Capacity-Fairness Controllable Scheduling for Uplink Single-carrier FDMA

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Abstract—**The well-known scheduling algorithms are Max-map, Max-Min, and Proportional fairness (PF)-map. Using the above scheduling algorithms, the capacity and the fairness among users vary according to the channel variation. Flexible system design is possible if the sum capacity and the fairness can be controllable. In this paper, assuming single-carrier frequency division multiple access (SC-FDMA) uplink, to control the trade-off between the capacity and the fairness among users, we present modified Max-map, Max-Min, and PF-map. We evaluate, by computer simulation, the sum capacity, the user capacity, and the fairness among users when using the modified scheduling algorithms. It is shown that the modified scheduling algorithms can control the trade-off relationship between the capacity and the fairness among users by changing the number of simultaneously accessing users.**

Keywords-component; SC-FDMA, scheduling

I. INTRODUCTION

The broadband wireless transmission performance suffers from the frequency-selective fading. The orthogonal frequency division multiple-access (OFDMA) [1] is robust against the frequency-selective fading. However, OFDMA has a disadvantage of high peak-to-average ratio (PAPR) and hence, it requires expensive wide-range linear transmit power amplifiers. Therefore, for the uplink transmissions, single carrier-frequency division multiple access (SC-FDMA) [2] has been attracting attention because of its lower PAPR than OFDMA.

When the number of active users is more than the number of available channels, the multi-user scheduling is required. By using multi-user scheduling, multi-user diversity gain is obtained as the number of active users increases [3]. There are 3 types of scheduling algorithm: Max-map scheduling [4], Proportional fairness (PF)-map scheduling [5, 6], and Max-Min scheduling [7]. The sum capacity and the fairness are in a trade-off relationship. The Max-map scheduling can achieve the highest sum capacity while the fairness among users is poor. The PF-map scheduling achieves a good balance between the sum capacity and the fairness. The Max-Min scheduling provides the highest fairness while the sum capacity degrades compared to the Max-map and PF-map scheduling.

Using the above scheduling algorithms, the number of simultaneously accessing users is not constant and varies according to the channel variation. Flexible system design is possible if the capacity and the fairness can be controllable. In this paper, to control the trade-off between the capacity and the fairness among users, we propose modified Max-map, PF-map, and Max-Min scheduling algorithms for SC-FDMA uplink. We evaluate, by computer simulation, the user capacity, the sum capacity, and the fairness among users when using the proposed algorithms. It is shown that the proposed scheduling algorithms can control the trade-off relationship between the capacity and the fairness among users by changing the number of simultaneously accessing users.

The rest of this paper is organized as follows. Section II describes the SC-FDMA uplink transmission model and derives the channel capacity expression. Section III overviews the conventional scheduling algorithms. Section IV presents the modified scheduling algorithms. Section V discusses the computer simulation results and Section VI concludes this paper.

II. SC-FDMA Uplink Transmission Model

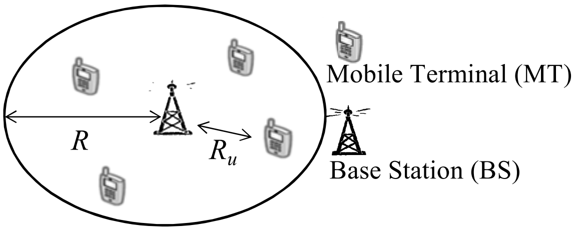

Fig. 1 System model

In this paper, we consider the single-cell and multi-user environment. Fig. 1 shows the system model. The cell radius is denoted by R. The distance between the uth (u=1,…,U) user and the base station (BS) is denoted by R_u. It is assumed that there are U active users in each cell and the scheduling algorithm selects U_t users from U active users for simultaneous access. The total number of available subcarriers is denoted by N_c. The number M of subcarriers allocated to each user is the same for all selected users (i.e., $M=N_c/U_t$).

The channel capacity $C_u(t)$ of the uth user at the tth time-slot can be computed using [8]

$$\begin{aligned} C_u(t) &= \frac{1}{N_c}\sum_{n=0}^{N_c-1} C_u(n,t) \\ &= \frac{1}{N_c}\sum_{n=0}^{N_c-1} \varepsilon_u(n,t)\log_2\left(1+\frac{P_{r,u}}{N}\,|H_u(n,t)|^2\right), \end{aligned} \tag{1}$$

where $C_u(n,t)$ denotes the capacity of the nth subcarrier to the uth user at the tth time-slot. $\varepsilon_u(n,t)$ in Eq. (1) takes 0 or 1 ("$\varepsilon_u(n,t)=1$" indicates that the nth subcarrier is allocated in tth time-slot and "$\varepsilon_u(n,t)=0$" indicates otherwise). $H_u(n,t)$ denotes the nth frequency complex-valued channel gain of the uplink between the uth user and the BS. $N = N_0/T_s$ is the noise power with N_0 and T_s being respectively the single-sided noise power spectrum density of additive white Gaussian noise (AWGN) and the symbol duration. $P_{r,u}$ is the local averaged received signal power (averaging is done over fading) associated with the uth user at BS and is given as

$$P_{r,u} = \overline{P}_t \cdot r_u^{-\alpha} \cdot 10^{-\eta_u/10} \quad , \qquad (2)$$

where $\overline{P}_t = P_t \cdot R^{-\alpha}$ is the normalized MT transmit power with P_t being the MT transmit power. α and η_u are respectively the path loss exponent and the shadowing loss in dB associated with uplink between the uth user and the BS.

III. CONVENTIONAL SCHEDULING METHODS

A. Max-map

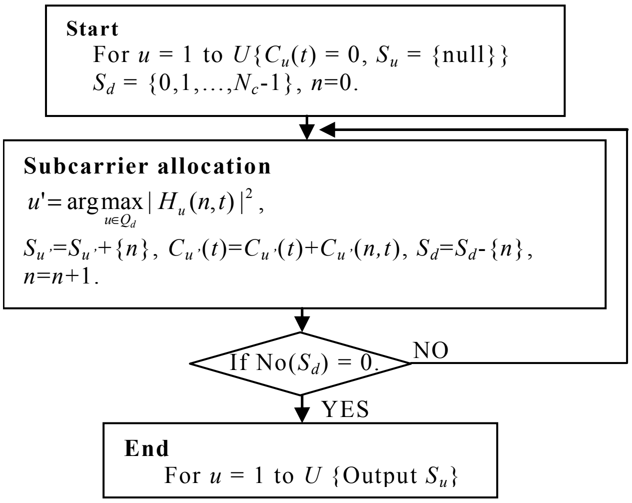

Fig. 2 Conventional Max-map

Max-map scheduling allocates each subcarrier to the users who have the highest channel gain. The Max-map scheduling is carried out based on the following sum capacity maximization problem:

$$\max \sum_{u=1}^{U} C_u(t) \, . \qquad (3)$$

Fig. 2 shows a flowchart of Max-map scheduling algorithm. S_u is a set of subcarriers which have already been allocated to the uth user (M subcarriers are allocated). S_d is a set of subcarriers which have not been allocated yet. Q_d is a set of users to whom the subcarrier allocation has not yet been completed. $\{x\}$ denotes a set of x. No(x) is the number of elements of set $\{x\}$. The user selection for the nth subcarrier is done as

$$u' = \arg\max_{u \in Q_d} |H_u(n,t)|^2 \, . \qquad (4)$$

B. PF-map

PF-map scheduling achieves a good balance between the sum capacity and the fairness among users. The subcarrier allocation is carried out based on the following maximization problem:

$$\max \sum_{u=1}^{U} \sum_{k=0}^{Nc-1} C'_u(n,t) \, . \qquad (5)$$

where $C'_u(n,t)$ denotes the normalized capacity given as [5, 6]

$$C'_u(n,t) = \frac{C_u(n,t)}{\overline{C}_u(t)} \, , \qquad (6)$$

In Eq. (6), $\overline{C}_u(t)$ is the channel capacity averaged over past slots as

$$\overline{C}_u(t) = \begin{cases} \left(1 - \dfrac{1}{T_c}\right)\overline{C}_u(t-1) & u \neq selected \;\; user \\ \left(1 - \dfrac{1}{T_c}\right)\overline{C}_u(t-1) + \dfrac{1}{T_c} C_u(t) & u = selected \;\; user \end{cases} \qquad (7)$$

where T_c is the equivalent averaging interval in slots.

C. Max-Min

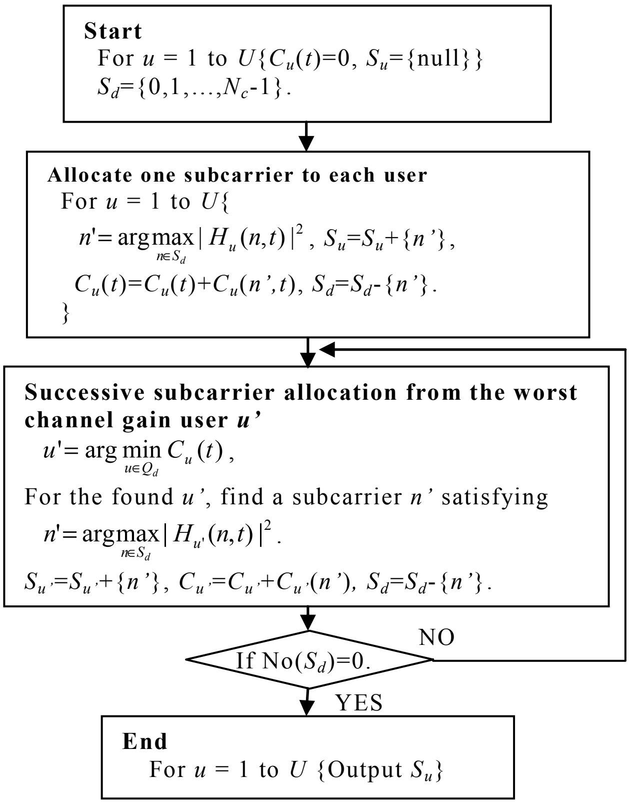

Fig. 3 Conventional Max-Min

Max-Min scheduling allocates subcarriers so as to maximize the fairness based on following maximization problem:

$$\max \min_{u \in U} C_u(t) \, . \qquad (8)$$

In this paper, we use the suboptimal algorithm [7] which nearly achieves the above optimization problem with low complexity. The flowchart of the conventional Max-Min algorithm is shown in Fig. 3.

Step 1: Allocate one subcarrier which has the highest channel gain to each user, i.e., the n'th subcarrier is allocated to uth user according to

$$n' = \arg\max_{n \in S_d} |H_u(n,t)|^2 \ . \tag{9}$$

The subcarrier which has been allocated to a user previously is not allocated to the other users to avoid duplication.

Step 2: Allocate another one subcarrier to the u'th user who satisfies Eq. (9) according to

$$u' = \arg\min_{u \in Q_d} C_u(t) \ . \tag{10}$$

Step 3: Repeat Step2 until all subcarriers are allocated to the users.

This algorithm can be used only when the number of simultaneously accessing users is smaller than the total number of available subcarriers.

IV. THE MODIFIED SCHEDULING ALGORITHM

By using the conventional scheduling algorithms, the capacity and the fairness among users vary according to the channel variation and hence, the trade-off relationship between the capacity and the fairness among users cannot be controlled. We expect that we can control the trade-off by choosing the number of simultaneously accessing users. In this paper, we propose the modified algorithms so as to control the trade-off between the capacity and the fairness among users by selecting the constant number of simultaneously accessing users. In each modified scheduling algorithm, we add the user selection operation and the adjustment operation of the number of allocated subcarriers to the conventional algorithms.

A. Modified Max-map

Figure 4 shows a flowchart of the modified Max-map scheduling algorithm. U_t is the number of simultaneously accessing users. The algorithm of the modified Max-map scheduling is summarized as follows.

Step 1 Conventional Max-map scheduling

Step 2 User-selection

Select the u_{MAX}th user according to

$$u_{MAX} = \arg\max_{u \in Q_d} C_u(t) \ . \tag{11}$$

Then, the u_{MAX}th user is removed from Q_d (i.e., $Q_d = Q_d - \{u_{MAX}\}$).

Step3 Adjustment of number of subcarriers

(a) If No($S_{u_{MAX}}$) is larger than M,

the n'th ($n' \in S_{u_{MAX}}$) subcarrier which satisfies

$$n' = \arg\min_{n \in S_{u_{MAX}}} |H_{u_{MAX}}(n,t)|^2 \tag{12}$$

is removed from the u_{MAX}th user (i.e., $S_{u_{MAX}} = S_{u_{MAX}} - \{n'\}$). The removed subcarrier is allocated to the vth user (i.e., $S_v = S_v + \{n'\}$, $v \in Q_d$) who satisfies

$$v = \arg\max_{v \in Q_d} |H_v(n',t)|^2 \ . \tag{13}$$

The above process is repeated until No($S_{u_{MAX}}$)=M.

(b) If No($S_{u_{MAX}}$) is smaller than M,

the n'th subcarrier ($n' \in (S_d - S_{u_{MAX}})$) which satisfies

$$n' = \arg\max_{n \in S_d - S_{u_{MAX}}} |H_{u_{MAX}}(n,t)|^2 \tag{14}$$

is allocated to the u_{MAX}th user (i.e., $S_{u_{MAX}} = S_{u_{MAX}} + \{n'\}$) from the vth user ($v \in Q_d$). Then, the n'th subcarrier is removed from S_v ($S_v = S_v - \{n'\}$). The above process is repeated until No($S_{u_{MAX}}$)=M.

Step 4 Repeat Step 2 and Step 3 until all subcarriers are allocated to U_t users (i.e., U-No(Q_d)=U_t).

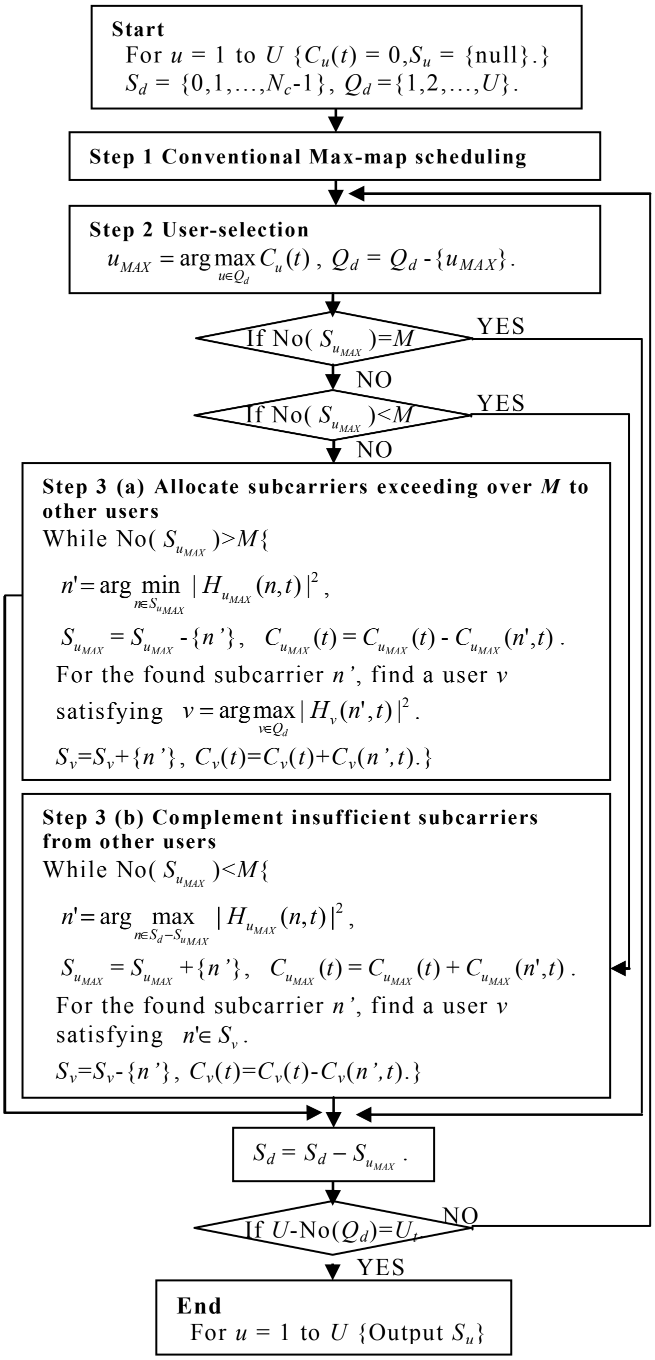

Fig. 4 Modified Max-map algorithm

B. *Modified PF-map*

The modified PF-map scheduling algorithm can be carried out in the same manner as the modified Max-map scheduling algorithm by replacing the conventional Max-map and $C_u(t)$ in Eq. (11) with the conventional PF-map and $C'_u(t)$ used for user-selection, respectively. $C'_u(t)$ is defined as

$$C'_u(t) = \frac{1}{N_c}\sum_{n=0}^{N_c-1} C'_u(n,t)\,. \tag{15}$$

C. *Modified Max-Min*

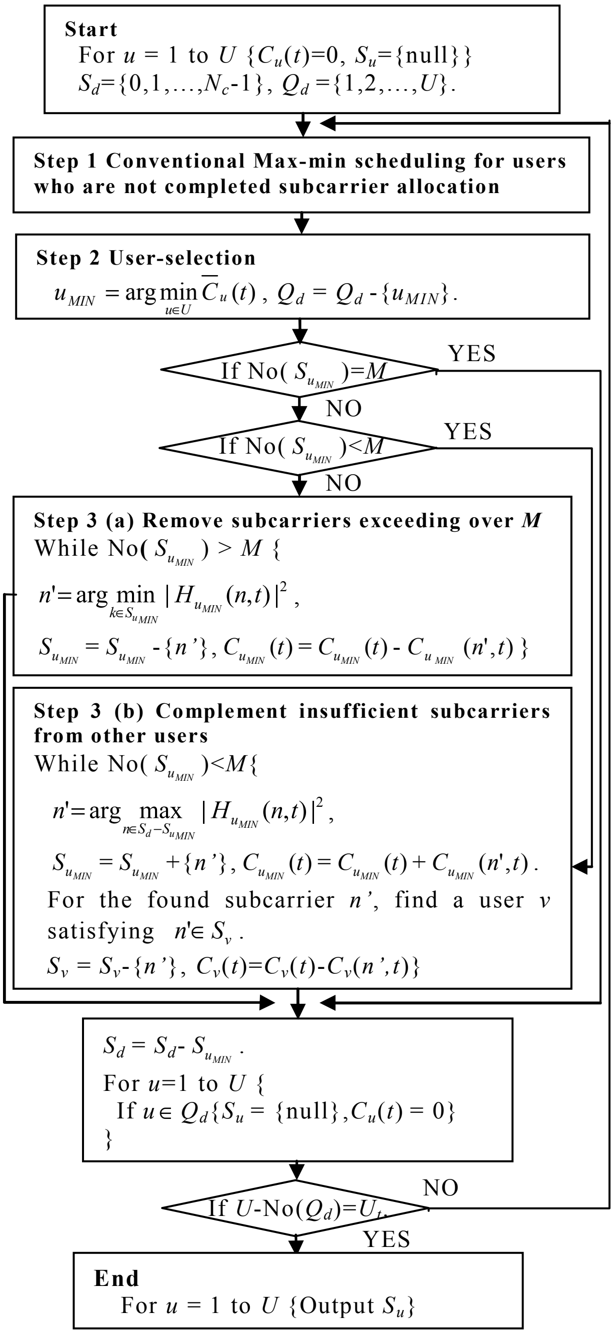

Fig. 5 Modified Max-Min algorithm

Figure 5 shows a flowchart of the modified Max-Min scheduling algorithm. The algorithm of the modified Max-Min scheduling algorithm is summarized as follows.

Step 1 Conventional Max-Min scheduling

Step 2 User-selection

Select the u_{MIN}th user according to

$$u_{MIN} = \arg\min_{u\in U} \overline{C}_u(t)\,. \tag{16}$$

Then, the u_{MIN}th user is removed from Q_d (i.e., Q_d=Q_d-{u_{MIN}}).

Step 3 Adjustment of subcarriers

(a) If No($S_{u_{MIN}}$) is larger than M, the n'th subcarrier satisfying Eq. (12) is removed from $S_{u_{MIN}}$ (i.e., $S_{u_{MIN}} = S_{u_{MIN}}$ -{n'}). The above process is repeated until No($S_{u_{MIN}}$)=M.

(b) If No($S_{u_{MIN}}$) is smaller than M, the n'th subcarrier ($n' \in (S_d - S_{u_{MIN}})$) satisfying Eq. (14) is allocated to the u_{MIN}th user (i.e., $S_{u_{MIN}} = S_{u_{MIN}} + \{n'\}$) from the vth user ($v \in Q_d$). Then, the n'th subcarrier is removed from S_v (i.e., S_v=S_v-{n'}). Above process is repeated until No($S_{u_{MIN}}$)=M.

After (a) or (b), all subcarriers which are allocated to users who have not been completed subcarrier allocation are removed from the users.

Step 4 Repeat Step 2 and Step 3 until all subcarriers are allocated to U_t users (i.e., U-No(Q_d)=U_t).

V. NUMERICAL EVALUATION

We evaluate, by Monte-Carlo numerical computation method, the cumulative distribution function of the channel capacity (CDF) and the fairness index [9]. The fairness index F is given as

$$F = \frac{\left(\sum_{u=0}^{U-1} \overline{C}_u(t)\right)^2}{U\cdot\sum_{u=0}^{U-1} \overline{C}_u(t)^2}\,. \tag{17}$$

As the fairness becomes higher, F approaches to 1. On the other hand, as the fairness becomes lower, F approaches to $1/U$.

TABLE I. NUMERICAL EVALUATION CONDITIONS

Fading type	Block Rayleigh fading
Power delay profile	Uniform
No. of paths	L=16
Time delay	τ=l, l=0~L-1
Total No. of users	U=128
Total no. of subcarriers	N_c=128
No. of simultaneously accessing users	U_t=1,2,4,8,16,32,64,128
Path loss exponent	α=3.5
Shadowing loss standard deviation	η=8.0(dB)
Normalized transmit SNR	$P_{r,u,M\rightarrow B}/N$=10(dB)

The numerical evaluation conditions are summarized in Table 1. The channel is assumed to be an L=16 path frequency-selective block Rayleigh fading channel. The total number of users and the total number of subcarriers are 128. The number of simultaneously accessing users is changed from 1 to 128.

A. Outage capacity and fairness

Figures 6 and 7 plot the relationship between the capacity and the fairness index with the number U_t of simultaneously accessing users as a parameter for the modified scheduling algorithms and the conventional scheduling algorithms. The x%- outage user capacity (fairness index) is the one below which the user capacity (fairness index) falls with a probability of x%. It is seen from Figs. 6 and 7 that, as U_t increases, the fairness consistently improves for the modified Max-map scheduling algorithm while, for the modified PF-map and Max-Min scheduling algorithms, the fairness first improves as U_t increases and then starts to deteriorate beyond U_t=N_c/8. The reason for this is explained as follows. The modified Max-map scheduling algorithm preferentially selects users in a good channel condition. When U_t is small, only users in a good channel condition are selected, resulting in a very low fairness. However, as U_t increases, the number of selected users in a bad channel condition gradually increases and therefore, the fairness among users consistently improves. On the other hand, the modified PF-Map and Max-Min scheduling algorithms preferentially select users in a bad channel condition. As U_t increases, the number of selected users in a good channel condition increases and the fairness among users improves. However, when U_t is too large (i.e., U_t>N_c/8), the number of users in a good channel condition who have higher capacity becomes larger than that of users in a bad channel condition and as a consequence, the fairness among users gets worse.

It is concluded from Figs. 6 and 7 that the trade-off relationship between the capacity and the fairness can be controlled by changing the number of simultaneously accessing users.

VI. CONCLUSION

In this paper, we presented modified Max-map, Max-Min, and PF-map scheduling algorithms for SC-FDMA uplink. The modified algorithms are designed to always choose the predetermined number of simultaneously accessing users irrespective of channel condition. We evaluated, by computer simulation, the user capacity, the sum capacity, and the fairness among users. We discussed the impacts of the number of simultaneously accessing users on the trade-off relationship between the capacity and the fairness. It was confirmed that by changing the number of simultaneously accessing users, the trade-off relationship between the capacity and the fairness can be controlled.

A comparison of our modified scheduling algorithms with other scheduling algorithms in terms of capacity-fairness tradeoff and computational complexity is left for our future study. An application of our modified scheduling algorithms to the OFDMA downlink is also left for our future study.

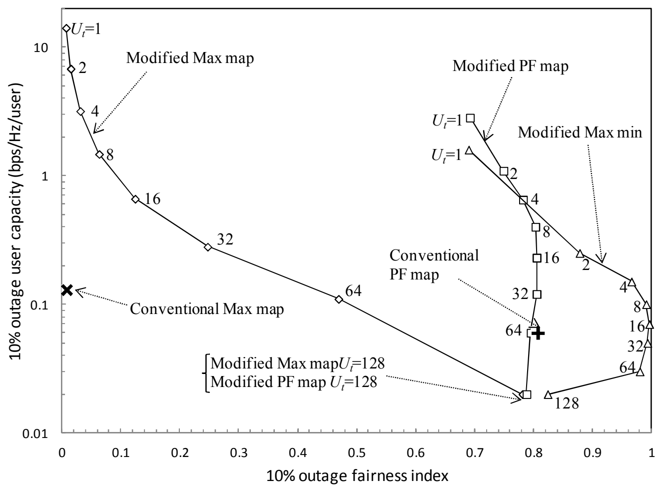

Fig. 6 10%-outage user capacity versus10%-outage fairness index.

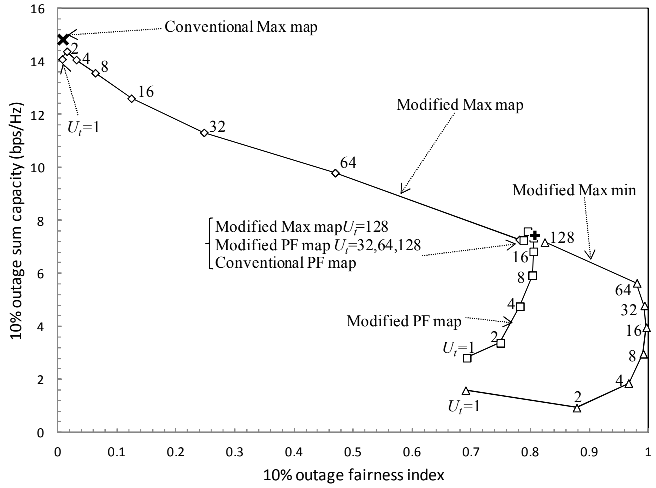

Fig. 7 10%-outage sum capacity versus 10%-outage fairness index.

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