Impact of Channel Estimation Error on IBI Cancellation and Circular Property Restoration for Broadband DS-CDMA Combining FDE without CP Insertion

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***Abstract*-In broadband DS-CDMA using frequency-domain equalization (FDE) without cyclic prefix (CP) insertion, the transmission performance degrades due to inter block interference (IBI) and circular property loss. In our previous study, we introduced the perfect IBI cancellation and circular property restoration to mitigate the problems arising from IBI and circular property loss. The perfect channel estimation was assumed. In this paper, assuming the Gaussian error model, we investigate the impact of channel estimating error on the bit error rate (BER). The requirement for practical channel estimation is also given based on the result.**

***Keywords*- DS-CDMA, MMSE-FDE, IBI cancellation, Circular property loss restoration, Gaussian error**

I. INTRODUCTION

Direct sequence code division multiple access (DS-CDMA) with coherent RAKE combining [1] is used in the present 3rd generation cellular mobile communication systems for high speed data services of up to around 10Mbps. For WCDMA using a chip rate of 3.84Mcps, the wireless channel is composed of a few resolvable propagation paths having different time delays and is frequency-selective [2]. The Rake combining maximizes the received signal-to-noise ratio (SNR) and works very well if the number of resolvable paths is not too large. However, if the chip rate is increased for providing broadband data services, the frequency-selectivity of the channels becomes severer and the achievable bit error rate (BER) performance with RAKE combining degrades due to severe inter-chip interference (ICI). For such a severe frequency-selective channel, the minimum mean square error (MMSE) based frequency domain equalization (MMSE-FDE) [3] can be used to achieve much better BER performance than the coherent RAKE combining [4].

Broadband DS-CDMA using FDE is a block transmission with the cyclic prefix (CP) insertion [5]. For FDE, the CP insertion is necessary to make the received chip block to become a circular convolution of the transmitted chip block and the channel. To avoid the inter block interference (IBI), the CP length must be longer than the maximum time delay of the channel [6]. However, the insertion of CP decreases the spectrum efficiency. The absence of CP produces IBI and the circular property loss. In our previous study [7], in order to mitigate the problems arising from IBI and circular property loss, we introduced the perfect IBI cancellation and circular property restoration to an MMSE-FDE receiver for broadband DS-CDMA uplink transmission using FDE without CP insertion. It was shown [7] that circular property restoration is more powerful than IBI cancellation.

Our previous study assumed the perfect channel estimation. However, in any practical channel estimation scheme, the channel estimation error occurs. In this paper, the Gaussian channel estimation error is assumed and the impact of the estimating accuracy on the achievable BER is discussed.

The rest of this paper is organized as follows. Section II presents the IBI and circular property loss problems in broadband DS-CDMA using FDE without CP. In Section III, IBI cancellation and circular property restoration are described. In Section IV, the Gaussian channel estimation error model is introduced. The BER performance in the presence of channel estimation error is evaluated by computer simulation and the impact of the channel estimation error on the BER performance is discussed in Section V. Finally the paper will be concluded in Section VI.

II. SYSTEM MODEL

The broadband DS-CDMA using FDE without CP insertion[8] is described in Section A. The IBI and circular property loss problems are presented in Section B.

A. DS-CDMA uplink transmission

Figure 1 shows the system model of multi-code DS-CDMA transmission. At the transmitter side, after data modulation for the u-th ($u = 0 \sim U-1$) data stream $d_u(t)$, the binary information sequence is spread by orthogonal spreading code $c_u(t)$ of spreading factor SF ($SF > U$). Then the sequence is multiplied by a scrambling sequence $c_{scr}(t)$ to generate the multi-code DS-CDMA data stream. At the receiver side, the signal is descrambled and de-spread after equalization. And the U parallel symbol streams are converted back to a sequence by a parallel/serial (P/S) converter before demodulation.

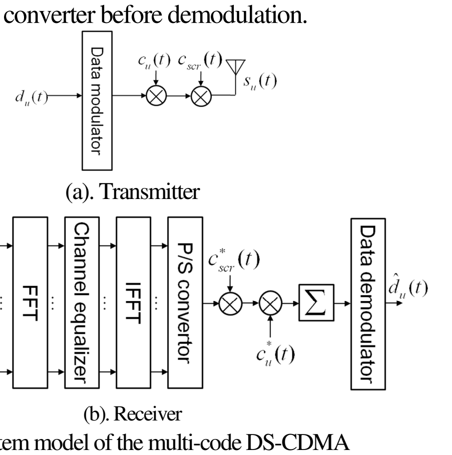


Fig.1 System model of the multi-code DS-CDMA

The multi-code DS-CDMA signal to be transmitted is expressed as

$$s(t) = \sqrt{\frac{2E_c}{T_c}} \sum_{u=0}^{U-1} d_u\left(\left\lfloor \frac{t}{SF} \right\rfloor\right) c_{scr}(t) c_u(t \bmod SF) \tag{1}$$

where E_c and T_c denote the chip energy and chip duration, respectively; SF is the spreading factor, u is the user index;

$d_u(t)$ is the data sequence of user u; $c_{scr}(t)$ is the channel-specific scrambling code; $c_u(t)$ is the user-specific spreading code; $\lfloor x \rfloor$ represents the largest integer less than or equal to x. The transmitted DS-CDMA signal propagates through a multi-path channel. At the receiver side, the base band equivalent received signal is given by

$$r(t)=\sum_{l=0}^{L-1} h_l s(t-\tau_l)+\eta(t), \tag{2}$$

where h_l is the l -th complex-valued path gain satisfying $\sum_{l=0}^{L-1} E\left[|h_l|^2\right]=1$ ($E[\cdot]$ denotes the ensemble average operation). In this study, integer chip-spaced multi-path delay is used and $\tau_l = l$. $\eta(t)$ is a zero-mean complex-valued additive white Gaussian noise (AWGN) with a variance of $2N_0/T_c$, where N_0 is the single-side noise power spectrum density.

B. Inter-block interference and circular property loss

Figure 2 shows the effect of multi-path fading on the received signal. Suppose that data block $\#M$ is under detection. Due to the multi-path delay, data block $\#M-1$ will "overlap" with data block $\#M$. On the other hand, the circular property is lost due to the absence of received CP replica. In order to detect block $\#M$, IBI should be removed and the circular property must be restored so that MMSE-FDE algorithm can be applied to the received block signal.

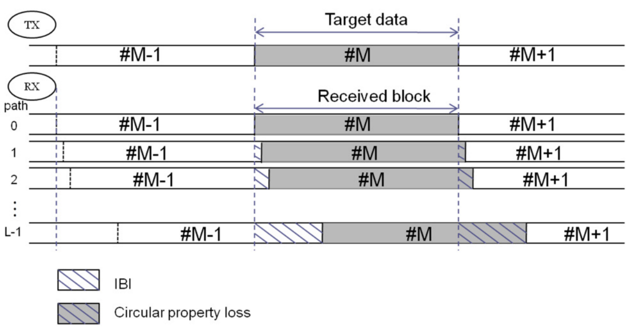


Fig.2 IBI and Circular property loss

III. PRINCIPLE OF IBI CANCELLATION AND CIRCULAR PROPERTY RESTORATION

The flow of the proposed MMSE-FDE receiver with IBI cancellation and circular property restoration is shown in Fig. 3.

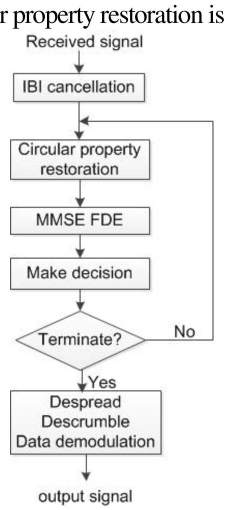


Fig. 3 Flow at receiver side

At first the IBI will be cancelled using the data decision of the previous block, then imperfect circular property restoration will be performed; in the next, MMSE FDE will be applied to the data block after IBI cancellation and imperfect circular property restoration. An improved circular property restoration can then be carried out by using the data decision after MMSE FDE. The performance of block detection can be improved in an iterative way until the termination condition is satisfied.

At the receiver side, the received DS-CDMA signal stream is divided into a sequence of N_c -chip blocks. The signal vector can be expressed using matrix form as

$$\begin{aligned}\mathbf{r} &= \mathbf{h}\mathbf{s}_0+\mathbf{v}+\boldsymbol{\eta} \\ &= \mathbf{h}\mathbf{s}_0+\mathbf{h}_{-1}(\mathbf{s}_{-1}-\mathbf{s}_0)+\boldsymbol{\eta} \\ &= \mathbf{h}\mathbf{s}_0+\mathbf{h}_{-1}\mathbf{s}_{-1}-\mathbf{h}_{-1}\mathbf{s}_0+\boldsymbol{\eta}\end{aligned} . \tag{3}$$

In the right hand side of the Equation (3), the first term contains the desired signal, the second term contains the IBI and the third term contains the power loss. $\mathbf{s}_0$, $\mathbf{s}_{-1}$ and $\boldsymbol{\eta}$ are respectively $N_c \times 1$ vectors given as

$$\begin{cases}\mathbf{s}_0=[s(0),s(1),\cdots,s(N_c-1)]^T \\ \mathbf{s}_{-1}=[s(-N_c),s(-N_c+1),\cdots,s(-1)]^T \\ \boldsymbol{\eta}=[\eta(0),\eta(1),\cdots,\eta(N_c-1)]^T\end{cases} . \tag{4}$$

$\mathbf{h}$ is the matrix of channel impulse response and $\mathbf{h}_{-1}$ is the matrix of channel impulse response to cause the interference. $\mathbf{h}_{-1}$ can be given as

$$\mathbf{h}_{-1}=\begin{bmatrix} & h_{L-1} & \cdots & h_1 \\ & & \ddots & \vdots \\ & & & h_{L-1} \\ \mathbf{0} & & & \end{bmatrix}_{N_c\times N_c} , \tag{5}$$

where, h_l is the complex valued path gain of the l th path.

Before giving the expressions for $\mathbf{h}$, we should notice that, since there is no CP, the circular property of $\mathbf{h}$ is lost. Therefore, the IBI cancellation and circular property restoration must be performed to recover the circular property. According to equation (3),

$$\begin{cases}\mathbf{v}_{-1}=\mathbf{h}_{-1}\mathbf{s}_{-1} \\ \mathbf{v}_0=\mathbf{h}_{-1}\mathbf{s}_0\end{cases} , \tag{6}$$

where, $\mathbf{v}_{-1}$ is the IBI component, which should be cancelled; and $\mathbf{v}_0$ is the circular property loss component, which should be restored.

To solve these two problems, perfect IBI cancellation, perfect circular property restoration and imperfect restoration will be discussed and compared in the following.

A. Perfect IBI cancellation

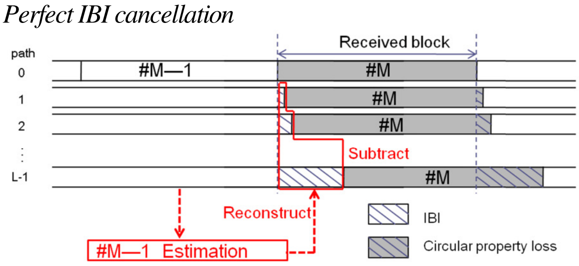


Fig. 4 Perfect IBI cancellation

To perform IBI cancellation, the estimation of the previous block $\#M-1$ is required. The IBI component is reconstructed based on Equation (6) and then subtracted from the target block $\#M$.

To observe the effect of perfect IBI cancellation, $\mathbf{s}_{-1}$, which is the previous block signal, is assumed to be perfectly restored. The channel estimation is also assumed to be perfect. Therefore, the IBI component can be perfectly cancelled by using $\mathbf{h}_{-1}$ and $\mathbf{s}_{-1}$.

B. Perfect circular property restoration

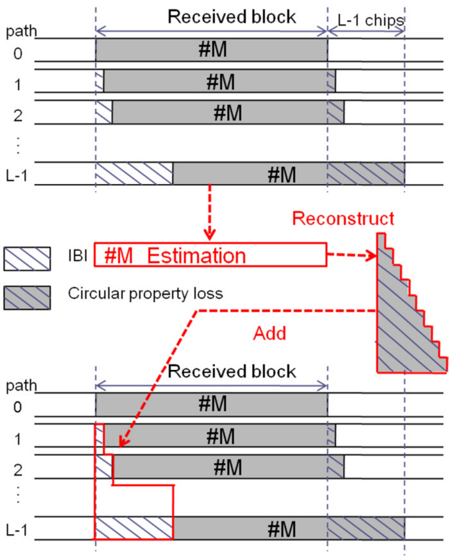


Fig. 5 Perfect circular property restoration

It can be seen from Figure 5 that, to perform circular property restoration, the estimation for the target block $\#M$ is required. The power loss component is reconstructed based on Equation (6) and then added to the head of the target block $\#M$.

Similarly, perfect $\mathbf{s}_0$ and $\mathbf{h}_{-1}$ will be used to observe the effect of perfect circular property restoration.

C. Imperfect circular property restoration [8]

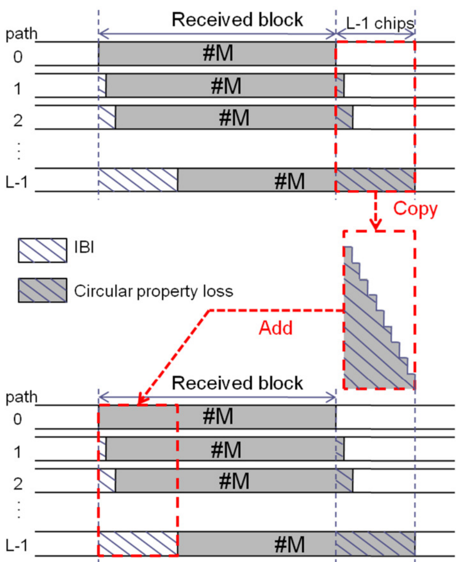


Fig. 6 Imperfect circular property restoration

Perfect $\mathbf{s}_0$ is used in Section B for perfect circular property loss. However, $\mathbf{s}_0$ is the target signal to be detected. In real case, it is unknown during the processing procedure. Therefore a scheme of imperfect circular property restoration is proposed, which will simply copy the head of the next block and add to the head of the target block, as shown in Figure 6.

After the IBI cancellation and circular property restoration, the circular property of the channel matrix $\mathbf{h}$ is recovered. The expression can be taken as a circular matrix,

$$\mathbf{h}=\begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ \vdots & \ddots & & & \ddots & \vdots \\ h_{L-2} & \vdots & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & \vdots & \vdots & h_0 & & \\ & \ddots & \vdots & \vdots & \ddots & \\ \mathbf{0} & & h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix}_{N_c\times N_c} . \tag{7}$$

Therefore, the received signal can be given as

$$\mathbf{r}=\mathbf{h}\mathbf{s}_0+\boldsymbol{\eta}\,, \tag{8}$$

MMSE-FDE is performed after the interference cancellation. The FDE is to apply N_c-point fast Fourier transform (FFT) to the signals. Frequency domain components are then equalized by weight $\{W(k)\}, k=0\sim N_c-1$, shown in Figure 7.

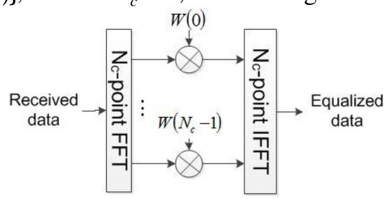


Fig.7 Frequency domain equalizer

By N_c-point FFT, the received signal $\mathbf{r}$ is transformed into the frequency domain signal $\mathbf{R}=[R(0),R(1),\cdots,R(N_c-1)]$, which can be given by

$$\mathbf{R}=\mathbf{F}\cdot\mathbf{r}=\mathbf{H}(\mathbf{F}\mathbf{s}_0)+\mathbf{F}\boldsymbol{\eta}\,, \tag{9}$$

where $\mathbf{H}=\mathbf{F}\mathbf{h}\mathbf{F}^H$, $\mathbf{F}$ is the $N_c\times N_c$ FFT matrix given as

$$\mathbf{F}=\frac{1}{\sqrt{N_c}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{1\cdot 1}{N_c}} & \cdots & e^{-j2\pi\frac{1\cdot(N_c-1)}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{(N_c-1)\cdot 1}{N_c}} & \cdots & e^{-j2\pi\frac{(N_c-1)(N_c-1)}{N_c}} \end{bmatrix}_{N_c\times N_c} . \tag{10}$$

Since $\mathbf{h}$ is a circular matrix, $\mathbf{H}$ becomes a diagonal matrix denoted as

$$\mathbf{H}=diag[H(0),\cdots,H(k),\cdots,H(N_c-1)], \tag{11}$$

where

$$H(k)=\sum_{l=0}^{L-1}h_l\exp\left(-j2\pi k\frac{\tau_l}{N_c}\right). \tag{12}$$

Then MMSE-FDE is applied to $\mathbf{R}$ so that $\hat{\mathbf{R}}=\mathbf{W}\cdot\mathbf{R}$, where $\mathbf{W}$ is the MMSE-FDE weight matrix. $\hat{\mathbf{R}}$ is given as

$$\hat{\mathbf{R}}=\mathbf{W}\mathbf{H}(\mathbf{F}\mathbf{s}_0)+\mathbf{W}\mathbf{F}\boldsymbol{\eta}\,, \tag{13}$$

According to the Wiener theory, for the given $\mathbf{h}$, $\mathbf{W}$ can be obtained as

$$\mathbf{W} = \mathbf{H}^H \left\{ \mathbf{H}\mathbf{H}^H + \left(U \frac{E_c}{N_0} \right)^{-1} \mathbf{I} \right\}^{-1}. \quad (14)$$

The right side of Equation (14) is a diagonal matrix, so $\mathbf{W}$ can also be expressed by a diagonal matrix as $\mathbf{W} = diag[W(0), \cdots, W(k), \cdots, W(N_c - 1)]$, resulting in one-tap MMSE-FDE. $W(k)$ is given by

$$W(k) = \frac{H^*(k)}{|H(k)|^2 + \left(U \frac{E_c}{N_0} \right)^{-1}}, \quad (15)$$

The frequency-domain signal $\hat{\mathbf{R}}$ after MMSE-FDE is transformed by an N_c-point inverse FFT (IFFT) back to the time-domain signal block as $\hat{\mathbf{r}} = \mathbf{F}^H \hat{\mathbf{R}}$. $\hat{\mathbf{r}}$ can be expressed as

$$\hat{\mathbf{r}} = \left(\frac{1}{N_c} tr[\mathbf{WH}] \right) \mathbf{s}_0 + \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\eta}}, \quad (16)$$

where the first term is the desired signal and, $\hat{\boldsymbol{\mu}}$, $\hat{\boldsymbol{\eta}}$ are the residual inter-chip interference (ICI) and noise component, respectively. $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\eta}}$ can be expressed as

$$\begin{cases} \hat{\boldsymbol{\mu}} = \left\{ \mathbf{F}^H (\mathbf{WH}) \mathbf{F} - \left(\frac{1}{N_c} tr[\mathbf{WH}] \right) \mathbf{I} \right\} \mathbf{s}_0 \\ \hat{\boldsymbol{\eta}} = -\mathbf{F}^H (\mathbf{WH}) \boldsymbol{\eta} \end{cases}. \quad (17)$$

Finally, the signal is de-spread, descrambled and decoded.

IV. CHANNEL MODEL

Channel estimation with Gaussian error is used to evaluate the effect of channel estimation accuracy on the IBI cancellation and circular property loss restoration.

The channel estimation of the impulse response series $\{h_l\}, l = 0 \sim L-1$ can be expressed as

$$\hat{h}_l = h_l + \Delta h_l, l = 0 \sim L-1. \quad (18)$$

where Δh_l is a Gaussian random variable with zero mean and variance of σ_e^2, $\Delta h_l \sim N(0, \sigma_e^2)$. Frequency domain channel response $\hat{\mathbf{H}}$ is then given by

$$\hat{\mathbf{H}} = diag[\hat{H}(0), \cdots, \hat{H}(k), \cdots, \hat{H}(N_c - 1)], \quad (19)$$

where

$$\hat{H}(k) = \sum_{l=0}^{L-1} \hat{h}_l \exp\left(-j2\pi k \frac{\tau_l}{N_c} \right). \quad (20)$$

V. SIMULATION RESULTS

A. Simulation parameters

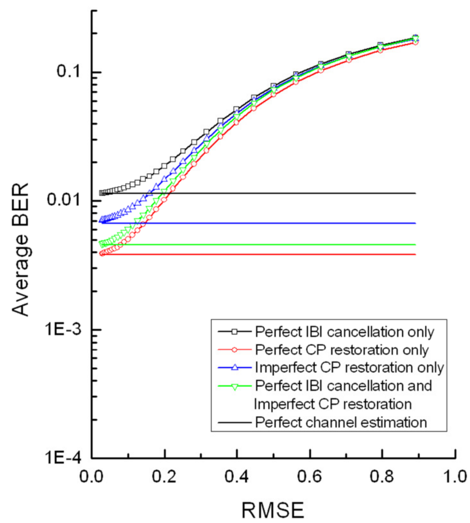
In this section, the performance of IBI cancellation and circular property loss restoration will be evaluated. The simulation parameters used are shown in table 1. QPSK transmission is assumed and the propagation channel is assumed to be a frequency-selective block Rayleigh fading channel, which means the channel remains unchanged during a block. The channel has a chip-spaced 16-path uniform power profile. Ideal channel estimation is assumed. Single user case is considered. The root mean squared error (RMSE) is used as the measurement of Gaussian error, namely σ_e. RMSE varies within a range of $[0,1]$.

Table 1 Simulation parameters

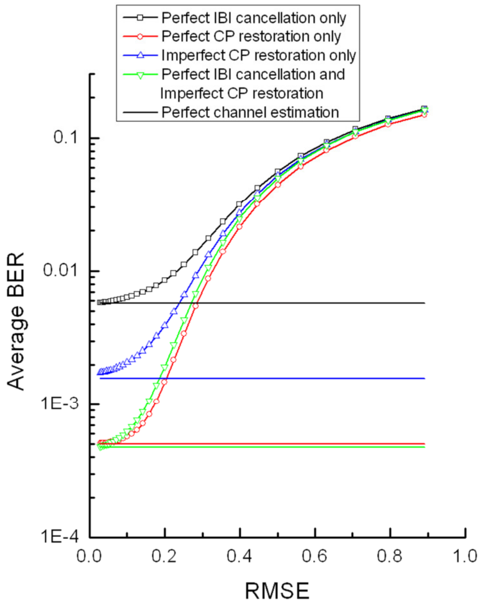
Transmitter	Data modulation	QPSK	
	Spreading sequence	Product of Walsh sequence and Long PN sequence	
	Spreading factor	SF=4	
	Number of user	Single user	
Channel model	Type	Frequency selective block Rayleigh fading	
	Number of path	L=16-path	
	Decay	Uniform power profile	
	Delay time	Chip-spaced	
Receiver	E_b/N_0	10, 20dB	
	Equalization	MMSE FDE	
	Channel estimation	Perfect CSI	Gaussian error
	RMSE	[0,1]	

B. Average BER performance affected by Gaussian error in channel estimation

Figure 8 shows the effect of channel estimation error on the average BER performance with $E_b / N_0 = 10dB$ and $E_b / N_0 = 20dB$, respectively. The solid lines are the results with perfect channel estimation. It can be observed that, in the region of higher channel estimation error ($RMSE \in [0.5,1]$), the BER performance for all the proposed schemes becomes poor and converges eventually. And the performance of block detection is mainly dominated by the channel estimation error. As the accuracy of the estimation improves, the BER performance becomes better. The superiority of circular property restoration over the IBI cancellation can still be observed; and when imperfect circular property restoration is combined with perfect IBI cancellation, a similar performance to perfect cyclic property loss restoration can be achieved, which is also the same conclusion as the perfect channel estimation case. When the channel estimation becomes more accurate, e.g., $RMSE < 0.1$, the curves are close to the perfect channel estimation case, which shows an almost-zero effect of channel estimation error.



(a) E_b/N_0=10dB



(b) E_b/N_0=20dB

Fig.8 Average BER performance vs RMSE

It can be learnt from the results that the proposed schemes are sensitive to the channel estimation error. Taking 10% BER increase from the perfect channel estimation case as an allowable error level, the corresponding Gaussian RMSEs for each scheme are listed in Table 2. From this table, it can be observed that perfect IBI cancellation has the best tolerance to the channel estimation error. While imperfect circular property restoration scheme exhibits the poorest robustness to the estimating error. However, if imperfect circular property restoration is combined with perfect IBI cancellation, the robustness improves obviously, it can be concluded that we should develop the scheme of imperfect circular restoration combining perfect IBI cancellation. Table 2 also shows the criterion that the practical channel estimation in the future work should satisfy to guarantee the performance of IBI cancellation and circular property restoration.

Table 2 Gaussian RMSE (10% BER increase)

E_b/N_0 (dB)	Perfect IBI	Perfect CP	Imperfect CP	Perfect IBI+ Imperfect CP
10	7.94%	5.62%	5.01%	5.62%
20	8.91%	8.91%	2.51%	6.30%

VI. Conclusion

In this paper, assuming the MMSE-FDE receiver, the impact of Gaussian error channel estimation on IBI cancellation and circular property restoration was discussed for broadband DS-CDMA uplink transmission using FDE without CP insertion. It was shown by computer simulation that the IBI cancellation is sensitive to the channel estimation error while the circular property restoration exhibits the poorest robustness against the estimating error. However, when combined with IBI cancellation, circular property restoration is more robust in the presence of channel estimation error. Therefore, we conclude that we should develop a combined IBI cancellation and circular property restoration scheme. Finally, a criterion to select a proper channel estimation method has also been discussed based on the simulation results.

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