Performance Analysis on FDE receiver for Single Carrier Transmission

Wei Peng and Fumiyuki Adachi
Department of Communication Engineering, Tohoku University
peng@mobile.ecei.tohoku.ac.jp, adachi@ecei.tohoku.ac.jp

Abstract- In this paper, single carrier (SC) transmission is considered. We focus on SC frequency domain equalization (FDE) receiver and analyze the signal to noise ratio (SNR) of the time domain output signal estimate and then, derive the achievable bit error rate (BER). For the purpose of comparison, we consider optimal time domain equalization (TDE) receiver as a reference. Our analytical results show the relationship between the FDE and optimal TDE.

Keywords: single carrier transmission; frequency domain equalization; time domain equalization; performance analysis

I Introduction

In the high speed data communication, the wireless channel exhibits strong frequency selectivity due to the multi-path delay and therefore, the inter-block interference (IBI) and inter-symbol interference (ISI) are produced at the receiver side [1]. It is well known that orthogonal frequency division multiplexing (OFDM) [2] is one promising technique to solve the problem by transmitting different symbols in orthogonal frequencies and avoid the interference. However, OFDM modulation will result in a high peak-to-average power ratio (PAPR) and requires a wide dynamic range of digital-to-analog converter (DAC) and analog-to-digital converter (ADC) devices at the transmitter and receiver, respectively. To avoid the high PAPR problem, single carrier (SC) block transmission with cyclic prefix (CP) [3] has been attracting the interests from both academic and industrial world. In the long term evolution (LTE) and LTE-advanced (LTE-A), SC transmission has been standardized for the uplink (link from user to base station) transmission.

In SC transmission, IBI can be easily avoided by CP removal at the receiver, and only ISI remains as a problem. Therefore, it necessary to equalize the received signal in order to recover the transmit signal. Time domain equalization (TDE) is a conventional way to combat the multi-path delay. The optimal TDE receiver is based on transversal equalization as well as feedback equalization [4] and relies on maximum likelihood (ML) detection. The computational complexity to realize the optimal TDE is prohibitively high. On the other hand, one-tap frequency domain equalization (FDE) has much lower computational complexity. The performance of SC-FDE has been compared with OFDM [5], by simulations, to show that SC-FDE receiver can achieve comparable performance to OFDM receiver. However, the theoretical analysis on the achievable bit error rate (BER) of the SC-FDE receiver has not been studied yet. In this paper, we focus on SC-FDE receiver. We analyze the signal-to-noise ratio (SNR) of the time domain output signal estimate and therefore derive the achievable BER. For the purpose of comparison, we also consider optimal SC-TDE receiver as a reference.

The rest of the paper is organized as follows: system model of SC-FDE transceiver is described in Section II, the performance analysis is given in Section III, then a comparison between SC-FDE receiver and SC-TDE receiver is given in Section IV and finally Section V concludes this paper.

II System Model

The transceiver structure of SC transmission with FDE receiver is shown in Fig. 1.

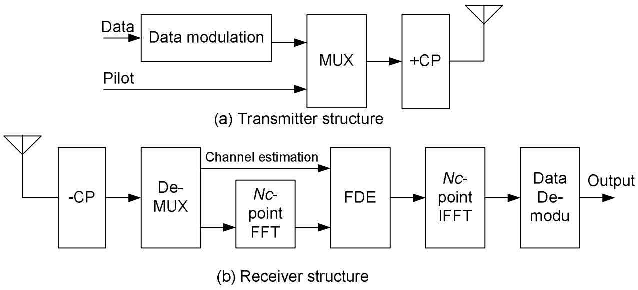

Figure 1 Single-user SC transmission with single antenna FDE receiver.

A block fading channel between the transmitter and the receiver is assumed, i.e., the channel remains unchanged during the transmission period of a block. In this paper, the symbol-spaced discrete time representation of the signal is used. Assuming an $L-$path channel, the impulse response of the channel can be expressed as

$$h(\tau)=\sum_{l=0}^{L-1} h_l \delta(\tau-\tau_l), \quad (1)$$

where h_l and τ_l are the path gain and time delay of the l^{th} path, respectively. h_l follows the complex Gaussian distribution (Rayleigh distribution) and satisfies $\sum_{l=0}^{L-1} E\{|h_l|^2\}=1$ where $E\{\cdot\}$ represents the expectation operation. It is assumed that the time delay τ_l is a multiple integer of the symbol duration and $\tau_l = l$. The cyclic-prefixed block signal transmission is used to avoid IBI and it is assumed that the CP is longer than the maximum path delay of the signal. In the following, we omit the insertion and removal of the CP for the purpose of simplicity.

The baseband equivalent received signal block $\{r(n); n=0 \sim N_c-1\}$ of N_c symbols at the receive antenna is given by

$$r(n)=\sqrt{P_t}\sum_{l=0}^{L-1} h_l s(n-l)+z(n) \quad (2)$$

where P_t and $s(n)$ are the transmit signal power and transmit signal respectively. And $z(n)$ is the additive white Gaussian noise (AWGN). According to (2), the frequency domain representation of the received signal on the k^{th} frequency is given by

$$R(k)=\sqrt{P_t}H(k)S(k)+Z(k), \quad (3)$$

where

$$\begin{cases} S(k)=\sum_{n=0}^{N_c-1} s(n)\exp\left(-j2\pi k\frac{n}{N_c}\right) \\ H(k)=\sum_{n=0}^{N_c-1} h(n)\exp\left(-j2\pi k\frac{n}{N_c}\right) \\ Z(k)=\sum_{n=0}^{N_c-1} z(n)\exp\left(-j2\pi k\frac{n}{N_c}\right) \end{cases}. \quad (4)$$

The k^{th} frequency signal after one-tap FDE weight control is given by [6]

$$\hat{R}(k)=W(k)R(k), \quad (5)$$

where $W(k)$ is the k^{th} FDE weight. Minimum mean square error (MMSE) criterion is commonly used and MMSE FDE weight $W(k)$ is given by

$$W(k)=\frac{H^*(k)}{|H(k)|^2+\sigma_k^2}, \quad (6)$$

where symbol * represents complex conjugate; σ_k^2 is the noise to transmit signal power ratio at frequency k which, according to the Parseval's theorem [7], has the same value as its counterpart in time domain, i.e., $\sigma_k^2=(P_t/N_0)^{-1}$ where N_0 is the power spectrum density of the AWGN. The time domain signal block estimate $\{\hat{r}(n); n=0 \sim N_c-1\}$ is then obtained by applying an N_c-point inverse FFT (IFFT) to $\hat{\mathbf{R}}=\left[\hat{R}(0),\cdots,\hat{R}(k),\cdots,\hat{R}(N_c-1)\right]$ as

$$\hat{r}(n)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(j2\pi k\frac{n}{N_c}\right). \quad (7)$$

III Performance Analysis on FDE Receiver

Submitting (4), (5) into (7) we will have

$$\begin{aligned}\hat{r}(n)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left(\sqrt{P_t}W(k)H(k)S(k)+W(k)Z(k)\right)\\&\cdot\exp\left(j2\pi k\frac{n}{N_c}\right)\\&=\frac{1}{N_c}\sum_{k=0}^{N_c}\sqrt{P_t}W(k)H(k)\left[s(n)\exp\left(-j2\pi k\frac{n}{N_c}\right)\right.\\&\left.+\sum_{i=0,i\neq n}^{N_c}s(i)\exp\left(-j2\pi k\frac{i}{N_c}\right)\right]\times\exp\left(j2\pi k\frac{n}{N_c}\right)+\hat{z}(n)\\&=\hat{s}(n)+I(n)+\hat{z}(n)\end{aligned} \quad (8)$$

which contains the desired signal component

$$\hat{s}(n)=s(n)\frac{\sqrt{P_t}}{N_c}\sum_{k=0}^{N_c-1}W(k)H(k), \quad (9)$$

the inter-symbol-interference (ISI) component

$$\begin{aligned}I(n)&=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\sqrt{P_t}W(k)H(k)\\&\cdot\sum_{i=0,i\neq n}^{N_c-1}s(i)\exp\left(-j2\pi k\frac{i}{N_c}\right)\exp\left(j2\pi k\frac{n}{N_c}\right)\end{aligned}, \quad (10)$$

and the noise component

$$\hat{z}(n)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}W(k)Z(k)\exp\left(j2\pi k\frac{n}{N_c}\right). \quad (11)$$

The power of the noise component is given by

$$\begin{aligned}&E\left\{\hat{z}(n)\hat{z}^*(n)\right\}\\&=\frac{1}{N_c^2}\sum_{k=0}^{N_c-1}\sum_{k'=0}^{N_c-1}W(k)W^*(k)E\left\{Z(k)Z^*(k')\right\}.\\&\cdot\exp\left(j2\pi k\frac{n}{N_c}\right)\exp\left(-j2\pi k'\frac{n}{N_c}\right)\end{aligned} \quad (12)$$

According to (4), $E\left\{Z(k)Z^*(k')\right\}$ in (12) can be calculated by

$$\begin{aligned}&E\left\{Z(k)Z^*(k')\right\}\\&=E\left\{\sum_{n=0}^{N_c-1}z(n)\exp\left(-j2\pi k\frac{n}{N_c}\right)\sum_{n'=0}^{N_c-1}z^*(n')\exp\left(-j2\pi k'\frac{n'}{N_c}\right)\right\}.\\&=N_0\sum_{n=0}^{N_c-1}\exp\left(-j2\pi(k-k')\frac{n}{N_c}\right)\\&=N_0N_c\delta(k-k')\end{aligned} \quad (13)$$

Substituting (14) into (12) we have

$$\begin{aligned}&E\left\{\hat{z}(n)\hat{z}^*(n)\right\}\\&=\frac{1}{N_c^2}\sum_{k=0}^{N_c-1}\sum_{k'=0}^{N_c-1}W(k)W^*(k)N_0N_c\delta(k-k')\\&\cdot\exp\left(j2\pi k\frac{n}{N_c}\right)\exp\left(-j2\pi k'\frac{n}{N_c}\right)\\&=\frac{N_0}{N_c}\sum_{k=0}^{N_c-1}\left|W(k)\right|^2\end{aligned}. \quad (14)$$

Let $\hat{a}(n)\equiv\hat{s}(n)+I(n)$, we have

$$\hat{a}(n)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\sqrt{P_t}W(k)H(k)S(k)\exp\left(j2\pi k\frac{n}{N_c}\right)\cdot \quad (15)$$

Then the power of $\hat{a}(n)$ is calculated by

$$\begin{aligned}&E\left\{\hat{a}(n)\hat{a}^*(n)\right\}=\\&\frac{P_t}{N_c^2}\sum_{k=0}^{N_c-1}\sum_{k'=0}^{N_c-1}W(k)W^*(k')H(k)H^*(k')\\&\cdot E\left\{S(k)S^*(k')\right\}\exp\left(j2\pi k\frac{n}{N_c}\right)\exp\left(-j2\pi k'\frac{n}{N_c}\right)\end{aligned}, \quad (16)$$

where

$$E\left\{S(k)S^*(k')\right\}=N_c\delta(k-k'). \quad (17)$$

Therefore, the power of $\hat{a}(n)$ is obtained by submitting (17) into (16) as

$$E\left\{\hat{a}(n)\hat{a}^*(n)\right\}=\frac{P_t}{N_c}\sum_{k=0}^{N_c-1}\left|W(k)\right|^2\left|H(k)\right|^2. \quad (18)$$

On the other hand, the power of the desired signal component $\hat{s}(n)$ is obtained by

$$E\left\{\hat{s}(n)\hat{s}^*(n)\right\} = \frac{P_t}{N_c^2}\left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2. \quad (19)$$

Then the power of ISI component is obtained by

$$\begin{aligned} &E\left\{I(n)I^*(n)\right\} \\ &= E\left\{a(n)a^*(n)\right\} - E\left\{\hat{s}(n)\hat{s}^*(n)\right\} \\ &= \frac{P_t}{N_c}\sum_{k=0}^{N_c-1}\left|W(k)\right|^2\left|H(k)\right|^2 - \frac{P_t}{N_c^2}\left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2 \end{aligned}. \quad (20)$$

Therefore, signal to interference plus noise ratio (SINR) of time-domain estimate $\hat{r}(n)$ can be obtained from (14), (19) and (20) as

$$\begin{aligned} &\Gamma_{SC-FDE} \\ &= \frac{E\left\{\hat{s}(n)\hat{s}^*(n)\right\}}{E\left\{\hat{I}(n)\hat{I}^*(n)\right\} + E\left\{\hat{z}(n)\hat{z}^*(n)\right\}} \\ &= \frac{\left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2}{\left(N_c\sum_{k=0}^{N_c-1}\left|W(k)\right|^2\left|H(k)\right|^2 - \left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2 + N_c\left(P_t/N_0\right)^{-1}\sum_{k=0}^{N_c-1}\left|W(k)\right|^2\right)} \end{aligned}. \quad (21)$$

By submitting (6) into (21), the SINR of SC MMSE FDE receiver output is

$$\Gamma_{SC\_FDE} = \frac{\left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2}{N_c\sum_{k=0}^{N_c-1} W(k)H(k) - \left(\sum_{k=0}^{N_c-1} W(k)H(k)\right)^2}. \quad (22)$$

By using the derived SINR in (22), the average bit error rate (BER) performance of the SC MMSE FDE receiver using QPSK modulation can be analytically evaluated by

$$P_b^{SC\_FDE} = \frac{1}{2\pi}\int_{\sqrt{\Gamma_{SC\_FDE}}}^{\infty} \exp\left(-t^2/2\right)dt. \quad (23)$$

The above analysis is verified by a comparison between the analytical BER and the computer simulation result, as shown in Fig. 2.

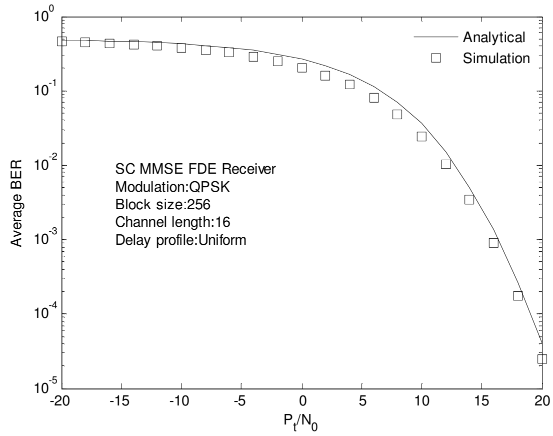

Figure 2 Analytical and simulation results of average BER performance of SC MMSE FDE receiver.

IV Comparison of Frequency Domain and Time Domain Equalization

In the following, we are going to show the relationship between the SC-FDE receiver and the SC-TDE receiver. In order to deal with the ISI problem, several types of TDE receivers have been widely used. One example is a linear receiver using transversal equalization [8] which simply obtains the transmit signal estimate by a weighted combination of the received signals on the delayed taps. Another example is a non-linear receiver using transversal equalization as well as the feedback equalization [9]. It has been shown in [4] that an optimal receiver, which has a receive filter impulse response of $h^*(-n)$ at each symbol-space sampling and frequency domain response of $H^*(k)$ on each frequency, can obtain an optimal SINR of $\Gamma_{TDE} = P_t/N_0$ at the output of the receiver.

In the next, we will compare the SC MMSE FDE receiver with optimal TDE receiver. According to (22), the SINR of SC MMSE FDE receiver output can be rewritten as

$$\begin{aligned} &\Gamma_{SC\_FDE} \\ &= \frac{\left(\sum_{k=0}^{N_c-1}\frac{\left|H(k)\right|^2}{\left|H(k)\right|^2+\sigma_k^2}\right)^2}{N_c\sum_{k=0}^{N_c-1}\frac{\left|H(k)\right|^2}{\left|H(k)\right|^2+\sigma_k^2} - \left(\sum_{k=0}^{N_c-1}\frac{\left|H(k)\right|^2}{\left|H(k)\right|^2+\sigma_k^2}\right)^2} \end{aligned}. \quad (24)$$

Although it is difficult to decide the relationship between the MMSE FDE receiver and the SC-TDE receiver from the above expression, we can consider two extreme cases instead.

Case 1) when the noise power is extremely significant, i. e., $\frac{P_t}{N_0} \to 0, \sigma_k^2 \to \infty$

In this case (24) can be approximated by

$$\lim_{\sigma_k^2 \to \infty} \Gamma_{SC-FDE} = \frac{N_c^2 / \sigma_k^4}{N_c^2 / \sigma_k^2 - N_c^2 / \sigma_k^4} \approx \frac{1}{\sigma_k^2} = \frac{P_t}{N_0}. \qquad (25)$$

It is obvious from (25) that the SINR of SC MMSE FDE receiver is equivalent to the optimal SC-TDE receiver.

Case 2) when the transmit signal to noise ratio is extremely high, i. e. , $\frac{P_t}{N_0} \to \infty, \sigma_k^2 \to 0$

In this case (24) can be evaluated by

$$\lim_{\sigma_k^2 \to 0} \Gamma_{SC-FDE} = \frac{N_c^2}{N_c \sum_{k=0}^{N_c-1} \frac{|H(k)|^2}{|H(k)|^2 + \sigma_k^2} - N_c^2} \to \infty. \qquad (26)$$

It can be concluded from (26) that if the input SNR increases to infinite, the output SINR of SC MMSE FDE receiver goes to infinite, which is similar to the optimal SC-TDE receiver. According to (25) and (26), it can be concluded that the SC-FDE receiver can achieve equivalent or comparable performance as the optimal SC-TDE receiver. However, please note that the computational complexity of SC-FDE receiver increases proportional to the block size, while the computational complexity of optimal SC-TDE receiver increases exponentially to the block size due to the use of ML detection.

V Conclusions

In this paper, the performance of SC-FDE receiver was analyzed and confirmed by the simulation results. Based on the analysis, the SNR of the time domain signal estimate at SC-FDE receiver output was compared with the optimal SC-TDE receiver. It was proved that the SC-FDE receiver can achieve equivalent performance as the SC-TDE receiver when at low transmit SNR; and the performance of SC-FDE is comparable to that of SC-TDE at high transmit SNR. On the other hand, the computational complexity of SC-FDE is much lower than that of SC-TDE.

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