Adaptive Sparse Channel Estimation Using Re-Weighted Zero-Attracting Normalized Least Mean Fourth

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***Abstract*—Accurate channel estimation problem is one of the key technical issues in broadband wireless communications. Standard normalized least mean fourth (NLMF) algorithm was applied to adaptive channel estimation (ACE). Since the channel is often described by sparse channel model, such sparsity could be exploited and then estimation performance could be improved by adaptive sparse channel estimation (ASCE) methods using zero-attracting normalized least mean fourth (ZA-NLMF) algorithm. However, this algorithm cannot exploit channel sparsity efficiently. By virtual of geometrical figures, we explain the reason why ℓ_1-norm sparse constraint penalizes channel coefficients uniformly. In this paper, we propose a novel ASCE method using re-weighted zero-attracting NLMF (RZA-NLMF) algorithm. Simulation results show that the proposed ASCE method achieves better estimation performance than the conventional one.**

Keywords*—normalized LMF (NLMF), adaptive sparse channel estimation (ASCE), re-weighted zero-attracting NLMF (RZA-NLMF).

I. Introduction

Broadband signal transmission is becoming one of the mainstream techniques in the next generation communication systems. Due to the fact that frequency-selective channel fading is unavoidable, accurate channel state information (CSI) is necessary at the receiver for adaptive coherent detection [1]. One of effective approaches is adopting adaptive channel estimation (ACE). A typical framework of ACE is shown in Fig. 1. It is well known that ACE using least mean fourth (LMF) algorithm outperforms the least mean square (LMS) algorithm in achieving a good balance between convergence and steady-state performances. However, standard LMF algorithm is unstable due to the fact that its stability depends on the following three factors: input signal power, noise power and weight initialization [2]. To improve the stability of LMF, stable normalized LMF (NLMF) algorithm was proposed in [3]. Recently, many channel measurement experiments have verified that broadband channels often exhibit sparse structure as shown in Fig. 2. In other words, sparse channel is consisted of a very few channel coefficients and most of them are zeros. Unfortunately, ACE using NLMF algorithm always neglects the inherent sparse structure information and it may degrade the estimation performance.

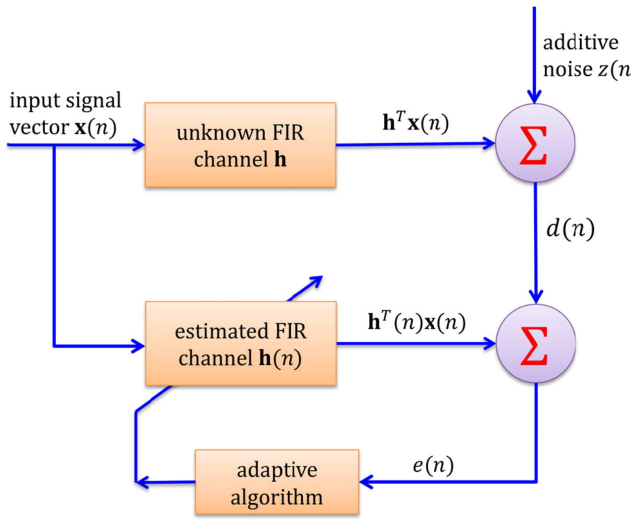

Fig. 1. ASCE for broadband communication systems.

To estimate such a channel, adaptive sparse channel estimation (ASCE) methods using sparse least mean fourth algorithm (ASCE-LMF) were proposed in [4]. However, the ASCE-LMF method is not stable except in low SNR regime (below 5dB). Based on the dense channel model, author in [3] proposed a normalized LMF (NLMF) algorithm to improve the stability, but stable NLMF algorithm cannot be applied directly in sparse channel estimation. Based on the method in [3], we proposed ASCE using zero-attracting NLMF (ZA-NLMF) algorithm [5]. From a geometrical perspective, we presented a detailed explanation of ℓ_1-norm zero-attracting based sparse channel estimation. Since ℓ_1-norm zero-attracting introduces a uniform sparse constraint on different magnitudes of channel coefficients, the performance of ZA-NLMF degrades.

Inspired by re-weighted ℓ_1-norm minimization algorithm in [6], in this paper, we propose an improved ASCE method using re-weighted zero-attracting NLMF (RZA-NLMF) algorithm. Unlike the ZA-NLMF algorithm [5], RZA-NLMF algorithm can penalize different magnitudes of channel coefficients with different sparse constraint strength. The effectiveness of our proposed method is confirmed by computer simulation.

The remainder of this paper is organized as follows. A system model is described and standard LMF and NLMF algorithms are introduced in Section II. In section III, sparse ASCE using ZA-NLMF algorithm is introduced and improved ACSE using RZA-NLMF algorithm is highlighted. Computer simulations are presented in Section IV in order to evaluate and compare performances of the proposed ASCE methods. Finally, we conclude the paper in Section V.

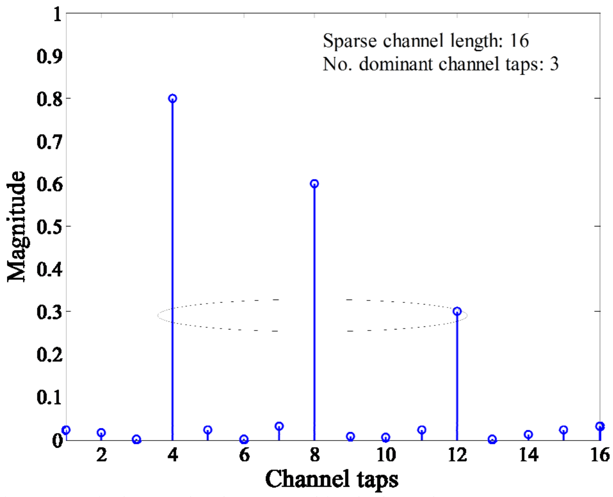

Fig. 2. A typical example of sparse multipath channel.

II. System Model and Standard LMF Algorithm

Consider a baseband frequency-selective fading wireless communication system where FIR sparse channel vector $\mathbf{h} = [h_0, h_1, \dots, h_{N-1}]^T$ is N-length and it is supported only by K nonzero channel taps. Assume that an input training signal $x(n)$ is input to probe the unknown sparse channel. At the receiver side, observed signal $y(n)$ is given by

$$y(n) = \mathbf{h}^T \mathbf{x}(n) + z(n), \tag{1}$$

where $\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-N+1)]^T$ denotes the vector of training signal $x(n)$, and $z(n)$ is the additive white Gaussian noise (AWGN) assumed to be independent with $x(n)$. The objective of ASCE is to adaptively estimate the unknown sparse channel estimator $\hat{\mathbf{h}}$ using the training signal $\mathbf{x}(n)$ and the observed signal $y(n)$. According to [2], we can apply standard LMF algorithm to adaptive channel estimation, with the cost function

$$G_1(n) = \frac{1}{4} e^4(n), \tag{2}$$

where $e(n) = y(n) - \mathbf{h}^T(n)\mathbf{x}(n)$ is n-th adaptive updated error. The update equation of the filter can be written as

$$\begin{aligned}\mathbf{h}(n+1) &= \mathbf{h}(n) + \mu \frac{\partial G_1(n)}{\partial \mathbf{h}(\mathrm{n})} \\ &= \mathbf{h}(n) + \mu e^3(n) x(n),\end{aligned} \tag{3}$$

where μ denotes the step-size of gradient descend. Unfortunately, channel estimation using standard LMF algorithm is not stable in adaptive updating process and hence it cannot be employed directly [2]. To improve the reliability of LMF, an stable LMF algorithms was proposed by virtual of normalization in [3], which was termed as normalized LMF (NLMF) algorithm. The update equation is given by

$$\begin{aligned}\mathbf{h}(n+1) &= \mathbf{h}(n) + \mu \frac{e^3(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2 \left(\|\mathbf{x}(n)\|_2^2 + e^2(n)\right)} \\ &= \mathbf{h}(n) + \mu_N \frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2},\end{aligned} \tag{4}$$

where

$$\mu_N = \frac{\mu e^2(n)}{\|\mathbf{x}(n)\|_2^2 + e^2(n)}, \tag{5}$$

denotes variable step-size of gradient descent and $\|\cdot\|_2$ is the Euclidean norm operator and $\|\mathbf{x}\|_2^2 = \sum_{i=0}^{N-1} |x_i|^2$.

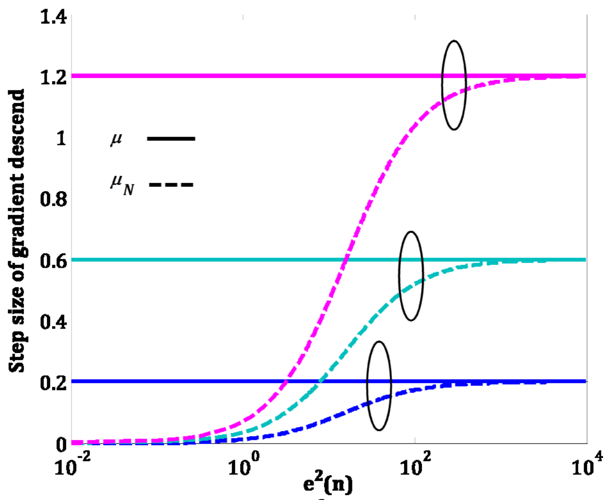

Fig. 3. Relations between μ_N and $e^2(n)$.

Unfortunately, interpretation of the relation between μ_N and $e^2(n)$ is not correct in [7]. Here, we observe that when $e^2(n) \gg \|x(n)\|_2^2$, then μ_N approaches to μ; when $e^2(n) \approx \|\mathbf{x}(n)\|_2^2$, then μ_N approaches to $\mu/2$; when $e^2(n) \ll \|\mathbf{x}(n)\|_2^2$, then μ_N approaches to 0. Fig. 3 illustrates the intuitive relation between μ_N and $e^2(n)$. For the standard NLMF algorithm, consider three step-sizes μ: 0.2, 0.6 and 1.2. As $e^2(n)$ increases, according to the depicted curves in Fig. 3, it is easy to find that the stability of NLMF approaches NLMS which stability was proven in [8]. Please also note that when $e^2(n)$

decreases, the step size μ_N also reduces to ensure stability.

III. Adaptive Sparse Channel Estimation

A. Geometrical interpretation of CS-based sparse channenl estimation

Consider a baseband system model for CS-based sparse channel estimation. Assume a N-length training sequence as $\boldsymbol{\phi} = [\phi_0, \phi_1, \dots \phi_{N-1}]^T$, and then the received signal model can be written as

$$\begin{aligned}\boldsymbol{\beta} &= [\beta_0, \dots, \beta_m, \dots, \beta_{M-1}]^T \\ &= \boldsymbol{\Phi}\mathbf{w} + \boldsymbol{\zeta}, \end{aligned} \tag{6}$$

where $\boldsymbol{\Phi} \in \mathbb{C}^{M\times N}$ is a Toeplitz training signal matrix with first row $\boldsymbol{\phi}$; $\boldsymbol{\zeta} = [\zeta_0, \dots, \zeta_m, \dots, \zeta_{M-1}] \in \mathbb{C}^{M\times 1}$ is an AWGN with distribution $\mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N)$ and $\mathbf{I}_M$ denotes an $M \times M$ identity matrix; $\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T$is an $N \times 1$ unknown channel vector which is same as $\mathbf{h}$ in Eq. (1). From the perspective of CS, the training signal matrix $\boldsymbol{\Phi}$ satisfies the restricted isometry property (RIP) [9] of order K with positive parameter $\delta_K \in (0,1)$, i.e., $\boldsymbol{\Phi} \in \mathrm{RIP}(K, \delta_K)$ if

$$(1-\delta_K)\|\mathbf{w}\|_2^2 \le \|\boldsymbol{\Phi}\mathbf{w}\|_2^2 \le (1+\delta_K)\|\mathbf{w}\|_2^2, \tag{7}$$

holds for all $\mathbf{w}$ having no more than K nonzero taps. Then, LASSO algorithm [10] based sparse channel estimator $\widehat{\mathbf{w}}$ is given by

$$\widehat{\mathbf{w}} = \arg\lim_{\mathbf{w}} \left\{ \frac{1}{2}\|\boldsymbol{\beta} - \boldsymbol{\Phi}\mathbf{w}\|_2^2 + \lambda_1 \|\mathbf{w}\|_1 \right\}, \tag{8}$$

where λ_1 denotes a regularization parameter which balances the mean-square error (MSE) term and sparsity of $\mathbf{w}$. In Fig. 4, geometrical interpretation of CS-based sparse channel estimation is depicted. When $\lambda_1 > 0$, as the figure shows, sparse channel estimator can be obtained from convex point between ℓ_1-norm sparse constraint and solution plane; When $\lambda_1 = 0$, however, there is no convex point between ℓ_2-norm constraint and solution plane. One can also extend the geometrical figure to explain adaptive sparse channel estimation using ZA-NLMF algorithm in [5].

B. ASCE using ZA-NLMF algorithm

Recall that the adaptive channel estimation method uses standard NLMF algorithm in Eq. (4), however, the proposed method does not take advantage of the channel sparsity. It was caused by its original cost function in (2) which does not utilize the sparse constraint or penalty function. According to LASSO algorithm in (7), we introduce ℓ_1-norm sparse constraint to the cost function in (4) and obtain a new cost function as

$$G_2(n) = \frac{1}{4}e^4(n) + \lambda_2 \|\mathbf{h}(n)\|_1, \tag{9}$$

where λ_2 denotes a regularization parameter which balances the mean-fourth error (MFE) term and sparsity of $\mathbf{h}$. The update equation of ZA-NLMF algorithm [5] is given by

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_N \frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2} + \gamma \mathrm{sgn}(\mathbf{h}(n)), \tag{10}$$

where $\gamma = \mu_N \lambda_2$ and $\mathrm{sgn}(\cdot)$ denotes sign function which is generated from

$$\mathrm{sgn}(\mathbf{h}(n)) = \frac{\partial \|\mathbf{h}(n)\|_1}{\partial \mathbf{h}(n)} = \begin{cases} 1, & h_i(n) > 0 \\ 0, & h_i(n) = 0, \\ -1, & h_i(n) < 0 \end{cases} \tag{11}$$

where $\mathbf{h}(n) = [h_0(n), h_1(n) \dots, h_{N-1}(n)]^T$ and $i \in \{0,1,\dots,N\}$. It is well known that ZA-NLMF uses ℓ_1-norm constraint to approximate the optimal sparse channel estimation [11].

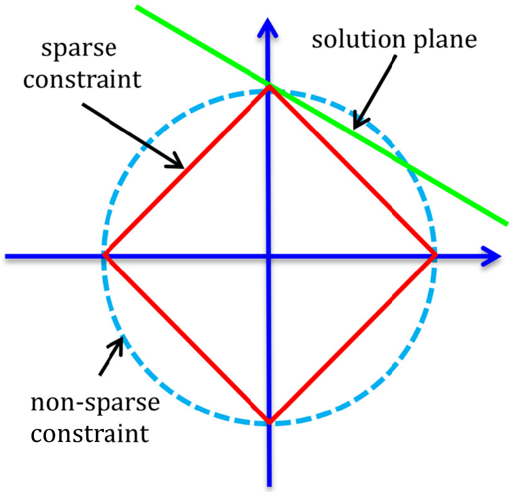

Fig. 4. Sparse channel estimation with ℓ_1-norm sparse constraint.

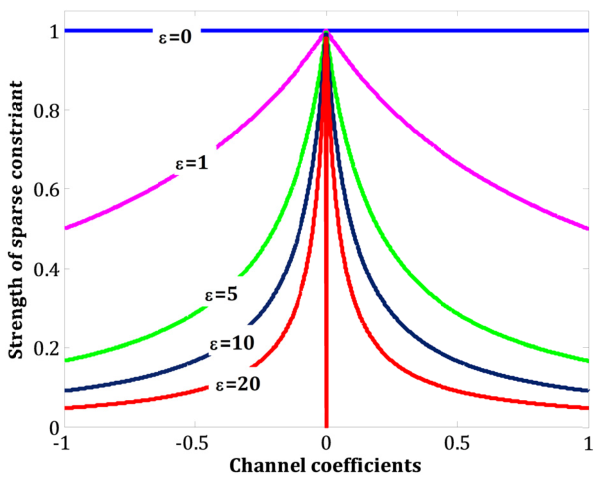

Fig. 5. Sparse constraint strength comparison between different channel coefficients by virtual of re-weighted ℓ_1-norm zero-attracting.

As shown in Fig. 5, however, ℓ_1-norm sparse constraint ($\varepsilon = 0$) on solution plane is uniform for each channel coefficient. In other words, ZA-NLMF cannot exploit the sparsity efficiently. If we introduce a re-weighted ℓ_1-norm sparse constraint function to the cost function in (4), adopt different re-weighted factors, e.g., $\varepsilon = 1,5,10$ and 20, then we are able to penalize different sparse constraint strength on different channel coefficients. In other words, RZA can penalize stronger constraint strength than smaller channel coefficients, and vice versa. Based on this idea, an improved ASCE using RZA-NLMF algorithm is proposed in the following.

C. ASCE using RZA-NLMFalgorithm

The ZA-NLMF cannot distinguish between zero taps and non-zero taps since all the taps are forced to zero uniformly as show in Fig. 5. Unfortunately, ZA-NLMF based approach will degrade amount of estimation performance. Motivated by reweighted ℓ_1-minimization sparse recovery algorithm [6] in CS [12,13], we proposed an improved ASCE method using RZA-NLMF algorithm. The cost function of RZA-NLMF is given by

$$G_3(n) = \frac{1}{4}e^4(n) + \lambda_3 \sum_{i=0}^{N-1} \log(1 + |h_i|/\varepsilon), \qquad (10)$$

where $\lambda_3 > 0$ is a regularization parameter which trades off the estimation error and channel sparsity. The corresponding update equation is

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_N \frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2} + \rho \frac{\mathrm{sgn}(\mathbf{h}(n))}{1 + \varepsilon|\mathbf{h}(n)|}, \qquad (11)$$

where $\rho = \mu_N \lambda_3/\varepsilon$ is a parameter which depends on step-size μ, regularization parameter λ_3 and threshold ε, respectively. In the second term of (11), if magnitudes of $h_i(n), i = 0,1,\dots,N-1$ are smaller than $1/\varepsilon$, then these small coefficients will be replaced by zeros in high probability.

D. Equivalence between CS-based sparse channel estimation and ASCE

According to LASSO algorithm, we explained the connections between CS-based sparse channel estimation method and ASCE. Here, we further interpretation their equivalence between them. It will be useful to enrich sparse signal processing theory. Let take LASSO and ZA-NLMF for an example. If each row of training matrix $\mathbf{\Phi}$ in system model (6) is $\boldsymbol{\phi}_m$, then by virtual of matrix vectorization, then it can be written as $[\boldsymbol{\phi}_0^T, \dots, \boldsymbol{\phi}_m^T, \dots, \boldsymbol{\phi}_{M-1}^T]^{\boldsymbol{T}}$. The m-th receive signal is obtained as $\beta_m = \boldsymbol{\phi}_m^T \mathbf{w} + \zeta_m$. Their corresponding functions of different vectors are listed in Tab. I.

TAB. I. EQUIVALENCE BETWEEN LASSO AND ZA-LNMF.

	LASSO	ZA-NLMF
Training signal	$\boldsymbol{\phi}_m$	$\mathbf{x}(n)$
Received signal	β_m	$y(n)$
Channel vector	$\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T$	$\mathbf{h} = [h_0, h_1, \dots, h_{N-1}]^T$
Sparse constraint	$\|\mathbf{w}\|_1$	$\mathrm{sign}(\mathbf{h})$
Iterative times	$m = \mathrm{mod}(n-1, M) + 1$	n

IV. COMPUTER SIMULATIONS

In this section, the proposed ASCE method using RZA-NLMF algorithm is evaluated. For achieving average performance, 1000 independent Monte-Carlo runs are adopted. The length of channel vector $\mathbf{h}$ is set as $N = 16$ and its number of dominant taps is set to $K = 1$ and 4, respectively. Each dominant channel tap follows random Gaussian distribution as $\mathcal{CN}(0, \sigma_{\mathbf{h}}^2)$ and their positions are randomly allocated within the length of $\mathbf{h}$ which is subject to $\mathrm{E}\{\|\mathbf{h}\|_2^2 = 1\}$. The received signal-to-noise ratio (SNR) is defined as $10\log(E_0/\sigma_n^2)$, where $E_0 = 1$ is the unit transmission power. Here, we set the SNR as 3dB and 5dB in computer simulation. All of the step sizes and regularization parameters are listed in Tab. II. The estimation performance is evaluated by average mean square error (MSE) which is defined by

$$\text{Avergae MSE}\{\mathbf{h}(n)\} = \mathrm{E}\{\|\mathbf{h} - \mathbf{h}(n)\|_2^2\}, \qquad (12)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator, $\mathbf{h}$ and $\mathbf{h}(n)$ are the actual channel vector and its n-th iterative adaptive channel estimator, respectively.

TAB. II. SIMULATION PARAMETERS.

Parameters	Values
Channel length	$N = 16$
No. of nonzero coefficients	$K = 1$ and 4
Step-size: μ	1.0 and 1.5
Regularization parameter: λ_2 and λ_3	$1e-5$
Re-weighted factor: ε	5, 10, 20

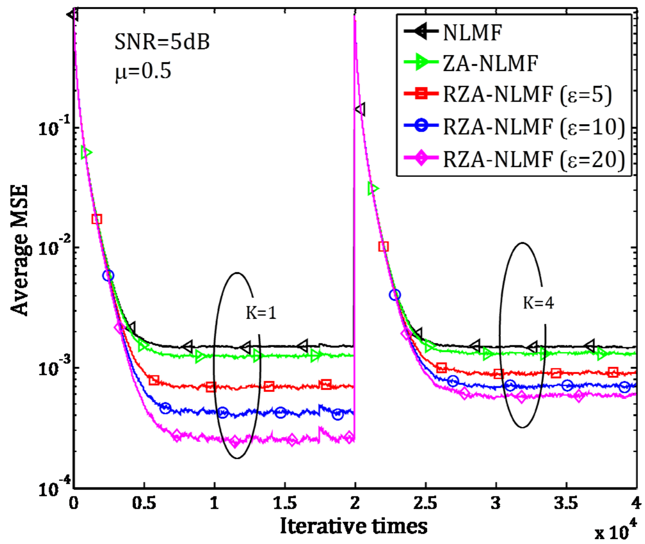

Fig. 6. Average MSE performance comparisons at SNR = 5dB and step-size $\mu = 0.5$.

Two experiments are considered in this section. In the first experiment, ASCE methods are evaluated in SNR = 5dB. Regularization parameters, i.e., λ_2 and λ_3, are set as $\lambda_2 = \lambda_3 = 1e-6$. Fig. 6-8 shows that proposed method can achieve better estimation performance than ZA-NLMF in different step-sizes, i.e., $\mu = 0.5$, 1.0 and 1.5. When the number of nonzero coefficients is $K = 1$, bigger re-weighted factor can obtain better estimation and vice versa. Please note that both ZA-NLMF and RZA-NLMF algorithms apply ASCE which can achieve better estimation performance for sparser channel. As shown in three figures, when $K = 1$, the depicted curves' gap between NLMF and either ZA-NLMF or RZA-NLMF larger than the case with $K = 4$. It is worth mentioning that bigger reweighted factor obtains better estimation performance, e.g., RZA-NLMF using $\varepsilon = 20$ achieves better estimation performance than $\varepsilon = 10$. It is mainly due to the bigger reweighted factor which can obtain variable sparse constraint for different magnitude of channel coefficients.

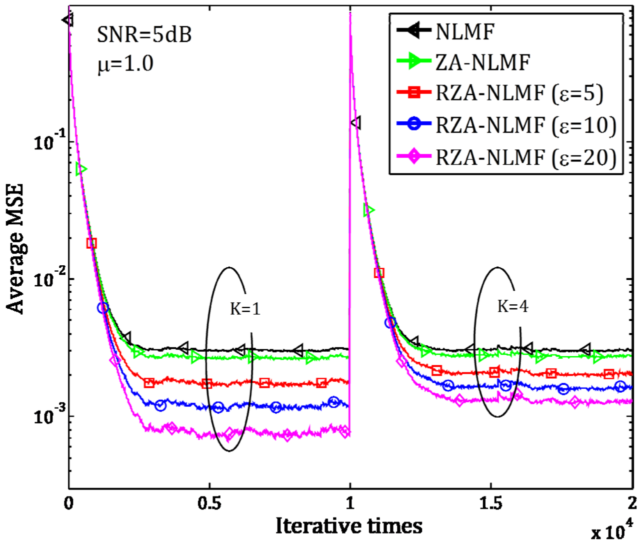

Fig. 7. Average MSE performance comparisons at SNR = 5dB and step-size $\mu = 1.0$.

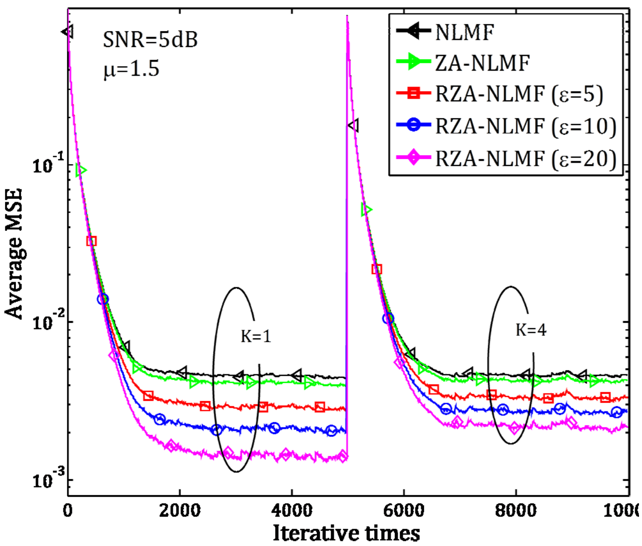

Fig. 8. Average MSE performance comparisons at SNR = 5dB and step-size $\mu = 1.5$.

To verify the flexibility of the proposed RZA-NLMF method at different SNR regimes, ASCE methods are evaluated in SNR = 10dB. Also, regularization parameters, i.e., λ_2 and λ_3, are set to $\lambda_2 = \lambda_3 = 5e-8$ to improve the estimation performance. According to Fig. 9, RZA-NLMF works well and achieves better estimation performance than ZA-NLMF and NLMF.

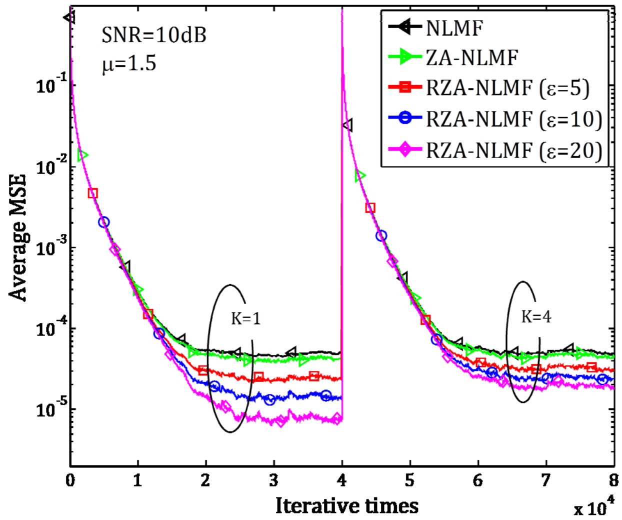

Fig. 9. Average MSE performance comparisons at SNR = 10dB and step-size $\mu = 1.5$.

V. CONCLSION

In this paper, the disadvantage of conventional ASCE method using ZA-NLMF algorithm was discussed from a geometrical perspective point of view. We found that ℓ_1-norm sparse constraint penalizes uniformly on different channel coefficients. Inspired by re-weighted ℓ_1-norm algorithm in CS, we proposed an improved ASCE using RZA-NLMF which penalizes smaller channel coefficients with stronger sparse constraint and vice versa. It was confirmed by computer simulation that the proposed ASCE using RZA-NLMF algorithm achieves better performance than either ZA-NLMF or NLMF approaches.

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REFERENCES

[1] F. Adachi, H. Tomeba, K. Takeda, and S. Members, "Introduction of frequency-domain signal processing to broadband single-carrier transmissions in a wireless channel," *IEICE Transactions on Communicationns*, vol. E92-B, no. 9, pp. 2789–2808, Sept. 2009.

[2] E. Walach and B. Widrow, "The least mean fourth (LMF) adaptive algorithm and its family," *IEEE Transactions on Information Theory*, vol. 30, no. 2, pp. 275–283, Mar. 1984.

[3] E. Eweda, "Global stabilization of the least mean fourth algorithm," *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1473–1477, Mar. 2012.

[4] G. Gui and F. Adachi, "Sparse LMF algorithm for adaptive channel estimation in low SNR region," *International Journal of Communication Systems*, DOI:10.1002/dac.2531, pp. 1-11, Apr. 2013.

[5] G. Gui and F. Adachi, "Sparse least mean forth filter with zero-attracting L1-norm constraint," Submitted for International Conference

on Information, Communications and Signal Processing (ICICS), Tainan, Taiwan, Dec. 10-13, 2013.

[6] E. J. Candès, M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted L1 minimization," *Journal of Fourier Analysis and Applications*, vol. 14, no. 5–6, pp. 877–905, Oct. 2008.

[7] E. Eweda and N. J. Bershad, "Stochastic analysis of a stable normalized least mean fourth algorithm for adaptive noise canceling with a white Gaussian reference," *IEEE Transactions on Signal Processing*, vol. 60, no. 12, pp. 6235–6244, Dec. 2012.

[8] B. Widrow and D. Stearns, *Adaptive signal processing*, New Jersey: Prentice Hall, 1985.

[9] E. J. Candes, "The restricted isometry property and its implications for compressed sensing," *Comptes Rendus Mathematique*, vol. 1, no. 346, pp. 589–592, May 2008.

[10] R. Tibshirani, "Regression shrinkage and selection via the Lasso," *Journal of the Royal Statistical Society (B)*, vol. 58, no. 1, pp. 267–288, Jan. 1996.

[11] D. L. Donoho and Y. Tsaig, "Fast solution of L1-norm minimization problems when the solution may be sparse," *IEEE Transactions on Information Theory*, vol. 54, no. 11, pp. 4789–4812, Nov. 2008.

[12] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[13] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles : exact signal reconstruction from highly incomplete frequency information," *IEEE Transctions on Information Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.