Recursive QR Packet Combining for Uplink Single-Carrier Multi-User MIMO HARQ Using Near ML Detection

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***Abstract*—QR decomposition based near maximum likelihood (ML) block detection significantly improves the transmission performance of uplink single-carrier (SC) multi-user multiple-input multiple-output (MU-MIMO). Hybrid automatic repeat request (HARQ) is an indispensable error control technique for high quality packet data transmission. The achievable diversity gain of HARQ depends on the packet combining strategy. In uplink MU-MIMO HARQ, received signal may consist of new packets and retransmitted packets as retransmission for each user acts independently. In this paper, a recursive QR packet combining scheme suitable for uplink SC MU-MIMO HARQ is proposed, which takes into account the number of retransmissions for each user in the detection order. The proposed scheme helps reduce the computational complexity and storage requirement significantly. Moreover, it improves the packet error rate (PER) performance significantly over the conventional bit-level log likelihood ratio (LLR) packet combining, as shown by our computer simulation results.**

Keywords-component; Single-carrier, MU-MIMO, HARQ, packet combining, QR decomposition, M-algorithm

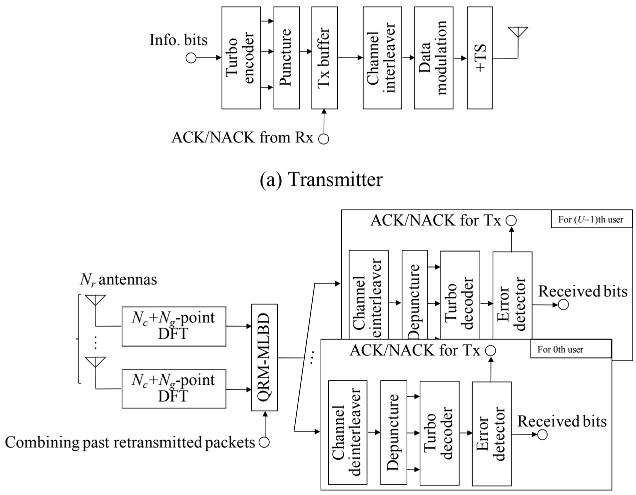
I. INTRODUCTION

In next generation mobile data communication systems, high data rate transmission is demanded. Multi-input multi-output (MIMO) spatial multiplexing [1] provides high data rate without increasing the signal bandwidth. For uplink (mobile terminal-to-base station (BS)) data communication, single-carrier (SC) transmission is suitable because of its lower peak-to-average power ration (PAPR) property [2, 3] compared to multi-carrier transmission, e.g., orthogonal frequency division multiplexing (OFDM) [4]. Hence, SC MIMO has been adopted as the uplink transmission technique for 3rd generation partnership project long term evolution (3GPP LTE) system [5].

Since the wireless channel becomes severely frequency-selective as the transmission data rate increases [6], SC MIMO suffers from inter-symbol interference (ISI) arising from the severe frequency-selectivity of the channel. QR decomposition and M-algorithm based near maximum likelihood (ML) block detection (QRM-MLBD) [7, 8] significantly improves the transmission performance of SC MIMO systems in a frequency-selective fading channel with significantly lower computational complexity compared to full ML detection. Multi-user (MU)-MIMO [9, 10] provides multiple users with high data rate without increasing the signal bandwidth. QRM-MLBD can be extended into the multi-user transmission environment and significantly improves the transmission performance of uplink SC MU-MIMO [11].

Packet access will be the core technology of the next generation mobile data communication systems. High-speed packet transmissions can be achieved by the joint use of MIMO systems and hybrid automatic repeat request (HARQ). Among several HARQ protocols, type I HARQ is relatively simple to implement as it requires less storage size [12]. In type I HARQ, if any error is detected in a received packet, the same packet is retransmitted until it is correctly received. The type I HARQ throughput is affected by a packet combining scheme as the achievable time-diversity gain depends on the combining scheme. The recursive QR packet combining [13] obtains the optimal packet combining gain as it optimally incorporates packet combining into QR decomposition. In [14], we proposed a near ML block detection with recursive QR packet combining and M-algorithm for single-user (SU)-MIMO. We showed that the recursive QR packet combining can significantly increase the throughput performance of SU-MIMO HARQ compared to the bit-level log likelihood ratio (LLR) packet combining [15]. Since a single channel encoder is assumed in [15], the received signal always consists of either new packet or retransmitted packet. On the other hand, in uplink MU-MIMO HARQ, retransmission for each user acts independently, hence the received signal may consist of new packets and retransmitted packets. How to perform the recursive QR packet combining for multi-user case is an important technical issue. In this paper, we present a recursive QR packet combining for SC MU-MIMO HARQ, which will reduce the computational complexity and storage requirement significantly over the conventional scheme. As HARQ is independently carried out among users, the received signal may consist of packets with different number of retransmission. In such a case, the packet combining gain differs among the transmitted signals. Hence, the detection order of transmitted signals may affect in the individual packet error rate (PER) performance. In this paper we will consider the effect of detection ordering between retransmitted and newly transmitted packets. We evaluate by computer simulation the PER performances of SC MU-MIMO HARQ with the QRM-MLBD using the recursive QR packet combining and show that significant performance improvement can be achieved by our proposed scheme, on top of the complexity and storage saving.

The rest of the paper is organized as follows. In Section II, system model of SC MU-MIMO HARQ is presented. Section III describes the recursive QRM-MLBD for MU-MIMO HARQ. In Section IV, we present the simulation results, and finally in Section V, we conclude the paper.



(a) Transmitter

(b) Receiver

Figure 1. System model.

II. SYSTEM MODEL

A. Transmission System

Figure 1 illustrates the system model of SC MU-MIMO HARQ with the recursive QRM-MLBD. In this paper, we consider SC block transmission with block size N_c and guard interval (GI) length N_g. The number of users is denoted by U, and each is equipped with a single transmit antenna. The BS is equipped with N_r receive antennas. In this paper, a synchronous slotted U-user MU-MIMO packet transmission is considered, where one packet is composed of multiple data blocks and the slot length is the same as the packet length. We assume that at every slot each user always has one packet to transmit.

All U users employ the type I HARQ. At the u-th user, u=0~(U−1), after channel coding and puncturing, the coded bit sequence is stored in the transmitter buffer. The coded bit sequence is transformed into a data-modulated symbol sequence, which is divided into a sequence of symbol blocks of N_c symbols each. The training sequence (TS) inserted SC block transmission [16, 17] is used instead of well-known cyclic prefix inserted SC (CP-SC) transmission as TS can reduce the detection complexity of QRM-MLBD [8]. Before the transmission, the TS of length N_g symbols is inserted into the GI placed at the end of each block. After the insertion of TS, the signal block is transmitted.

The transmit block of each user propagates through different channel. At the BS, a superposition of U users' transmitted blocks is received by N_r receive antennas. The received signal block is transformed by (N_c+N_g)-point discrete Fourier transform (DFT) into the frequency domain signal. The joint QRM-MLBD and packet combining is carried out to output LLR. Finally, channel decoding is performed by using the LLR output to recover each user's transmit data sequence. For a particular user, if a packet is received successfully, then a positive acknowledgment (ACK) signal is sent back to the user over an error- and delay-free feedback channel to request the new packet transmission. If any error is detected in a received packet, a negative acknowledgment (NACK) is sent back to request for retransmission of the same packet.

B. Transmit Signal and Received Signal Representations

A packet consists of coded bit sequence. The coded bit sequence within the packet is grouped in a sequence of N_c-symbol blocks. The data symbol block of the u-th user can be expressed using the vector form as $\mathbf{d}_u=[d_u(0),\ldots,d_u(t),\ldots,d_u(N_c-1)]^T$, where $(.)^T$ expresses the transposition. After TS insertion, the block $\mathbf{s}_u=[s_u(0),\ldots,s_u(t),\ldots,s_u(N_c+N_g-1)]^T$ to be transmitted is expressed as

$$\mathbf{s}_u=[d_u(0),\ldots,d_u(N_c-1),x_u(0),\ldots,x_u(N_g-1)]^T \\ =\begin{bmatrix}\mathbf{d}_u\\ \mathbf{x}_u\end{bmatrix}, \quad (1)$$

where $\mathbf{x}_u=[x_u(0),\ldots,x_u(t),\ldots,x_u(N_g-1)]^T$ denotes the TS vector which is identical for all blocks.

Let us consider the p-th slot. Suppose the u-th user transmits the n_u-th packet. The frequency-domain received signal vector at the n_r-th receive antenna $\mathbf{Y}_{n_r}^{(p)}=[Y_{n_r}^{(p)}(0),\ldots,Y_{n_r}^{(p)}(k),\ldots,Y_{n_r}^{(p)}(N_c+N_g-1)]^T$, n_r=0~(N_r−1), after (N_c+N_g)-point DFT is expressed as [8]

$$\mathbf{Y}_{n_r}^{(p)}=\sum_{u=0}^{U-1}\sqrt{2S_u}\mathbf{H}_{u,n_r}^{(p)}\mathbf{F}\mathbf{s}_u^{(n_u)}+\mathbf{N}_{n_r}^{(p)}, \quad (2)$$

where S_u is the average received signal power of the u-th user, $\mathbf{F}$ is the DFT matrix of size $(N_c+N_g)\times(N_c+N_g)$ [8], $\mathbf{s}_u^{(n_u)}$ is the u-th user's data block of the n_u-th packet, and $\mathbf{N}_{n_r}^{(p)}$ is the $(N_c+N_g)\times 1$ noise vector whose element is the zero-mean additive white Gaussian noise (AWGN) having the variance $2\sigma^2$. $\mathbf{H}_{u,n_r}^{(p)}$ is the frequency-domain channel matrix between the u-th user and the n_r-th receive antenna in the p-th slot [8].

From Eq. (2), the $N_r(N_c+N_g)\times 1$ overall frequency-domain received signal $\mathbf{Y}^{(p)}=[\{\mathbf{Y}_0^{(p)}\}^T,\ldots,\{\mathbf{Y}_{N_r-1}^{(p)}\}^T]^T$ is given by

$$\mathbf{Y}^{(p)}=\mathbf{G}^{(p)}\begin{bmatrix}\mathbf{s}_0^{(n_0)}\\ \vdots\\ \mathbf{s}_{U-1}^{(n_0)}\end{bmatrix}+\mathbf{N}^{(p)}, \quad (3)$$

where $\mathbf{G}^{(p)}$ is an equivalent channel matrix of size $N_r(N_c+N_g)\times U(N_c+N_g)$, which is a concatenation of the space and frequency-domain channel and DFT given by

$$\mathbf{G}^{(p)}=\begin{bmatrix}\sqrt{2S_0}\mathbf{H}_{0,0} & \cdots & \sqrt{2S_{U-1}}\mathbf{H}_{U-1,0}\\ \vdots & \ddots & \vdots\\ \sqrt{2S_0}\mathbf{H}_{0,N_r-1} & \cdots & \sqrt{2S_{U-1}}\mathbf{H}_{U-1,N_r-1}\end{bmatrix}\begin{bmatrix}\mathbf{F} & & \\ & \ddots & \\ & & \mathbf{F}\end{bmatrix}. \quad (4)$$

$\mathbf{N}^{(p)}$ is the $N_r(N_c+N_g)\times 1$ overall noise vector.

III. RECURSIVE QR PACKET COBINING FOR MU-MIMO HARQ

In this section, the recursive QR packet combining for SC MU-MIMO HARQ is presented. For simplicity, only two users (U=2) are assumed (The approach, however, can be easily extended to systems with more than two users). Two cases are considered: Case 1 when both users retransmit same packet P

times, and Case 2 when one user retransmits same packet P times and the other user transmits a new packet.

A. Case 1

In this case, both users retransmit same packet P times. In this case, the recursive QR packet combining can be derived in a similar way to the SU-MIMO system [14].

Suppose that the u-th user, u=0, 1 transmits the 0th packet in the p-th slot, p=0~P. The overall frequency-domain received signal vector in the p-th slot $\mathbf{Y}^{(p)}$ is expressed as

$$\mathbf{Y}^{(p)} = \mathbf{G}^{(p)}\begin{bmatrix}\mathbf{s}_0^{(0)}\\ \mathbf{s}_1^{(0)}\end{bmatrix} + \mathbf{N}^{(p)}. \tag{5}$$

1) Expanded QR Packet Combining

First, let us define the expanded received signal vector $\overline{\mathbf{Y}}^{(P)}$ of size $(P+1)N_r(N_c+N_g)\times 1$ as

$$\begin{aligned}\overline{\mathbf{Y}}^{(P)} &= \begin{bmatrix}\mathbf{Y}^{(0)}\\ \vdots\\ \mathbf{Y}^{(P)}\end{bmatrix} = \begin{bmatrix}\mathbf{G}^{(0)}\\ \vdots\\ \mathbf{G}^{(P)}\end{bmatrix}\begin{bmatrix}\mathbf{s}_0^{(0)}\\ \mathbf{s}_1^{(0)}\end{bmatrix} + \begin{bmatrix}\mathbf{N}^{(0)}\\ \vdots\\ \mathbf{N}^{(P)}\end{bmatrix},\\ &= \overline{\mathbf{G}}^{(P)}\begin{bmatrix}\mathbf{s}_0^{(0)}\\ \mathbf{s}_1^{(0)}\end{bmatrix} + \overline{\mathbf{N}}^{(P)}\end{aligned} \tag{6}$$

where $\overline{\mathbf{G}}^{(P)}$ is the expanded channel matrix of size $(P+1)N_r(N_c+N_g)\times U(N_c+N_g)$.

To achieve the optimal packet combining gain, expanded packet combining [14], which is a straightforward application of the QR decomposition to all the received retransmitted packets can be used. In expanded QR packet combining, QR decomposition is applied to the expanded channel matrix $\overline{\mathbf{G}}^{(P)}$ to obtain $\overline{\mathbf{G}}^{(P)} = \overline{\mathbf{Q}}^{(P)}\overline{\mathbf{R}}^{(P)}$, where $\overline{\mathbf{Q}}^{(P)}$ is a $(P+1)N_r(N_c+N_g)\times U(N_c+N_g)$ unitary matrix and $\overline{\mathbf{R}}^{(P)}$ is an $U(N_c+N_g)\times U(N_c+N_g)$ upper triangular matrix. The transformed received signal $\hat{\mathbf{Y}}^{(P)}$ is obtained as

$$\begin{aligned}\hat{\mathbf{Y}} &= \{\overline{\mathbf{Q}}^{(P)}\}^H\overline{\mathbf{Y}}^{(P)}\\ &= \overline{\mathbf{R}}^{(P)}\begin{bmatrix}\mathbf{s}_0^{(0)}\\ \mathbf{s}_1^{(0)}\end{bmatrix} + \{\overline{\mathbf{Q}}^{(P)}\}^H\overline{\mathbf{N}}^{(P)}.\end{aligned} \tag{7}$$

From Eq. (7), the ML solution is equivalent to the selection of the path with the minimum Euclidean distance in the tree diagram which is composed of $U(N_c+N_g)$ stages. In each stage, the best M surviving paths are selected based on the squared Euclidean distance from all the paths and are passed to the next stage. The obtained bit LLRs are used as the input to the channel decoder. When M-algorithm is used, however, the LLR values cannot be directly computed since surviving paths at the last stage may not contain both 1 and 0 for every coded bit. In this paper, we applied the LLR computation method proposed in [18].

Although this expanded QR packet combining can achieve the optimal packet combining gain, it needs to store all the retransmitted packets and the associated channel matrices and therefore, the memory size of the receiver increases. Furthermore, the size of the expanded channel matrix to which the QR decomposition is applied becomes $(P+1)N_r(N_c+N_g)\times U(N_c+N_g)$ and therefore, the computational complexities required for the QR decomposition and generation of the transformed signal grow with the number of retransmissions.

2) Recursive QR Packet Combining

The recursive QR packet combining is derived below. The expanded channel matrix in the Pth slot $\overline{\mathbf{G}}^{(P)}$ can be expressed by using $\overline{\mathbf{G}}^{(P-1)}$ and $\mathbf{G}^{(P)}$ as

$$\overline{\mathbf{G}}^{(P)} = \begin{bmatrix}\overline{\mathbf{G}}^{(P-1)}\\ \mathbf{G}^{(P)}\end{bmatrix}. \tag{8}$$

Furthermore, $\overline{\mathbf{G}}^{(P)}$ can be modified, using the relationship $\overline{\mathbf{G}}^{(P-1)} = \overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}^{(P-1)}$, as

$$\overline{\mathbf{G}}^{(P)} = \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{G}^{(P)}\end{bmatrix} = \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{G}^{(P)}\end{bmatrix}, \tag{9}$$

where $\mathbf{I}_A$ is the $A\times A$ identity matrix.

Applying QR decomposition to $\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{G}^{(P)}\end{bmatrix}$ in Eq. (9), we have

$$\begin{bmatrix}\overline{\mathbf{R}}^{(P-1)}\\ \mathbf{G}^{(P)}\end{bmatrix} = \mathbf{Q}^{(P)}\mathbf{R}^{(P)}. \tag{10}$$

Now, $\mathbf{Q}^{(P)}$ is an $(N_r(N_c+N_g)+U(N_c+N_g))\times U(N_c+N_g)$ unitary matrix and $\mathbf{R}^{(P)}$ is a $U(N_c+N_g)\times U(N_c+N_g)$ upper triangular matrix. Finally, from Eqs. (8)-(10), we have

$$\overline{\mathbf{G}}^{(P)} = \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\mathbf{Q}^{(P)}\mathbf{R}^{(P)}. \tag{11}$$

Since $\mathbf{Q}^{(P)}$ and $\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}$ are both unitary, $\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\mathbf{Q}^{(P)}$ is also unitary. Therefore, the expanded received signal vector $\overline{\mathbf{Y}}^{(P)}$ of Eq. (6) can be rewritten as

$$\begin{aligned}\hat{\mathbf{Y}}^{(P)} &= \left\{\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\mathbf{Q}^{(P)}\right\}^H\begin{bmatrix}\overline{\mathbf{Y}}^{(P-1)}\\ \mathbf{Y}^{(P)}\end{bmatrix}\\ &= \{\mathbf{Q}^{(P)}\}^H\begin{bmatrix}\{\overline{\mathbf{Q}}^{(P-1)}\}^H\overline{\mathbf{Y}}^{(P-1)}\\ \mathbf{Y}^{(P)}\end{bmatrix} = \{\mathbf{Q}^{(P)}\}^H\begin{bmatrix}\hat{\mathbf{Y}}^{(P-1)}\\ \mathbf{Y}^{(P)}\end{bmatrix},\end{aligned} \tag{12}$$

which implies that the transformed signal vector $\hat{\mathbf{Y}}^{(P)}$ in Eq. (6) can be obtained recursively using the previously obtained transformed vector $\hat{\mathbf{Y}}^{(P-1)}$ and the present overall frequency-domain received signal vector $\mathbf{Y}^{(P)}$. Only the previously obtained upper triangular matrix and the present channel matrix are required for the QR decomposition.

The memory size can be reduced because this recursive QR packet combining does not need to store all of the received retransmitted packets and equivalent channel matrices unlike the expanded QR packet combining. The computational complexity of QR decomposition can also be reduced compared to the expanded QR packet combining when the number of retransmissions is more than two. The size of a matrix to which the QR decomposition is applied is always $(N_r(N_c+N_g)+U(N_c+N_g))\times U(N_c+N_g)$ irrespective of the number of retransmissions when $P\geq 1$. It should be noted that this recursive QR packet combining obtains the same time diversity gain as the expanded QR packet combining without the increase of the memory size and complexity. After the recursive packet combining, M-algorithm is performed.

B. Case 2

Let us suppose that the 0-th user retransmits the same packet P times while the 1-st user transmits a new packet in the P-th slot.

The overall frequency-domain received signal vector in the p-th slot, p=0~P–1, is the same as Eq. (5) while in the P-th slot $\mathbf{Y}^{(P)}$ is expressed as

$$\mathbf{Y}^{(P)} = \mathbf{G}^{(P)}\begin{bmatrix}\mathbf{s}_0^{(0)} \\ \mathbf{s}_1^{(1)}\end{bmatrix} + \mathbf{N}^{(P)}. \tag{13}$$

The expanded received signal vector $\overline{\mathbf{Y}}^{(P)}$ can be given by

$$\overline{\mathbf{Y}}^{(P)} = \begin{bmatrix}\mathbf{Y}^{(0)} \\ \vdots \\ \mathbf{Y}^{(P-1)} \\ \mathbf{Y}^{(P)}\end{bmatrix} = \begin{bmatrix}\mathbf{G}_0^{(0)} & \mathbf{G}_1^{(0)} & \mathbf{0} \\ \vdots & \vdots & \vdots \\ \mathbf{G}_0^{(P-1)} & \mathbf{G}_1^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P-1)} & \mathbf{0} & \mathbf{G}_1^{(P)}\end{bmatrix}\begin{bmatrix}\mathbf{s}_0^{(0)} \\ \mathbf{s}_1^{(0)} \\ \mathbf{s}_1^{(1)}\end{bmatrix} + \begin{bmatrix}\mathbf{N}^{(0)} \\ \vdots \\ \mathbf{N}^{(P)}\end{bmatrix}, \tag{14}$$

where $\mathbf{G}^{(p)}$ is partitioned into the $N_r(N_c+N_g)\times(N_c+N_g)$ matrices $\mathbf{G}_u^{(p)}$, which corresponds to the u-th user's equivalent channel matrix.

Since $\mathbf{s}_1^{(0)}$ was successfully received, $\mathbf{s}_1^{(0)}$ can be cancelled as

$$\widetilde{\mathbf{Y}}^{(P)} = \overline{\mathbf{Y}}^{(P)} - \begin{bmatrix}\mathbf{G}_1^{(0)} \\ \vdots \\ \mathbf{G}_1^{(P-1)} \\ \mathbf{0}\end{bmatrix}\mathbf{s}_1^{(0)} = \begin{bmatrix}\mathbf{G}_0^{(0)} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{G}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix}\begin{bmatrix}\mathbf{s}_0^{(0)} \\ \mathbf{s}_1^{(1)}\end{bmatrix} + \begin{bmatrix}\mathbf{N}^{(0)} \\ \vdots \\ \mathbf{N}^{(P)}\end{bmatrix}. \tag{15}$$

The recursive QR packet combining is derived below. The QR decomposition in the (P–1)-th slot is given by

$$\begin{aligned}\overline{\mathbf{G}}^{(P-1)} &= \begin{bmatrix}\overline{\mathbf{G}}_0^{(P-1)} & \overline{\mathbf{G}}_1^{(P-1)}\end{bmatrix} \\ &= \overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}^{(P-1)} = \overline{\mathbf{Q}}^{(P-1)}\begin{bmatrix}\overline{\mathbf{R}}_0^{(P-1)} & \overline{\mathbf{R}}_1^{(P-1)}\end{bmatrix},\end{aligned} \tag{16}$$

where $\overline{\mathbf{G}}^{(p)}$ is partitioned into the $pN_r(N_c+N_g)\times(N_c+N_g)$ matrices $\overline{\mathbf{G}}_u^{(p)}$ and $\overline{\mathbf{R}}^{(p)}$ is partitioned into the $U(N_c+N_g)\times(N_c+N_g)$ matrices $\overline{\mathbf{R}}_u^{(p)}$. From Eq. (16), the expanded channel matrix in the Pth slot $\overline{\mathbf{G}}^{(P)}$ can be expressed by using $\overline{\mathbf{G}}_0^{(p)}$ and $\mathbf{G}_u^{(P)}$ as

$$\begin{aligned}\overline{\mathbf{G}}^{(P)} &= \begin{bmatrix}\mathbf{G}_0^{(0)} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{G}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix} = \begin{bmatrix}\overline{\mathbf{G}}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix} \\ &= \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix} \\ &= \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{R}}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix}\end{aligned}. \tag{17}$$

Applying QR decomposition to $\begin{bmatrix}\overline{\mathbf{R}}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix}$ in Eq. (17) as

$$\begin{bmatrix}\overline{\mathbf{R}}_0^{(P-1)} & \mathbf{0} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)}\end{bmatrix} = \mathbf{Q}^{(P)}\mathbf{R}^{(P)}. \tag{18}$$

Finally, from Eqs. (17) and (18), we have

$$\overline{\mathbf{G}}^{(P)} = \begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\mathbf{Q}^{(P)}\mathbf{R}^{(P)}. \tag{19}$$

The expanded received signal vector after the cancellation of successfully received data $\widetilde{\mathbf{Y}}^{(P)}$ of Eq. (15) can be transformed as

$$\begin{aligned}\hat{\mathbf{Y}}^{(P)} &= \left\{\begin{bmatrix}\overline{\mathbf{Q}}^{(P-1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_r(N_c+N_g)}\end{bmatrix}\mathbf{Q}^{(P)}\right\}^H \widetilde{\mathbf{Y}}^{(P)} \\ &= \{\mathbf{Q}^{(P)}\}^H\begin{bmatrix}\{\overline{\mathbf{Q}}^{(P-1)}\}^H(\overline{\mathbf{Y}}^{(P-1)} - \overline{\mathbf{G}}_1^{(P-1)}\mathbf{s}_1^{(0)}) \\ \mathbf{Y}^{(P)}\end{bmatrix} \\ &= \{\mathbf{Q}^{(P)}\}^H\begin{bmatrix}\hat{\mathbf{Y}}^{(P-1)} - \overline{\mathbf{R}}_1^{(P-1)}\mathbf{s}_1^{(0)} \\ \mathbf{Y}^{(P)}\end{bmatrix} = \mathbf{R}^{(P)}\begin{bmatrix}\mathbf{s}_0^{(0)} \\ \mathbf{s}_1^{(1)}\end{bmatrix}\end{aligned}. \tag{20}$$

It can be understood from Eq. (20) that in Case 2, the transformed signal vector $\hat{\mathbf{Y}}^{(P)}$ can also be obtained recursively using $\hat{\mathbf{Y}}^{(P-1)}$ and $\overline{\mathbf{R}}^{(P-1)}$ from the previous time slot, and, the received signal vector $\mathbf{Y}^{(P)}$, and the channel matrix $\mathbf{G}^{(P)}$ from the present slot.

C. Detection Ordering

In the previous subsection, we assumed that the transmit symbols are ordered so that new packet is detected first. However, this order can be arbitrarily changed at the receiver by swapping the corresponding columns in matrix $\overline{\mathbf{G}}^{(p)}$. In M-algorithm, the path selection at early stages significantly affects the performance of QRM-MLBD as the M-algorithm successively reduces the number of paths stage-by-stage from the symbols located at the bottom of the transmit symbol vector. It can be expected that better performance can be achieved by detecting the combined packet first since combined packets can obtain higher time diversity order. Therefore, we introduce the detection ordering in which the combined packet is detected first.

Ordering can be applied to the expanded received signal vector after the cancellation of successfully received data $\tilde{\mathbf{Y}}^{(P)}$ of Eq. (15) as

$$\tilde{\mathbf{Y}}^{(P)} = \begin{bmatrix} & \overline{\mathbf{G}}_0^{(P-1)} \\ \mathbf{G}_1^{(P)} & \mathbf{G}_0^{(P)} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{(1)} \\ \mathbf{s}_0^{(0)} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(0)} \\ \vdots \\ \mathbf{N}^{(P)} \end{bmatrix}. \qquad (21)$$

Similar to the Eq. (17), the expanded channel matrix in the Pth slot $\overline{\mathbf{G}}^{(P)}$ can be modified as

$$\begin{aligned} \overline{\mathbf{G}}^{(P)} &= \begin{bmatrix} \mathbf{0} & \overline{\mathbf{G}}_0^{(P-1)} \\ \mathbf{G}_1^{(P)} & \mathbf{G}_0^{(P)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \overline{\mathbf{Q}}^{(P-1)}\overline{\mathbf{R}}_0^{(P-1)} \\ \mathbf{G}_0^{(P)} & \mathbf{G}_1^{(P)} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0} & \overline{\mathbf{Q}}^{(P-1)} \\ \mathbf{I}_{N_r(N_c+N_g)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{G}_1^{(P)} & \mathbf{G}_0^{(P)} \\ \mathbf{0} & \overline{\mathbf{R}}_0^{(P-1)} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0} & \overline{\mathbf{Q}}^{(P-1)} \\ \mathbf{I}_{N_r(N_c+N_g)} & \mathbf{0} \end{bmatrix} \mathbf{Q}^{(P)}\mathbf{R}^{(P)} \end{aligned} \quad . \qquad (22)$$

The expanded received signal vector after the cancellation of successfully received data $\tilde{\mathbf{Y}}^{(P)}$ of Eq. (21) can be transformed as

$$\begin{aligned} \hat{\mathbf{Y}}^{(P)} &= \left\{ \begin{bmatrix} \mathbf{0} & \overline{\mathbf{Q}}^{(P-1)} \\ \mathbf{I}_{N_r(N_c+N_g)} & \mathbf{0} \end{bmatrix} \mathbf{Q}^{(P)} \right\}^H \tilde{\mathbf{Y}}^{(P)} \\ &= \{\mathbf{Q}^{(P)}\}^H \begin{bmatrix} \mathbf{Y}^{(P)} \\ \hat{\mathbf{Y}}^{(P-1)} - \overline{\mathbf{R}}_1^{(P-1)}\mathbf{s}_1^{(0)} \end{bmatrix} = \mathbf{R}^{(P)} \begin{bmatrix} \mathbf{s}_1^{(1)} \\ \mathbf{s}_0^{(0)} \end{bmatrix} \end{aligned} . \qquad (23)$$

TABLE I. COMPUTER SIMULATION CONDITION

Transmitter	Data modulation	16QAM
	No. of users	U=2
	Data symbol block length	N_c=64
	TS lengths	N_g=16
Turbo coding	Packet size	2048
	Encoder	(13, 15) RSC encoders
	Decoder	Log-MAP decoding with 6 iterations
	Coding rate	R=3/4
HARQ	Type	Type I
Channel	Fading type	Frequency-selective block Rayleigh
	Power delay profile	L=16-path uniform power delay profile
Receiver	No. of receive antennas	N_r=2
	Channel estimation	Ideal

IV. COMPUTER SIMULATION

The performance of SC MU-MIMO HARQ using the recursive QRM-MLBD is evaluated by computer simulation. The simulation condition is summarized in Table I. 16QAM is assumed for data modulation. We assume U=2, N_r=2, N_c=64, N_g=16, and L=16-path frequency-selective block Rayleigh fading channel with uniform power delay profile. Independent channel is assumed for each retransmission. The ideal channel estimation is assumed at the receiver. A rate 1/3 turbo encoder using two (13, 15) recursive systematic convolutional (RSC) component encoders is used for channel encoder. The two parity sequences from the turbo encoder are punctured to obtain rate-3/4 turbo codes. Log-MAP decoding with 6 iterations is used. The packet size is set to 2048 bits. An error-free ACK/NACK feedback is assumed from the receiver to each user. Slow transmit power control is considered such that the average received powers of all the users are assumed to be the same.

The average PER performance of SC MU-MIMO HARQ achieved by the QRM-MLBD with the recursive QR packet combining is plotted as a function of the average received E_s/N_0. The number M of surviving paths in M-algorithm is set to M=4, 16, and 64. The same packet is assumed to be transmitted P=2 times. The PER performance achieved by the conventional QRM-MLBD with bit-level LLR combining is also plotted for comparison. As in Fig. 2, the recursive QR packet combining can achieve significantly better PER performance compared to the bit-level LLR combing. This is because the optimal packet combing gain can be obtained. The recursive QR packet combining can reduce the required E_s/N_0 to achieve PER=10^{-2} by about 5.5dB for M=4 compared to the bit-level LLR combining. Fig. 2 also shows that the achievable PER of SC MU-MIMO with the QRM-MLBD using the recursive QR packet combining is almost the same regardless of the value of M. This is because large packet combining gain can be achieved. As a result, the probability of removing the correct path at early stages can be reduced. This implies that in the retransmission, the receiver can reduce the value of M.

Figure 3 shows the average PER performance achieved by the QRM-MLBD using the recursive QR packet combining with M=16 for case 2 (One user retransmits the same packet and another user transmits a new packet). Two detection ordering schemes are considered; one is the combined packet is detected first and the other is the new packet is detected first. It can be seen from Fig. 3 that for new packet, better PER performance can be achieved by detecting the combined packet first. This is because the probability of correct detection is increased since the new packet is located at the later stage of the M-algorithm. On the other hand, for combined packet, no performance difference among ordering schemes exists. This is because packet combining gain can be achieved.

Figure 4 shows the comparison between the recursive QR packet combining and the bit-level LLR combining for case 2. The same packet is assumed to have been transmitted P=2 times. It can be seen from Fig. 4 that the recursive QR packet combining provides significantly better PER performance than the bit-level LLR combing for both new packet and combined packet. The recursive QR packet combining can reduce the required E_s/N_0 to achieve PER=10^{-2} by about 4.5dB and 3.5dB for combined packet and new packet, respectively.

V. CONCULUSION

In this paper, we presented the QRM-MLBD with the recursive QR packet combining for MU-MIMO HARQ. The recursive QR packet combining for MU-MIMO HARQ, where the received signal may consist of both new packet and retransmitted packet was derived. In the case when the received signal consists of new packet and retransmitted packet, we considered two detection ordering, in which the retransmitted packet is detected first. We showed by computer simulation that when received signal consists of both new packet and re-transmitted packet, detecting the retransmitted packet before the new packets improves the PER performance of new packets. We also showed that the recursive QR packet combining provides significantly better PER performance than the bit-level LLR combing.

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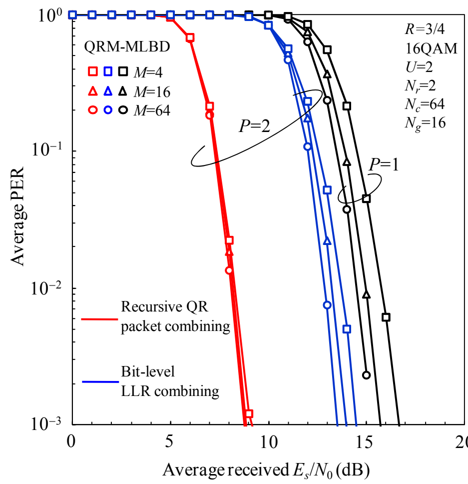

Figure 2. Average PER performance (Case 1).

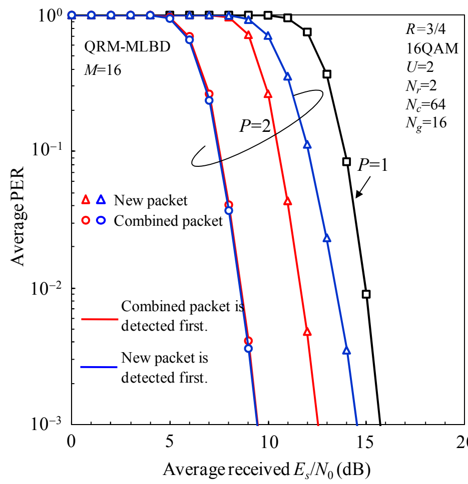

Figure 3. Comparison of ordering schemes (Case 2).

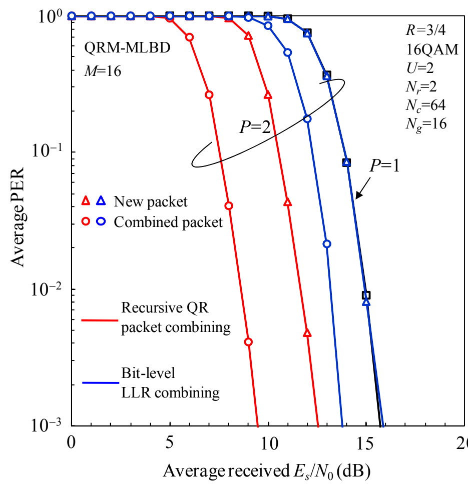

Figure 4. Comparison between recursive QR packet combining and the bit-level LLR combining (Case 2).