

2-Step Frequency-Domain Channel Estimation for Training Sequence Inserted Single-Carrier Block Transmission

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Abstract—In this paper, we propose a 2-step frequency-domain channel estimation scheme suitable for training sequence inserted single-carrier (TS-SC) block transmission using frequency-domain equalization (FDE). In the first step, the received training sequence having cyclic property is constructed for frequency-domain channel estimation to obtain the instantaneous channel estimate. Improved channel estimate is obtained by simply averaging the instantaneous channel estimates over several blocks. In the second step, the maximum likelihood channel estimation is carried out iteratively by using both the estimated symbol sequence and the training sequence. The BER performance with the proposed frequency-domain channel estimation scheme is evaluated by computer simulation. It is shown that the proposed channel estimation scheme achieves a BER performance close to the perfect channel estimation.

Keywords—component; Single-carrier, frequency-domain equalization, training sequence, channel estimation

I. INTRODUCTION

Since the mobile wireless channel is composed of many propagation paths with different time delays, the channel becomes severely frequency-selective for broadband data transmissions. The bit error rate (BER) performance of single-carrier (SC) transmission significantly degrades due to intersymbol interference (ISI) [1]. In a frequency-selective fading channel, one-tap frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can improve the BER performance of cyclic prefix inserted SC (CP-SC) block transmission [2, 3].

CP-SC is a block transmission. Instead of CP insertion, a known training sequence insertion [4-6] can be used. The training sequence in the previous block acts as the CP in the present block. Furthermore, the training sequence can be utilized for channel estimation and therefore, no pilot block is needed unlike CP-SC block transmissions. For MMSE-FDE, accurate estimation of channel transfer function is necessary. Some studies on channel estimation for training sequence inserted SC (TS-SC) block transmissions are found in [5, 7-8]. By using a training sequence of length equal to $2L$ symbols, where L denotes the channel length, the channel can be estimated by exploiting the circular property of the training sequence. However, the transmission efficiency reduces.

In this paper, we propose a 2-step frequency-domain channel estimation scheme which requires a training sequence of length equal to L symbols. In the first step, the received training sequence having cyclic property is constructed for frequen-

cy-domain channel estimation to obtain the instantaneous channel estimate. Improved channel estimate is obtained by simply averaging the instantaneous channel estimates over several blocks. Noise power is also estimated in the frequency-domain. In the second step, the maximum likelihood channel estimation [9] is carried out iteratively by using both the estimated symbol sequence and the training sequence.

The remainder of this paper is organized as follows. In Sect. II, transmission system model of TS-SC with MMSE-FDE is presented. The proposed channel estimation scheme is present in Sect. III. In Sect. IV, the BER performance with the proposed frequency-domain channel estimation scheme is evaluated by computer simulation. Sect. V offers some concluding remarks.

II. TS-SC BLOCK TRANSMISSION WITH MMSE-FDE

A. Transmission System Model

Transmission system model of TS-SC using MMSE-FDE is illustrated in Fig. 1. Throughout the paper, symbol-spaced discrete time representation is used. At the transmitter, information bit sequence is transformed into a data-modulated symbol sequence. Then, the data-modulated symbol sequence is divided into a sequence of symbol blocks of N_c symbols each. The m -th data symbol block is expressed using the vector form as $\mathbf{d}^{(m)} = [d^{(m)}(0), \dots, d^{(m)}(n), \dots, d^{(m)}(N_c-1)]^T$, where $(\cdot)^T$ expresses the transposition. Before the transmission, a training sequence of length N_g ($\geq L$) symbols is appended at the end of each block. The m -th block $\mathbf{s}^{(m)} = [s^{(m)}(0), \dots, s^{(m)}(n), \dots, s^{(m)}(N_c+N_g-1)]^T$ to be transmitted is expressed using the vector form as

$$\mathbf{s}^{(m)} = \begin{bmatrix} \mathbf{d}^{(m)} \\ \mathbf{u} \end{bmatrix}, \quad (1)$$

where $\mathbf{u} = [u(0), \dots, u(n), \dots, u(N_g-1)]^T$ denotes the training sequence vector which is identical for all blocks. TS-SC block structure is illustrated in Fig. 2. The difference from CP-SC block transmission is that CP is replaced by training sequence. In order to let a training sequence to play the role of CP, block size for discrete Fourier transform (DFT) must be N_c+N_g symbols for TS-SC block transmission.

The signal block is transmitted over a frequency-selective fading channel. Channel estimation is carried out over N_B received symbol blocks, during which the channel gains are assumed to stay constant. The received signal is transformed by N_c+N_g -point DFT into the frequency-domain signal and then,

MMSE-FDE is carried out. After MMSE-FDE, inverse DFT (IDFT) is applied to obtain the time-domain received signal block for data de-modulation.

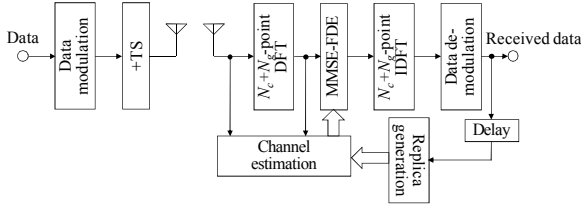


Figure 1. System model of TS-SC block transmission using MMSE-FDE.

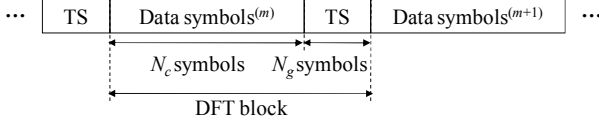


Figure 2. Block structure.

B. Signal Representation

The propagation channel is assumed to be a frequency-selective fading channel composed of L distinct symbol-spaced propagation paths. The channel impulse response $h(\tau)$ is given by

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \quad (2)$$

where h_l and τ_l are respectively the complex-valued path gain with $E[\sum_{l=0}^{L-1} |h_l|^2] = 1$ and the time delay of the l -th path. We assume the l -th path has a time delay of l symbols, i.e. ($\tau_l = l$). The m -th received signal block $\mathbf{y}^{(m)} = [y^{(m)}(0), \dots, y^{(m)}(n), \dots, y^{(m)}(N_c + N_g - 1)]^T$ can be expressed using the vector form as

$$\mathbf{y}^{(m)} = \sqrt{2S} \mathbf{h} \mathbf{s}^{(m)} + \mathbf{z}^{(m)}, \quad (3)$$

where S is the average received signal power, \mathbf{h} is the $(N_c + N_g) \times (N_c + N_g)$ channel impulse response matrix given as

$$\mathbf{h} = \begin{bmatrix} h_0 & & & & h_{L-1} & \cdots & h_1 \\ h_1 & h_0 & & \mathbf{0} & & \ddots & \vdots \\ \vdots & h_1 & \ddots & & & & h_{L-1} \\ h_{L-1} & \vdots & \ddots & \ddots & & & \\ & h_{L-1} & \ddots & \ddots & \ddots & & \\ \mathbf{0} & & h_{L-1} & \cdots & h_1 & h_0 \end{bmatrix} \quad (4)$$

and $\mathbf{z}^{(m)} = [z^{(m)}(0), \dots, z^{(m)}(n), \dots, z^{(m)}(N_c + N_g - 1)]^T$ is the noise vector. The n -th element, $z^{(m)}(n)$, of \mathbf{z} is the zero-mean additive white Gaussian noise (AWGN) having the variance $2N_0/T_s$ with N_0 and T_s being the one-sided noise power spectrum density and symbol duration, respectively.

C. MMSE-FDE

The frequency-domain signal block obtained by $(N_c + N_g)$ -point DFT $\mathbf{Y}^{(m)} = [Y^{(m)}(0), \dots, Y^{(m)}(k), \dots, Y^{(m)}(N_c + N_g - 1)]^T$ is expressed as

$$\mathbf{Y}^{(m)} = \mathbf{F}_{N_c + N_g} \mathbf{y}^{(m)} = \mathbf{H} \mathbf{S}^{(m)} + \mathbf{Z}^{(m)}, \quad (5)$$

where \mathbf{F}_K is the DFT matrix of size $K \times K$ given by

$$\mathbf{F}_K = \frac{1}{\sqrt{K}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi \frac{1 \times 1}{K}} & \cdots & e^{-j2\pi \frac{1 \times (K-1)}{K}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi \frac{(K-1) \times 1}{K}} & \cdots & e^{-j2\pi \frac{(K-1) \times (K-1)}{K}} \end{bmatrix}. \quad (6)$$

$\mathbf{S}^{(m)} = [S^{(m)}(0), \dots, S^{(m)}(k), \dots, S^{(m)}(N_c + N_g - 1)]^T = \mathbf{F}_{N_c + N_g} \mathbf{s}^{(m)}$ is the frequency-domain transmit symbol vector, $\mathbf{Z}^{(m)} = [Z^{(m)}(0), \dots, Z^{(m)}(k), \dots, Z^{(m)}(N_c + N_g - 1)]^T = \mathbf{F}_{N_c + N_g} \mathbf{z}^{(m)}$ is the frequency-domain noise vector, and $\mathbf{H} = \sqrt{2S} \mathbf{F}_{N_c + N_g} \mathbf{h} \mathbf{F}_{N_c + N_g}^H$ is the channel gain matrix. $(\cdot)^H$ is the Hermitian transpose operation. Due to the circulant property of \mathbf{h} , the channel gain matrix \mathbf{H} is diagonal. The k -th diagonal element of \mathbf{H} is given by

$$H(k) = \sqrt{2S} \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \frac{\tau_l}{N_c + N_g}\right). \quad (7)$$

MMSE-FDE is carried out to obtain

$$\hat{\mathbf{Y}}^{(m)} = \mathbf{W} \mathbf{Y}^{(m)}, \quad (8)$$

where $\mathbf{W} = \text{diag}[W(0), \dots, W(k), \dots, W(N_c + N_g - 1)]$ is the MMSE weight matrix. The k -th diagonal element of \mathbf{W} is given by [6]

$$W(k) = \frac{\tilde{H}^*(k)}{|\tilde{H}(k)|^2 + 2\tilde{\nu}}, \quad (9)$$

where $\tilde{H}(k)$ represents the channel gain estimate, $\tilde{\nu}$ is the noise power estimate, and $(\cdot)^*$ denotes the complex conjugate operation.

$\hat{\mathbf{Y}}^{(m)}$ is transformed into a time-domain symbol block $\hat{\mathbf{s}}^{(m)}$ by $(N_c + N_g)$ -point IDFT to obtain the decision variable vector $\hat{\mathbf{d}}^{(m)} = [\hat{s}^{(m)}(0), \dots, \hat{s}^{(m)}(t), \dots, \hat{s}^{(m)}(N_c - 1)]^T$.

III. 2-STEP FREQUENCY-DOMAIN CHANNEL ESTIMATION

Figure 3 shows the block diagram of the proposed 2-step frequency-domain channel estimation scheme, which uses N_B received symbol blocks. In the first step, only a received training sequence is used. The received training sequence having cyclic property is constructed for frequency-domain channel estimation to obtain the instantaneous channel estimate. Improved channel estimate is obtained by simply averaging the instantaneous channel estimates over N_B blocks. Noise power is also estimated. MMSE-FDE and tentative symbol decision are carried out to generate the N_B transmitted symbol block replicas. Then, in the second step, the maximum likelihood channel estimation [9] is carried out iteratively by using both the estimated symbol sequences and the training sequences to obtain improved channel estimate.

A. First Step

First, the received training sequence having the cyclic property is constructed [10] by using first $L-1$ symbols and last N_g ($\geq L$) symbols of the received signal block as

$$\tilde{y}^{(m)}(n) = \begin{cases} y^{(m)}(n) + y^{(m)}(n + N_c) & n = 0 \sim L - 2 \\ y^{(m)}(n + N_c) & n = L - 1 \sim N_g - 1 \end{cases}. \quad (10)$$

Eq. (10) can be rewritten by using the vector form as

$\{\tilde{S}^{(m,i-1)}(k); k=0\sim N_c+N_g-1\}$ is the k -th frequency component of the transmit symbol block replica $\tilde{\mathbf{S}}^{(m,i-1)} = [\tilde{S}^{(m,i-1)}(0), \dots, \tilde{S}^{(m,i-1)}(k), \dots, \tilde{S}^{(m,i-1)}(N_c+N_g-1)]^T$ in the previous iteration stage and is given as

$$\tilde{\mathbf{S}}^{(m,i-1)} = \mathbf{F}_{N_c+N_g} \begin{bmatrix} \tilde{\mathbf{d}}^{(m,i-1)} \\ \mathbf{u} \end{bmatrix}, \quad (23)$$

where $\tilde{\mathbf{d}}^{(m,i-1)} = [\tilde{d}^{(m,i-1)}(0), \dots, \tilde{d}^{(m,i-1)}(t), \dots, \tilde{d}^{(m,i-1)}(N_c-1)]^T$ is the soft symbol replica block [12-14].

In the i -th ($i>0$) iteration stage, the delay time-domain windowing technique is used to reduce the noise. N_c+N_g -point IDFT is applied to $\{\tilde{H}^{(i)}(k); k=0\sim N_c+N_g-1\}$ to obtain the channel impulse response estimate $\{\tilde{h}^{(i)}(\tau); \tau=0\sim N_c+N_g-1\}$. The channel impulse response is present only within the training sequence length, while the noise is spread over an entire delay-time range. Replacing $\tilde{h}^{(i)}(\tau)$ with zero's for $N_g \leq \tau \leq N_c+N_g-1$ and applying N_c+N_g -point DFT, the improved channel gain estimates $\{\tilde{H}^{(i)}(k); k=0\sim N_c+N_g-1\}$ can be obtained.

$\tilde{\mathbf{v}}$ is also obtained as [15]

$$\tilde{\mathbf{v}}^{(i)} = \frac{1}{2} \frac{1}{N_B} \frac{1}{N_c+N_g} \sum_{m=0}^{N_B-1} \sum_{k=0}^{N_c+N_g-1} |Y^{(m)}(k) - \tilde{H}^{(i)} \tilde{S}^{(m,i-1)}(k)|^2. \quad (24)$$

It should be noted that $\tilde{\mathbf{v}}^{(i)}$ includes the contribution from the channel estimation error.

A series of channel estimation, MMSE-FDE and soft decision is repeated a sufficient number of times.

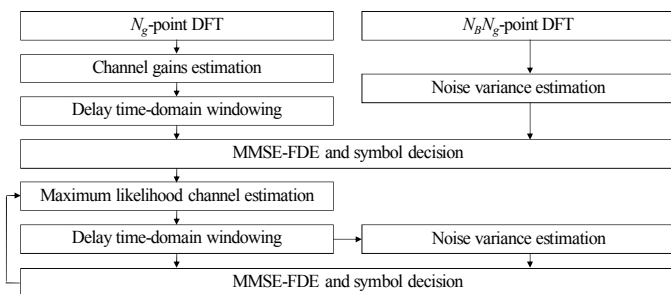


Figure 3. 2-step frequency-domain channel estimation.

TABLE I. COMPUTER SIMULATION CONDITION

		COMPUTER SIMULATION CONDITION
Transmitter	Data modulation	QPSK
	Data symbol block length	$N_c=64$
	Training sequence lengths	$N_g=16$
	Training sequence	Chu sequence [16]
Channel	Fading type	Frequency-selective block Rayleigh
	Power delay profile	$L=16$ path uniform power delay profile
Receiver	Signal detection	MMSE-FDE
	Channel estimation	2-stage frequency-domain channel estimation

IV. COMPUTER SIMULATION

The BER performance with the proposed 2-step frequency-domain channel estimation is evaluated by computer simulation.

The simulation parameters are summarized in Table I. Quadrature phase shift keying (QPSK) data modulation, $N_c=64$, $N_g=16$, and 16-path frequency-selective block Rayleigh fading channel having uniform power delay profile are assumed. Chu sequence [16] is used for the training sequence.

Figure 4 plots the BER performance using 2-step frequency-domain channel estimation as a function of the average received $E_b/N_0(=(E_s/N_0)(1+N_g/N_c)/2)$. Only one iteration is used (i.e., $I=1$). We have assumed the normalized Doppler frequency $f_D T(=f_D(N_c+N_g)T_s) \rightarrow 0$. It can be seen from Fig. 4 that the BER performance improves by increasing the number N_B of the symbol blocks to be used. This is because as N_B increases, the noise and interference from the data symbol block can be better suppressed.

Figure 5 plots the BER performance using 2-step frequency-domain channel estimation with the number I of iterations as a parameter when $N_B=16$. It can be seen from Fig. 5 that by increasing I , the BER performance improves and approaches that with perfect channel estimation.

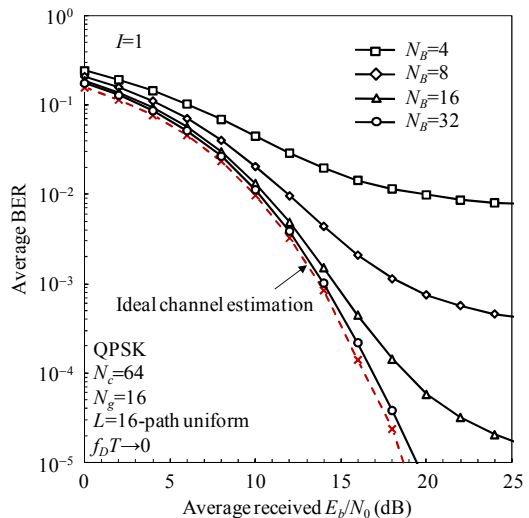


Figure 4. BER performance using 2-step frequency-domain channel estimation when $I=1$.

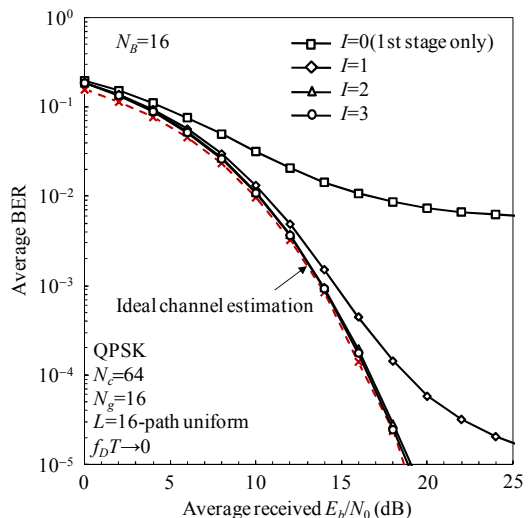


Figure 5. BER performance using 2-step frequency-domain channel estimation when $N_B=16$.

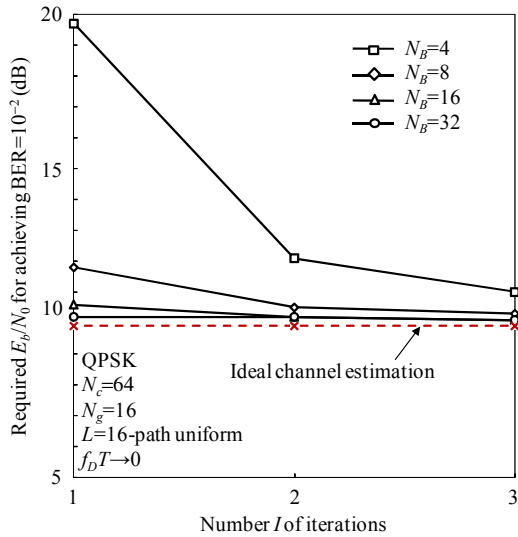


Figure 6. Required E_b/N_0 for achieving $\text{BER}=10^{-3}$.

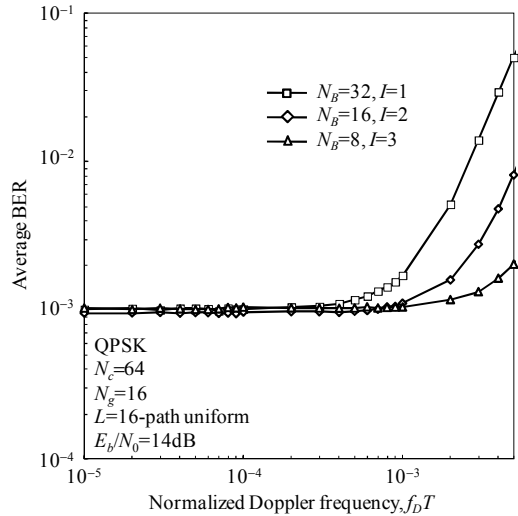


Figure 7. Impact of normalized Doppler frequency.

Figure 6 plots the required E_b/N_0 for achieving $\text{BER}=10^{-2}$ as a function of I . By increasing N_B , smaller I can be used to reduce the E_b/N_0 gap from the perfect channel estimation within 0.5dB; it is $I=1, 2$, and 3 for $N_B=32, 16$, and 8 , respectively.

So far, we have considered the case where the channel gains stay constant over the period of N_B blocks (i.e., a quasi-static fading). Below, we will consider the case where the channel gains stay constant over each block but vary block-by-block. Figure 7 shows the impact of Doppler frequency on the average BER. Assuming 20MHz signal bandwidth at the carrier frequency 2GHz, the normalized Doppler frequency $f_D T=1 \times 10^{-3}$ corresponds to a travelling speed of 135km/h. Three cases of $(N_B, I)=(32, 1), (16, 2)$, and $(8, 3)$ are plotted (the E_b/N_0 gap from the perfect channel estimation for achieving $\text{BER}=10^{-3}$ is within 0.5dB). It can be seen from Fig. 7 that the BER is almost constant when $f_D T \leq 3 \times 10^{-4}$, $f_D T \leq 9 \times 10^{-4}$, and $f_D T \leq 1 \times 10^{-3}$ for $N_B=32, 16$, and 8 , respectively. By increasing N_B , smaller number of iterations can be used due to better suppression of the noise and the interference from the data symbol

block, but the BER starts to increase because the tracking ability against fading tends to be lost. When either $(N_B, I)=(16, 2)$ or $(8, 3)$ is used, the proposed 2-step frequency-domain channel estimation achieves a performance close to the perfect channel estimation even in a fast fading environment.

V. CONCLUSION

In this paper, we proposed 2-step frequency-domain channel estimation scheme suitable for TS-SC block transmission with MMSE-FDE. It was demonstrated by the computer simulation that the proposed channel estimation scheme achieves a BER performance close to the perfect channel estimation case.

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