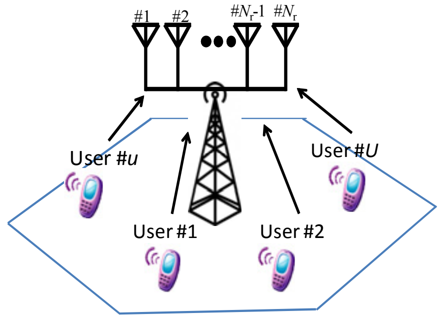

Fig. 1 Uplink Communication

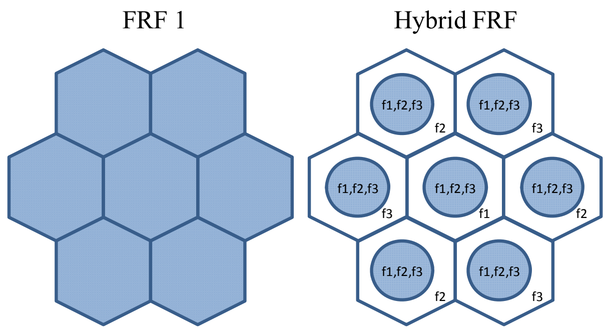

Fig. 2 Structure of fixed FRF and hybrid FRF

ISI and the multiuser interference (MUI) [3]. The AAA weight at each frequency or subcarrier is controlled so as to maximize the signal-to-interference plus noise ratio (SINR).

In cellular systems, there exists a trade-off between CCI and spectrum efficiency. It has been reported in [5-6] that for a system using multiple receive antennas, the use of fractional frequency reuse factor (FRF) can increase the cellular capacity compared to the fixed FRF scheme. In this paper, the uplink cellular capacity (bps/Hz/BS) is investigated for a cellular system with proposed hybrid FRF using FDAAA receiver. The cellular capacity is evaluated by computer simulation. It is shown that hybrid FRF is able to increase the cellular capacity compared to the fixed FRF.

The rest of the paper is organized as follows. The cellular system model and the FDAAA will be described in Sections II and III, respectively. The uplink capacity when using proposed Hybrid FRF is investigated in Section IV. The computer simulation results are presented in Section V. Finally, the paper will be concluded in Section VI.

II. CELLULAR SYSTEM MODEL

A. Uplink Transmission Model

Uplink communication model is shown in Fig. 1. It is assumed that the base station (BS) in the center of each cell is equipped with N_r antennas. There are U users in each cell and each is equipped with one transmit antenna. We assume that the channel is unchanged during one transmission block. Two cellular structures are shown in Fig. 2. On the left is the conventional cellular structure using fixed single FRF (FRF 1), on the right is the cellular structure using hybrid FRF which will be further explained in Section IV.

The channel impulse response between the u-th user and the BS can be expressed as

$$\mathbf{h}_u(\tau)=\sum_{l=0}^{L-1}\mathbf{h}_{u,l}\delta(\tau-\tau_l), \qquad (1)$$

where $\mathbf{h}_{u,l}$ and τ_l are the l-th path gain vector and time delay respectively, $\sum_{l=0}^{L-1}E\left\{\left|h_{m,l}\right|^2\right\}=1$ where $h_{m,l}$ is the m-th element of the gain vector $\mathbf{h}_{u,l}$ and $E\{\}$ denotes statistical expectation operator. The path delay is assumed to be integer multiples of symbol duration and $\tau_l=l$. Cyclic prefix (CP) is used and its length is assumed to be longer than the maximum path delay so that inter block interference (IBI) can be avoided.

The baseband received signal vector $\mathbf{r}(n)=\left[r_0(n),r_1(n),\ldots,r_{N_r-1}(n)\right]^T,(n=1,\cdots,N_c)$ is given by

$$\begin{aligned}\mathbf{r}(n)=&\sqrt{P_0\hat{d}_0^{\;-\alpha}10^{-\xi_0/10}}\sum_{l=0}^{L-1}\mathbf{h}_0 s_0(n-\tau_l)\\&+\sum_{u=1}^{U-1}\sqrt{P_u\hat{d}_u^{\;-\alpha}10^{-\xi_u/10}}\sum_{l=0}^{L-1}\mathbf{h}_u s_u(n-\tau_l)\\&+\sum_{i=0}^{I-1}\sum_{u=0}^{U-1}\sqrt{P_{u,i}\hat{d}_{u,i}^{\;-\alpha}10^{-\xi_{u,i}/10}}\sum_{l=0}^{L-1}\mathbf{h}_{u,i}s_{u,i}(n-\tau_l)\\&+z(n).\end{aligned} \qquad (2)$$

U and I are the number of users per cell and number of co-channel cells respectively; P_u and $P_{u,i}$ represent the transmit power of the u-th user at the desired cell and i-th co-channel cell respectively; $s_u(n)$ is the transmit signal of the u-th user within the desired cell; α represents the path loss exponent; $\hat{d}_{u,i}$ and $\xi_{u,i}$ represent the normalized distance and shadowing loss between the user at the i-th co-channel cell and the desired BS respectively. $\mathbf{z}(n)=\left[z_0(n)\;\; z_1(n)\;\;\ldots\;\; z_{N_r-1}(n)\right]^T$ is the vector of additive white Gaussian noise (AWGN).

The frequency domain received signal on the k-th frequency is then expressed as

$$\mathbf{R}(k)=\mathbf{H}_0(k)S_0(k)+\sum_{u=1}^{U-1}\mathbf{H}_u(k)S_u(k)+\sum_{i=0}^{I-1}\sum_{u=0}^{U-1}\mathbf{H}_{u,i}(k)S_{u,i}(k)+\mathbf{Z}(k), \qquad (3)$$

where $\mathbf{H}_u=\left[H_{u,0}(k)\;\; H_{u,1}(k)\;\;\cdots\;\; H_{u,N_r-1}(k)\right]^T$, $S_u(k)$, and $\mathbf{Z}(k)=\left[Z_0(k)\;\; Z_1(k)\;\;\ldots\;\; Z_{N_r-1}(k)\right]^T$ are respectively the frequency domain channel response, transmit data and noise component, given by equation (4). The first term in equation (3) comes from the desired user, the second term comes from the multi-users interference (MUI) users in the same cell, the third term comes from CCI users, and the last term is the AWGN component.

$$\begin{cases}S_u(k)=\sqrt{P_u\hat{d}_u^{\;-\alpha}10^{-\xi_u/10}}\sum_{n=0}^{N_c-1}s_u(n)\exp\left(-j2\pi n\frac{k}{N_c}\right)\\ H_{u,m}(k)=\sum_{l=0}^{L-1}\sum_{n=0}^{N_c-1}h_{u,l,m}\exp\left(-j2\pi n\frac{k}{N_c}\right)\\ Z_m(k)=\sum_{n=0}^{N_c-1}z_m(n)\exp\left(-j2\pi n\frac{k}{N_c}\right).\end{cases} \qquad (4)$$

B. Propagation Model of Adaptive Antenna Array

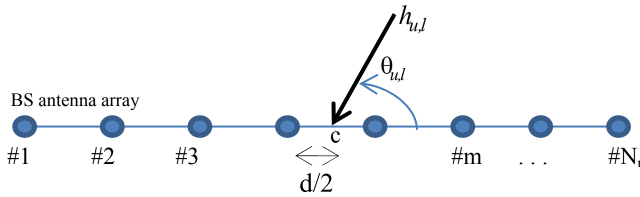

Fig. 3 Linear antenna array with N_r antennas

The propagation model of a linear AAA is shown in Fig. 3. $h_{u,l}$ represents the l-th delay-path gain from the u-th user at the center of the array and no spread between the un-resolvable paths is used. $\lambda/2$ antenna separation is used. We assume that the reference point of the received signal at antenna array is point "c", the geometric center of the array, as shown by Fig. 3.

The nominal angle of arrival (AOA) of $h_{u,l}$ is denoted by $\theta_{u,l}$ and the AOA spread of $\theta_{u,l}$ is uniformly distributed within a range of Δ (in this study, all direction AOA is used so that $\Delta=2\pi$). The l-th path gain observed at the m-th antenna element is then expressed as

$$h_{u,l,m}=h_{u,l}\,exp\left(-j2\pi\frac{(0.5N_r-m+0.5)}{\lambda}d\cos\theta_{u,l}\right), \qquad (5)$$

where m = 1,2,3,..,N_r and λ is the carrier wavelength.

III. FDAAA ALGORITHM

FDAAA receiver has been investigated in [3] and [4]. The transceiver structure of FDAAA receiver is shown in Fig.4. The output of received signal after applying AAA weight is expressed as

$$\hat{R}(k)=\mathbf{W}_{FDAAA}^H(k)\mathbf{R}(k), \qquad (6)$$

where the AAA weight $\mathbf{W}_{FDAAA}(k)=\left[W_{FDAAA,0}(k),W_{FDAAA,1}(k),\ldots,W_{FDAAA,N_r-1}(k)\right]^T$ minimizes the mean square error (MMSE) between $\hat{R}(k)$ and the desired signal $S_0(k)$. The AAA weight is calculated following MMSE criterion as follows [7]

$$\mathbf{W}_{FDAAA}(k)=\mathbf{X}(k)^{-1}\mathbf{p}(k), \qquad (7)$$

where $\mathbf{X}(k)=E\left\{\mathbf{R}(k)\mathbf{R}(k)^H\right\}$ is the auto-correlation matrix

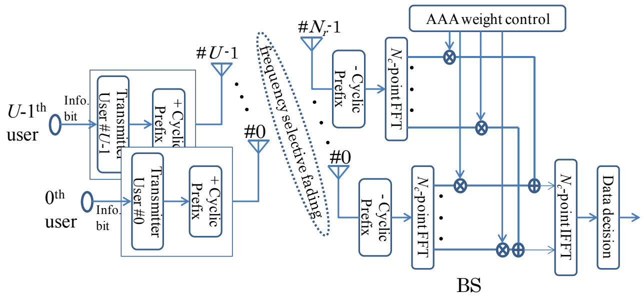

Fig. 4 The FDAA uplink transmission

of the received signal vector, $\mathbf{p}(k)=E\{\mathbf{R}(k)S_0^*(k)\}$ is the cross correlation between the received signal and the reference signal, superscript H denotes Hermitan transposition (i.e. operation of transposition with complex conjugate) and $*$ denotes the complex conjugate operation. We assume that the transmit signals from different users are independent and the noise component is also independent. The auto-correlation matrix $\mathbf{X}(k)$ is a square matrix with N_r x N_r dimension and the (m,n)-th element of $\mathbf{X}(k)$ is given by

$$\begin{aligned} X_{m,n}(k) &= E\{R_m(k)R_n^*(k)\} \\ &= E\{H_{0,m}(k)S_0(k)S_0^*(k)H_{0,n}^*(k)\} \\ &+ \sum_{u=1}^{U-1} E\{H_{u,m}(k)S_u(k)S_u^*(k)H_{u,n}^*(k)\} \\ &+ \sum_{i=0}^{I-1}\sum_{u=0}^{U-1} E\{H_{u,i,m}(k)S_{u,i}(k)S_{u,i}^*(k)H_{u,i,n}^*(k)\} \\ &+ E\{Z_m(k)Z_n^*(k)\}. \end{aligned} \quad (8)$$

The expectation value is taken over desired signal, MUI signal, CCI signal, and noise power. Channel state information (CSI) is only available for the users within the same cell of interest. Therefore (8) can be simplified as

$$\begin{aligned} X_{m,n}(k) &= B_0 H_{0,m}(k)H_{0,n}^*(k) + \sum_{u=1}^{U-1} P_{MUI,u} H_{u,m}(k)H_{u,n}^*(k) \\ &+ \sum_{i=0}^{I-1}\sum_{u=0}^{U-1} diag[P_{CCI,u,i}]_{N_r\times N_r} + \sigma^2\mathbf{I}. \end{aligned} \quad (9)$$

The cross correlation for the m-th antenna element can also be derived as

$$\begin{aligned} p_m(k) &= E\{R_m(k)S_0^*(k)\} \\ &= E\{H_{0,m}(k)S_0(k)S_0^*(k)\} \\ &= B_0 H_{0,m}(k). \end{aligned} \quad (10)$$

where B_0 is received power from the desired user; $P_{MUI,u}$ and $P_{CCI,u}$ represent MUI and CCI power from the u-th user respectively .

IV. HYBRID FRF CALCULATION AND CAPACITY ANALYSIS

A. Hybrid FRF

Hybrid FRF adopts FRF=1 and FRF=3 adaptively according to location and instantaneous channel gain of user to maximize the bandwidth efficiency [5-6]. Different from traditional cellular system which uses the same bandwidth for all the area within a cell, hybrid FRF [5] uses FRF 1 which employs the whole bandwidth for area near the cell center and FRF 3 which employs one third of the bandwidth for area near the cell edge. FRF 3 is applied at cell edge area to mitigate the strong CCI while FRF 1 keeps the maximal spectrum efficiency. Therefore, two data rates coexist within a cell. The same target E_s/N_0 among users is required to guarantee equal quality of service (QoS). The relation between energy and power is shown in equation (11).

$$\frac{E_s}{N_0} = \frac{P_{\text{target}} \times T_s}{N_0}$$

$$\frac{P_{\text{target}}}{N_0} = \begin{cases} BW \times \left(\frac{E_s}{N_0}\right)_{\text{target}}, & \text{cell center}, \\ \frac{BW}{3} \times \left(\frac{E_s}{N_0}\right)_{\text{target}}, & \text{cell edge} \end{cases} \quad (11)$$

where P_{target} is the target receive signal power, T_s is symbol period, and BW is the bandwidth. Receive signal power at the desired BS from the u-th user in the desired cell and the u-th user in the i-th co-channel cell are given by equations (12) and (13) respectively.

$$\frac{B_u}{N_0} = \left(\frac{P_{\text{target}}/N_0}{\hat{d}_u^{-\alpha}10^{-\xi_u/10}}\right)\hat{d}_u^{-\alpha}10^{-\xi_u/10}$$

$$B_u = \begin{cases} (E_s)_{\text{target}} \times BW, & \text{cell center} \\ (E_s)_{\text{target}} \times \frac{BW}{3}, & \text{cell edge} \end{cases} \quad (12)$$

$$\frac{B_{u,i}}{N_0} = \left(\frac{P_{\text{target}}/N_0}{\hat{r}_{u,i}^{-\alpha}10^{-\eta_{u,i}/10}}\right)\hat{d}_{u,i}^{-\alpha}10^{-\xi_{u,i}/10}$$

$$B_{u,i} = \left(\frac{P_{\text{target}}}{\hat{r}_{u,i}^{-\alpha}10^{-\eta_{u,i}/10}}\right)\hat{d}_{u,i}^{-\alpha}10^{-\xi_{u,i}/10}$$

$$B_{u,i} = \begin{cases} \left(\frac{(E_s)_{\text{target}} \times BW}{\hat{r}_{u,i}^{-\alpha}10^{-\eta_{u,i}/10}}\right)\hat{d}_{u,i}^{-\alpha}10^{-\xi_{u,i}/10}, & \text{cell center} \\ \left(\frac{(E_s)_{\text{target}} \times \frac{BW}{3}}{\hat{r}_{u,i}^{-\alpha}10^{-\eta_{u,i}/10}}\right)\hat{d}_{u,i}^{-\alpha}10^{-\xi_{u,i}/10}, & \text{cell edge} \end{cases} \quad (13)$$

$\hat{r}_{u,i}$ and $\eta_{u,i}$ represent normalized distance and shadowing loss between the u-th user at the i-th co-channel cell and its communicating BS, respectively. $\hat{d}_{u,i}$ and $\xi_{u,i}$ are respectively normalized distance and shadowing loss between the u-th user at the i-th co-channel cell and the desired BS.

B. System capacity analysis

Since FRF 1 and FRF 3 are both used within a cell, capacity (bps/Hz/BS) calculation depends on the desired user's location. That is given by

$$C = \begin{cases} \log_2(1+SINR), & \text{FRF 1 area} \\ \frac{1}{3}\log_2(1+SINR), & \text{FRF 3 area} \end{cases} \quad (14)$$

Signal to interference plus noise ratio ($SINR$) is the power ratio of desired signal and interference power plus noise.

SINR for FDAAA receiver can be calculated by

$$SINR = \frac{\sum_{k=0}^{Nc-1} \mathbf{W}_{FDAAA}^{H}(k)\mathbf{R}_{s}(k)\mathbf{W}_{FD-AAA}(k)}{\sum_{k=0}^{Nc-1} \mathbf{W}_{FDAAA}^{H}(k)\mathbf{R}_{NI}(k)\mathbf{W}_{FD-AAA}(k)} \quad (15)$$

where $\mathbf{W}_{FDAAA}$ is the AAA weight given by (7), $\mathbf{R}_s(k)$ is the covariant matrix of received signal from the desired user and $\mathbf{R}_{NI}(k)$ is the covariant matrix of received signal from interference as well as the noise part.

Interference power from the *u*-th interferer depends on the desired user's location and its own location. Several cases are considered as follows.

1). Desired user is located inside FRF 1 area.

a. *Multi user interference (MUI) power*:

$$P_{MUI,u} = \begin{cases} (E_s)_{target} \times BW, & \text{cell center} \\ (E_s)_{target} \times \frac{BW}{3}, & \text{cell edge.} \end{cases} \quad (16)$$

b. *Co-channel interference (CCI) power*:

$$P_{CCI,u} = \begin{cases} \left(\frac{(E_s)_{target} \times BW}{\hat{r}_{u,i}^{-\alpha} 10^{-\eta_{u,i}/10}} \right) \hat{d}_{u,i}^{-\alpha} 10^{-\xi_{u,i}/10}, & \text{cell center} \\ \left(\frac{(E_s)_{target} \times \frac{BW}{3}}{\hat{r}_{u,i}^{-\alpha} 10^{-\eta_{u,i}/10}} \right) \hat{d}_{u,i}^{-\alpha} 10^{-\xi_{u,i}/10}, & \text{cell edge.} \end{cases} \quad (17)$$

2). Desired user is located outside FRF 1 area

a. *Multi user interference (MUI) power*:

$$P_{MUI,u} = \begin{cases} (E_s)_{target} \times \frac{BW}{3}, & \text{cell center} \\ (E_s)_{target} \times \frac{BW}{3}, & \text{cell edge.} \end{cases} \quad (18)$$

b. *Co-channel interference (CCI) power*:

$$P_{CCI,u} = \begin{cases} \left(\frac{(E_s)_{target} \times \frac{BW}{3}}{\hat{r}_{u,i}^{-\alpha} 10^{-\eta_{u,i}/10}} \right) \hat{d}_{u,i}^{-\alpha} 10^{-\xi_{u,i}/10}, & \text{cell center} \\ \left(\frac{(E_s)_{target} \times \frac{BW}{3}}{\hat{r}_{u,i}^{-\alpha} 10^{-\eta_{u,i}/10}} \right) \hat{d}_{u,i}^{-\alpha} 10^{-\xi_{u,i}/10}, & \text{cell edge.} \end{cases} \quad (19)$$

V. COMPUTER SIMULATION RESULTS

Optimal FRF 1 area is observed at first. System capacity as a function of FRF 1 area radius is observed to find the optimal radius for FRF 1 area. 10^6 runs of simulation are carried out for each normalized radius. Cellular capacity in bps/Hz/BS for multiple users will be shown later. The effectiveness of hybrid FRF to improve the system capacity compared with fixed FRF will be investigated as well.

Table 1 Simulation Conditions

Transmitter	Data Modulation	QPSK
	FFT size	N_c =256
	TPC	Slow TPC
	Number of user per cell	U = 2 - 8
	No. of CCI cell	I = 18
	Target receive E_s/N_0 per antenna	10 dB
Channel	Channel model	Frequency-selective block Rayleigh fading
	Power delay profile	L = 16 uniform power delay profile
	Angle spread of irresolvable paths	δ=0 degrees
	Angle spread of resolvable paths (AOA)	△= 360 degrees
	Path loss exponent	α= 3.5
	Standard deviation of shadowing losses	ξ= 6 dB
Receiver	No. of antennas	N_r = 8
	Antenna separation	λ/2
	Channel estimation	Ideal

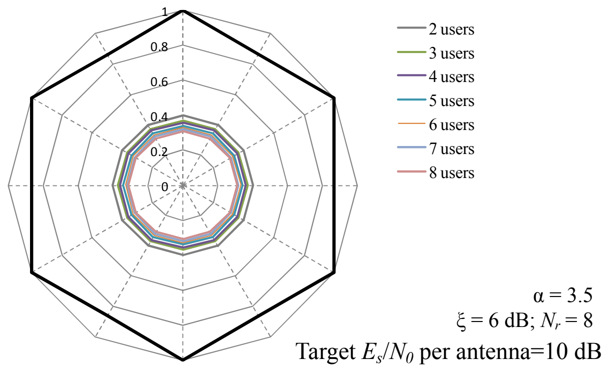

Fig. 5 Optimal FRF 1 area as a function of the number of users.

A. Optimal FRF 1 area

Optimal FRF 1 area gives the maximal 1% and 10% outage capacities. Circle lines in Fig. 5 shows the bound between FRF 1 and FRF 3 area with various numbers of users in each cell. Since multi-user interference can be efficiently suppressed by FDAAA receiver, the number of user is up to the number of antennas. It is observed that the FRF 1 area becomes smaller when the number of users in each cell increases. This is because stronger CCI interference occurs with larger amount of active users. In other words, the number of CCI users has a significant effect on optimal FRF 1 area.

B. System Capacity of Hybrid FRF

System capacity of uplink communication system in bps/Hz/BS can be calculated following equation (14). It is assumed that the user is selected randomly without scheduling. Scheduling method will be interesting as our future work. Fig. 6 shows 1% and 10% outage capacity system. System capacity of FRF 1 increases slowly while the number of users increases. The capacity increase is limited by the increasing interference level while the number of users increases. For the hybrid FRF case, the

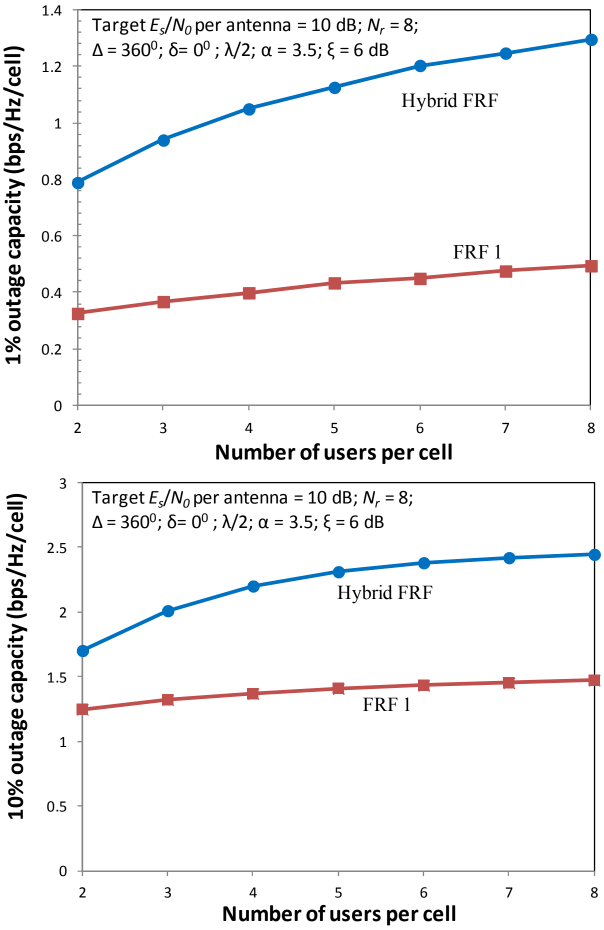

Fig. 6 Outage capacity of hybrid FRF and FRF=1

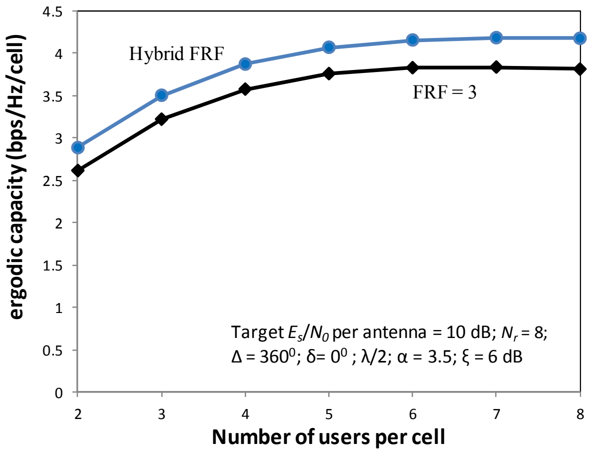

Fig. 7 Ergodic capacity of hybrid FRF and FRF=3

system capacity increases more significantly as the number of user increases. This is because the CCI is properly controlled by the hybrid FRF scheme. However, the increment of system capacity becomes less significant as the number of users increases. This is limited by the fact that FDAAA receiver can only support the number of users up to the number of antennas. The system ergodic capacity in Fig. 7 shows that the hybrid FRF scheme using optimal FRF 1 area for 1% and 10% outage capacity can also improve the system ergodic capacity.

Thus it can be concluded that hybrid FRF optimizes the trade-off between CCI and spectrum efficiency in cellular system by using FDAAA receiver. As a result, both outage and ergodic capacities can be improved.

VI. CONCLUSION

In this paper, the uplink capacity of a cellular system using FDAAA and hybrid FRF was evaluated by computer simulations. It was shown that the use of hybrid FRF can greatly improve the 1% and 10% outage capacities if the FRF=1 area is optimized. The optimum FRF=1 area shrinks due to stronger CCI as the number of users per cell increases. The application of FDAAA to a cellular system with hybrid FRF in more practical situation is left for future study.

REFERENCES

[1] W. Peng and F. Adachi, "Frequency Domain Adaptive Antenna Array for Broadband Single-Carrier Uplink Transmission", *IEICE TRANS. COMMUN.*, vol. E94-B, No. 7 July 2011.

[2] J. G. Proakis, *Digital Communications*, New York: McGraw Hill, 2001.

[3] W. Peng and F. Adachi, "Frequency Domain Adaptive Antenna Array Algorithm for Single-carrier Uplink Transmission", *IEEE PIMRC 2009*, pp.1-5, Sept.2009.

[4] W. Peng and F. Adachi, "Single-carrier Frequency Domain Adaptive Antenna Array for Cellular system," *2010 IEEE 72nd Vehicular Technology Conference (VTC-Fall)*, Ottawa, Canada, 6-9 Sept. 2010.

[5] W. Peng and F. Adachi, "Hybrid Frequency Reuse Scheme for Cellular MIMO system", *IEICE TRANS. COMMUN.* Vol.E92-B No.5, pp.1641-1647, May 2009.

[6] W. Peng and F. Adachi, "Multi-user hybrid FRF algorithm for downlink cellular MIMO systems," *The 20th Personal, Indoor and Mobile Radio Communications Sympsdium (PIMRC2009)*, Tokyo, Japan, 13-16 Sept. 2009.

[7] J. C. Liberti and J. S. Rappaport, *Smart Antenna for Wireless Communication: IS-95 and Third Generation CDMA Applications*, New Jersey: Prentice Hall, 1995.