

Bayesian Sparse Channel Estimation and Data Detection for OFDM Communication Systems

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Abstract—Channel state information (CSI) is required at receiver in orthogonal frequency division modulation (OFDM) communication systems due to the fact that frequency-selective fading channel leads to inter-symbol interference (ISI) over data transmission. Broadband channel model is often described by very few dominant channel taps and they can be probed by sparse channel estimation (SCE) methods, e.g., subspace pursuit (SP) algorithm, can take the advantage of sparse structure effectively in broadband channels as for prior information. However, these developed methods are vulnerable to both noise, interference and column coherence of training signal matrix. In other words, the primary objective of these conventional methods is to catch the dominant channel taps without a report of posterior channel uncertainty. To improve the estimation performance, we proposed a Bayesian sparse channel estimation (BSCE) method which not only exploits the channel sparsity but also mitigates the unexpected channel uncertainty. The proposed method can reveal potential ambiguity among multiple channel estimators that are ambiguous due to observation noise or correlation interference among columns in the training matrix. Computer simulations show that our technique can improve the estimation performance with comparable computational complexity when comparing with conventional SCE methods.

Keywords—Bayesian sparse channel estimation (BSCE), data detection, OFDM system, sparse channel representation (SCE).

I. INTRODUCTION

In broadband wireless communication systems using orthogonal frequency division modulation (OFDM), frequency-selective fading is incurred by the reflection, diffraction and scattering of the transmitted signals due to the buildings, large moving vehicles, mountains, etc. Such fading phenomenon distorts received signals and poses critical challenges in the design of communication systems for high-rate and high-mobility wireless communication applications. Hence, accurate channel estimation becomes a fundamental problem of such communication systems. In recent years, various linear estimation methods have been proposed based on the assumption of rich multipath channel model. However, recently, most of physical channel measurements verified that

the channel taps exhibit sparse distribution. A typical example of sparse multipath channel is shown in Fig. 1. To improve the estimation performance, extra sparse structure information can be exploited as prior information. Thanks to the development of compressive sensing [1], [2], many sparse channel estimation (CCE) methods have been proposed for exploiting the channel sparsity. In [3], an orthogonal matching pursuit (OMP) algorithm was proposed with application to sparse multipath channel estimation in the OFDM systems. In [4][5], sparse channel estimation methods have been proposed using a compressive sampling matching pursuit (CoSaMP) algorithm [6] in frequency-selective and doubly-selective channel fading communication systems. In [7], to further reduce the computational complexity, sparse channel estimation using smooth L_0 -norm (SL0) algorithm [8] has been proposed. Compared to traditional linear methods, sparse channel estimation methods have two obvious advantages: spectral efficiency and lower performance bound. On one hand, the spectral efficiency is improved by utilizing less training sequence and we achieve the same estimation performance as linear methods. On the other hand, we obtain a lower performance bound by exploiting channel sparsity due to the fact that less active channel degree of freedom is acquired [9].

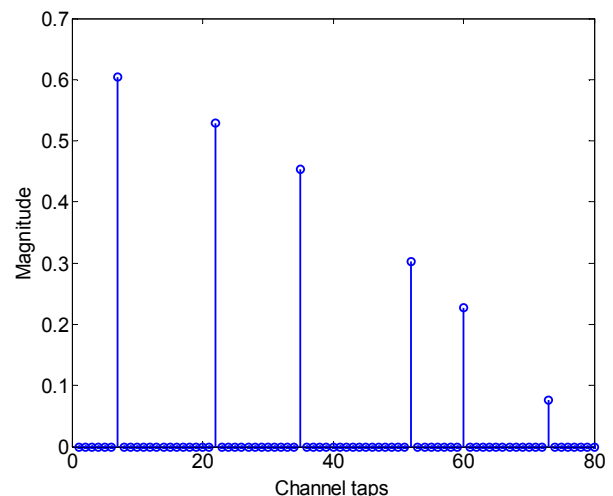


Fig. 1. A typical example of sparse multipath channel.

Conventional sparse channel estimation methods have a cardinal objective that try to probe the dominant channel taps as much as accurate, while neglect the posterior information report from additive noise received signal. These proposed channel estimation methods are termed as model selection or basis selection. Unfortunately, their estimation performances are often degraded due to the neglecting of channel model uncertainty [10]. Unlike the conventional methods, in this paper, we propose a Bayesian sparse channel estimation (BSCE) method using fast Bayesian matching pursuit (FBMP) algorithm [10]. In general, Bayesian approach provides model uncertainty which reveals uncertainty among multiple candidate channel estimators that are ambiguous due to observation noise or correlation among columns in the training matrix. Furthermore, the complexity of the proposed method is comparable with conventional methods. Simulation results are provided to verify performance and complexity. Please note that estimation performance is evaluated by mean-square-error (MSE) and bit-error rate (BER) standard and computational complexity is measured coarsely by CPU time of computer.

The remainder of this paper is organized as follows. An OFDM system model is described and problem formulation is given in Section II. In section III, the BSCE method is proposed in OFDM systems. Computer simulation results are given in Section IV in order to evaluate and compare performance of the BSCE method with conventional methods. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL

Consider a frequency-selective multipath channel whose impulse response is given by

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \quad (1)$$

where L is the number of multipaths, and h_l and τ_l are the (complex) channel gain and the delay spread, respectively, of path l at time t . Hence, the L -length discrete channel vector can be written as $\mathbf{h} = [h_0, h_1, \dots, h_{L-1}]^T$. OFDM system uses size- N discrete Fourier transform (DFT) and its number of pilot subcarriers is N_p . To avoid inter-symbol interference (ISI), we assume that the length N_g of the zero-padding cyclic prefix (CP) in the OFDM symbols is larger than maximum delay spread τ_{\max} , where $\tau_{\max} \geq \tau_l$, $l = 0, 1, \dots, L-1$. Suppose that $X(i)$ denotes the i -th subcarrier in an OFDM symbol, where $i = 0, 1, \dots, N-1$. If the coherence time of the channel is much larger than the OFDM symbol duration T , then the channel can be considered quasi-static over an OFDM symbol. Let $\bar{\mathbf{y}}$ be the vector of received signal samples in one OFDM symbol after DFT, then

$$\bar{\mathbf{y}} = \bar{\mathbf{X}}\mathbf{h} + \bar{\mathbf{z}} = \bar{\mathbf{X}}\mathbf{F}\mathbf{h} + \bar{\mathbf{z}} = \mathbf{X}\mathbf{h} + \mathbf{z}, \quad (2)$$

where $\bar{\mathbf{X}} = \text{diag}\{X(0), X(1), \dots, X(N-1)\}$ denotes the diagonal subcarrier matrix, $\bar{\mathbf{h}}$ is the channel frequency response (CFR) in frequency-domain, $\bar{\mathbf{z}}$ is assumed to be additive white Gaussian noise (AWGN) with variance σ_n^2 . \mathbf{F} is an $N \times L$ partial DFT matrix with its k -th row which is easily given by $1/\sqrt{N} [0, e^{-j2\pi k/N}, \dots, e^{-j2\pi k(L-1)/N}]$ and \mathbf{h} is the time-domain channel vector which is denoted by $\mathbf{h} = [h_0, h_1, \dots, h_{L-1}]^T$. Since $\bar{\mathbf{h}} = \mathbf{F}\mathbf{h}$, hence, the frequency-

domain channel impulse response $\bar{\mathbf{h}}$ lies within the time-delay spread domain.

Consider the sparse channel model and assume that the time-domain channel vectors \mathbf{h} are generated from a random Gaussian mixture density (RGMD) function according to

$$\mathbf{h}|\mathbf{g} \sim \mathcal{CN}(\boldsymbol{\mu}_g, \mathbf{V}_g), \quad (3)$$

where the conditional covariance matrix \mathbf{V}_g is determined by a discrete random vector $\mathbf{g} = [g_0, g_1, \dots, g_{L-1}]^T$ of mixture parameters. To a better understanding, we take \mathbf{V}_g to be diagonal with $[\mathbf{V}_g]_{ll} = \sigma_l^2$, implying that $\{h_l|g_l\}_{l=0}^{L-1}$ are independent with random Gaussian distribution $h_l|g_l \sim \mathcal{CN}(0, \sigma_l^2)$. Assume that the RGMD parameters $\{g_l\}_{l=0}^{L-1}$ are satisfied Bernoulli distribution with probability p_1 , then the probability of nonzero and zero channel taps of channel vector \mathbf{h} can be defined as

$$\text{nonzero taps: } \Pr\{g_l = 1\} = p_1, \quad (4a)$$

$$\text{zero taps: } \Pr\{g_l = 0\} = 1 - p_1. \quad (4b)$$

Let channel taps satisfying distribution as $(\mu_0, \sigma_0^2) = (0, 0)$, when $g_l = 0$ implies $h_l = 0$, and $g_l = 1$ represents $h_l \neq 0$. In addition, to model a sparse channel vector \mathbf{h} , we choose $\sigma_0^2 = 0$ and $p_1 \ll 1$, which ensures that the channel vector \mathbf{h} has relatively few dominant channel taps. In other words, sparseness of channel vector \mathbf{h} depends on the probability p_1 . To get a better understanding, a typical example is depicted in Fig. 2. Assume that channel length is $L = 80$, relationship between number of nonzero taps and probability p_1 can be defined as $p_1 L$. In the figure shown, the smaller probability $\Pr\{g_l = 1, l = 0, 1, \dots, L-1\} = p_1$ implies sparser channel vector \mathbf{h} and vice versa.

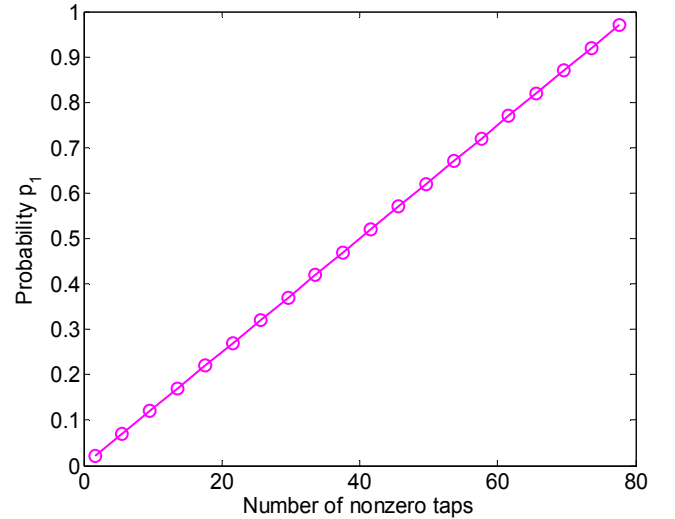


Fig.2. Sparseness of channel vector depends on the probability p_1 .

The research objective of this paper is to estimate the sparse channel vector \mathbf{h} using received signal vector $\bar{\mathbf{y}}$ and equivalent training signal matrix \mathbf{X} . Hence, the system model is assumed to satisfy prior distribution as

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{h} \end{bmatrix} | \mathbf{g} \sim \mathcal{CN} \left(\mathbf{0}, \begin{bmatrix} \mathbf{C}_g & \mathbf{XV}_g \\ \mathbf{V}_g \mathbf{X}^H & \mathbf{V}_g \end{bmatrix} \right), \quad (5)$$

where $\mathbf{C}_g := \mathbf{XV}_g \mathbf{X}^T + \sigma_n^2 \mathbf{I}_M$ is a covariance matrix which depends on input signal matrix \mathbf{X} , conditional covariance matrix \mathbf{V}_g and noise variance $\sigma_n^2 \mathbf{I}_M$.

III. BAYESIAN SPARSE CHANNEL ESTIMATION

According to the well-known Bayesian rules, the posterior of discrete random vector \mathbf{g} can be written

$$P(\mathbf{g} | \mathbf{y}) = \frac{P(\mathbf{y} | \mathbf{g}) P(\mathbf{g})}{\sum_{\mathbf{g}' \in \mathbf{G}} P(\mathbf{y} | \mathbf{g}') P(\mathbf{g}')}, \quad (6)$$

where $\mathbf{G} = \{0, 1\}^L$, which shows that estimating $\{P(\mathbf{g} | \mathbf{y})\}_{\mathbf{g} \in \mathbf{G}}$ reduces to estimating $\{P(\mathbf{y} | \mathbf{g}) P(\mathbf{g})\}_{\mathbf{g} \in \mathbf{G}}$. Due to the extremely computational complexity, the size of \mathbf{G} makes it impractical to compute $P(\mathbf{g} | \mathbf{y})$ or $\{P(\mathbf{y} | \mathbf{g}) P(\mathbf{g})\}$ for all $\mathbf{g} \in \mathbf{G}$. However, the set \mathbf{G}_* responsible for dominant posteriors can be quite small and therefore practical to compute. Since the probability density function (PDF) $P(\mathbf{y} | \mathbf{g})$ can be written

$$P(\mathbf{y} | \mathbf{g}) = \frac{1}{\pi^M \det(\mathbf{C}_g)} \exp \left[-(\mathbf{y} - \mathbf{X}(\mu_g))^H \mathbf{C}_g^{-1} (\mathbf{y} - \mathbf{X}(\mu_g)) \right], \quad (7)$$

By transformed it in log-domain, then the log-domain PDF $V(\mathbf{g}, \mathbf{y})$ can be given by

$$\begin{aligned} V(\mathbf{g}, \mathbf{y}) &= \log(P(\mathbf{y} | \mathbf{g})) \\ &= -(\mathbf{y} - \mathbf{X}(\mu_g))^H \mathbf{C}_g^{-1} (\mathbf{y} - \mathbf{X}(\mu_g)) \\ &\quad - M \ln \pi - \ln[\det(\mathbf{C}_g)] \\ &\quad + \|\mathbf{g}\|_0 \ln \frac{p_1}{1 - p_1} + L \ln(1 - p_1). \end{aligned} \quad (8)$$

Due to the positive exponent relationship $P(\mathbf{g} | \mathbf{y}) = e^{V(\mathbf{g}, \mathbf{y})}$, $V(\mathbf{g}, \mathbf{y})$ in (8) can be considered as measure function on dominant channel taps of $P(\mathbf{g}, \mathbf{y})$. According to the derivation in [11], the mathematical expectation of the $V(\mathbf{g}, \mathbf{y})$ is given by

$$\begin{aligned} E\{V(\mathbf{g}, \mathbf{y})\} &= 2M + L p_1 (1 - p_1) \cdot \\ &\quad (\ln[(\sigma_1^2 / \sigma^2 + 1)(1 - p_1) / p_1])^2. \end{aligned} \quad (9)$$

For a given pair $\{\mathbf{g}', \mathbf{y}\}$, the measure function $V(\mathbf{g}', \mathbf{y})$ can be used to compare the mean $E\{V(\mathbf{g}, \mathbf{y})\}$ and standard deviation $\sqrt{\text{var}\{V(\mathbf{g}, \mathbf{y})\}}$ in order to get a rough evaluation of $\{\mathbf{g}', \mathbf{y}\}$. Here, we define $[\mathbf{g}]_l = q$ and $[\mathbf{g}']_l = q'$, and then describe an efficient method to compute

$$d(\mathbf{g}, \mathbf{y}) \triangleq V(\mathbf{g}', \mathbf{y}) - V(\mathbf{g}, \mathbf{y}) \begin{cases} \beta_{n,q'} \left| \mathbf{c}_n^H (\mathbf{y} - \mathbf{X} \mu_g) + \mu_g / \sigma_{q',q}^2 \right| - |\mu_{q',q}|^2 / \sigma_{q',q}^2 \\ \quad + \ln(\beta_{n,q'} / \sigma_{q',q}^2) + \ln(\lambda_{q'} / \lambda_q), & \sigma_{q',q}^2 \neq 0 \\ 2 \text{Re}\{\mu_{q',q}^* \mathbf{c}_n^H (\mathbf{y} - \mathbf{X} \mu_g)\} - |\mu_{q',q}|^2 \mathbf{c}_n^H \mathbf{x}_n \\ \quad + \ln(\lambda_{q'} / \lambda_q), & \sigma_{q',q}^2 = 0 \end{cases} \quad (10)$$

TABLE I. PROPOSED METHOD USING FBMP ALGORITHM.

| |
|--|
| $v^0 = -1/\sigma^2 \ \mathbf{y}\ _2^2 - M \ln(\sigma^2 \pi) + N \ln \lambda_0$ |
| For $n = 0: N - 1$ $\mathbf{c}_n^0 = 1/\sigma^2 \mathbf{x}_n$ $v^0 = \sigma_1^2 (1 + \sigma_1^2 \mathbf{x}_n^H \mathbf{c}_n^0)^{-1}$ |
| For $q = 0: Q - 1$ $v_{n,q}^0 = v^0 + \ln \beta_n^0 / \sigma_1^2 + \beta_n^0 (\mathbf{c}_n^0)^H \mathbf{y} + \mu_q / \sigma_1^2 ^2$ $\quad + \ \mathbf{y}\ _2^2 - \mu_q / \sigma_1^2 + \ln \lambda_1 / \lambda_0$ |
| End |
| End |
| For $d = 1: D_{\max}$ |
| $\mathbf{n} = []$; $\mathbf{q} = []$; $\hat{\mathbf{s}}^{(d,0)} = \mathbf{0}$; $\mathbf{z} = \mathbf{y}$; $\mathbf{c}_n = \mathbf{c}_n^0$; $\beta_n = \beta_n^0$ |
| For $q = 1: Q - 1$ $v_{n,q} = v_{n,q}^0$ |
| End |
| For $p = 1: P$ $(n_*, q_*) = (n, q)$ denotes largest coefficient position in $\{v_{n,q}\}_{n=0:N-1}^{q=0:Q-1}$, which leads to an as-of-yet unexplored node $v_{n,q} = v_{n_*,q_*}$; $\hat{\mathbf{s}}^{(d,p)} = \hat{\mathbf{s}}^{(d,p-1)} + q_* \delta_{n_*}$; $\mathbf{n} \leftarrow [\mathbf{n}, n_*]^T$; $\mathbf{q} \leftarrow [\mathbf{q}, q_*]^T$; $\mathbf{z} \leftarrow \mathbf{z} - \mathbf{x}_{n_*} \mu_{q_*}$; |
| For $n = 0: N - 1$ $\mathbf{c}_n \leftarrow \mathbf{c}_n - \beta_{n_*} \mathbf{c}_{n_*} \mathbf{c}_{n_*}^H \mathbf{x}_n$; $\beta_n = \sigma_1^2 (1 + \sigma_1^2 \mathbf{x}_n^H \mathbf{c}_n)^{-1}$; |
| For $q = 0: Q - 1$ $v_{n,q} = v^{(d,p)} + \ln \beta_n / \sigma_n^2 + \beta_n \mathbf{c}_n^H \mathbf{z} + \mu_q / \sigma_1^2 ^2$ $\quad - \mu_q / \sigma_1^2 + \ln \lambda_1 / \lambda_0$ |
| End |
| End $\hat{\mathbf{h}}^{(d,q)} = \sum_{k=1}^p \sigma_{[n]_k} [\sigma_1^2 \mathbf{c}_{[n]_k}^H + \mu_{[n]_k}]$; $\hat{\Sigma}^{(d,q)} = \sigma_1^2 \sum_{k=1}^p \sum_{j=1}^p \delta_{[n]_k} [\sigma_{[n]_k - [n]_j} - \sigma_1^2 \mathbf{c}_{[n]_k}^H \mathbf{x}_{[n]_j} \delta_{[n]_j}^T]$; |
| End |
| If $\max\{v^{(d,p)}\}_{p=1:P} > v_{\text{thresh}}$, then break; |
| End |

IV. COMPUTER SIMULATIONS

In this section, the proposed BSCE estimator using 1000 independent Monte-Carlo runs for averaging. The length of channel vector \mathbf{h} is set as $N = 80$. Values of dominant channel taps follow Gaussian distribution and their positions are randomly allocated within the length of \mathbf{h} which is subjected to $E\{\|\mathbf{h}\|_2^2\} = 1$. The received signal-to-noise ratio (SNR) is defined as $10 \log(E_0 / \sigma_n^2)$, where $E_0 = 1$ is normalized power of input signal.

The proposed method is compared to five conventional sparse channel estimation methods using algorithms: OMP [12], CoSaMP [6], BCS [13], BCS-LAP [14] and SL0 [15]. It is worth mentioning that these simulation parameters were chosen in accordance with original papers provided by the authors. The parameters of FBMP algorithm were initialized

as $\lambda_1 = 0.01$, $\mu_1 = 0$, $\sigma^2 = 0.05$, and $\sigma_1^2 = 2$. Computer simulation parameters are listed in Tab. II.

TABLE II. SIMULATION PARAMETERS.

| | | |
|---------------|-----------------------|---------------------------|
| Transmitter | Data modulation | BPSK |
| | No. of subcarrier | $N = 512$ |
| | No. of pilot symbol | $M = 40$ |
| | Length of CP | $N_g = 128$ |
| | Pilot sequence | Random Gaussian sequence |
| Channel model | Fading | Frequency-selective block |
| | No. of channel taps | $L = 80$ |
| | Prob. of nonzero taps | $p = 0.1$ |
| | Power delay profile | Uniform |
| Receiver | Channel estimation | BSCE |
| | Data detection | Zero forcing |

A. MSE versus SNR

The estimation performance is evaluated by average mean square error (MSE) standard which is defined by

$$\text{Average MSE}\{\hat{\mathbf{h}}\} = E\{\|\mathbf{h} - \hat{\mathbf{h}}\|_2^2\}, \quad (11)$$

where $E\{\cdot\}$ denotes the expectation operator, \mathbf{h} and $\hat{\mathbf{h}}$ are the actual channel vector and its channel estimator, respectively. In Fig. 3, we compare the average MSE performance of the proposed channel estimator with traditional sparse channel estimators. The lower bound is given by least square (LS) method which utilizes the channel position information. We observe that the proposed method obtains a better MSE performance than conventional methods. In other words, if the proposed estimator is applied in data detection, better BER performance can be achieved when compared with conventional methods.

B. BER versus SNR

By using above channel estimators, data detection performances are evaluated as shown in Fig. 4. From this figure, average BER performance curves are depicted with respect to SNR for binary phase shift keying (BPSK) data. We can see that the BER performance of the proposed method is close to lower bound which is given by ideal channel estimator whose nonzero taps' positions are known.

C. Complexity evaluation

To compare the computational complexity of the proposed method with other methods, CPU time is adopted for evaluation standard as shown in Fig. 5. It is worth mentioning that although the CPU time is not an exact measure of complexity, it can still give us a rough estimation. Our simulation is performed in MATLAB 2012 environment using a 2.90GHz Intel i7 processor with 8GB of memory and under Microsoft Windows 8 64 bit operating system. This figure shows that the complexity of the proposed method is close to OMP and SL0-based methods and lower than CoSaMP, BCS

and BCS-LAP based methods. Please note that the complexity of OMP and SL0 is very low for sparse channel estimation.

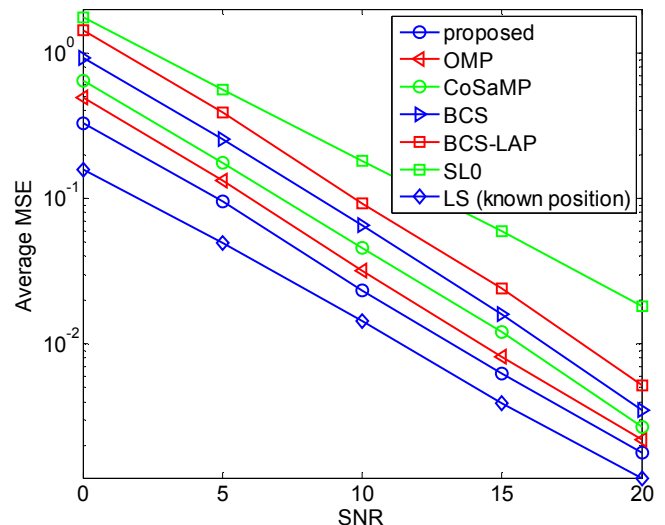


Fig. 3. Average MSE performance verses SNR.

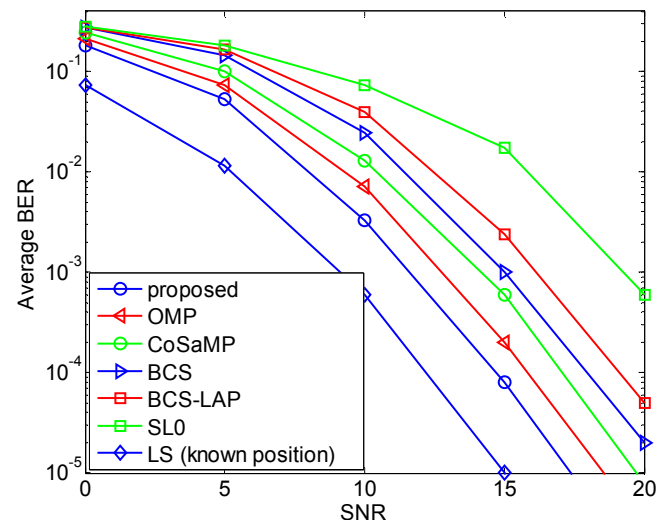


Fig. 4. Average BER performance verses SNR.

V. CONCLUSION

Traditional sparse channel estimation methods are vulnerable to noise and column coherence interference in training matrix. Their primary aim is to try to exploit sparse structure information without a report of posterior channel uncertainty. In this paper, we introduced a fast Bayesian matching pursuit algorithm with application to sparse channel estimation which not only exploited the channel sparsity but also mitigated the unexpected interferences in training matrix. In addition, the propose method revealed a potential ambiguity among multiple channel estimators that are ambiguous due to observation noise or correlation among columns in the training signal. Computer simulation results verified that our propose method improved the estimation performance with comparable

computational complexity when compared to traditional methods.

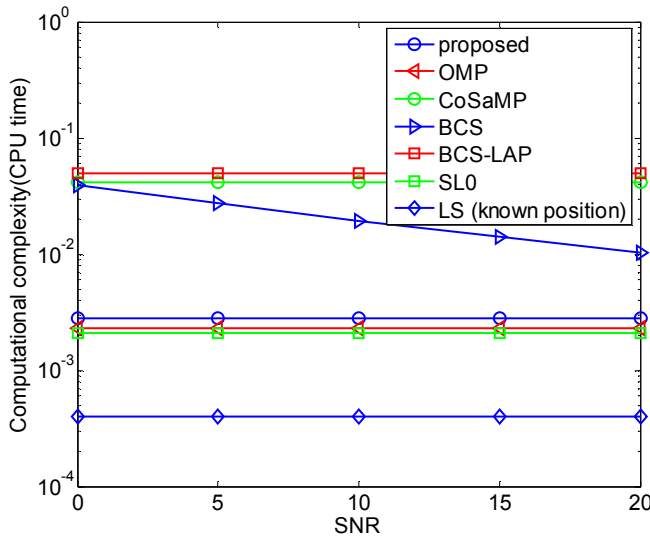


Fig. 5. Computational complexity comparison via CPU time.

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