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Least Mean Square/Fourth Algorithm for Adaptive Sparse Channel Estimation

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***Abstract*—Broadband signal transmission over frequency-selective fading channel often requires accurate channel state information at receiver. One of the most attracting adaptive channel estimation (ACE) methods is least mean square (LMS) algorithm. However, its performance is often degraded by random scaling of input training signal. To overcome this degradation, in this paper we consider the use of standard *least mean square/fourth* (LMS/F) algorithm. Since the broadband channel is often described by sparse channel model, such sparsity could be exploited as prior information. First, we propose an adaptive sparse channel estimation (ASCE) method with *zero-attracting LMS/F* (ZA-LMS/F) algorithm by introducing an ℓ_1-norm sparse constraint into the cost function. Then, to exploit the sparsity more effectively, an improved ASCE with *reweighted zero-attracting LMS/F* (RZA-LMS/F) algorithm is proposed. For different channel sparsity, we propose a Monte Carlo method for a regularization parameter selection in RA-LMS/F and RZA-LMS/F to achieve better steady-state estimation performance. Simulation results show that the proposed ASCE methods achieve better estimation performance than the conventional one.**

Keywords—least mean square fourth (LMS/F), adaptive sparse channel estimation (ASCE), zero-attracting least mean square/fourth (ZA-LMS/F), re-weighted zero-attracting least mean square/fourth (RZA-LMS/F).

I. INTRODUCTION

Broadband signal transmission is becoming one of the mainstream techniques in the next generation communication systems [1]. Accurate channel state information (CSI) of frequency-selective fading channel is necessary at receiver for coherent detection. One of effective approaches is an adaptive channel estimation (ACE) using standard least mean square (LMS) algorithm [2]. A typical framework of ACE is shown in Fig. 1. The merit of the LMS algorithm is its low complexity and easy implementation at the receiver. However, it cannot achieve good steady-state estimation performance due to the fact that it depends highly on random scaling of *input training signal*, *signal transmit power* and *noise power* [2]. It is well known that ACE using least mean fourth (LMF) algorithm outperforms the LMS algorithm by balancing convergence speed and steady-state performance [3].

To fully benefit from the aforementioned merits of LMS and LMF, the least mean square/fourth (LMS/F) algorithm is proposed in [4][5]. The LMS/F algorithm improves the mean square error (MSE) estimation performance of the LMS algorithm without sacrificing the simplicity and stability properties.

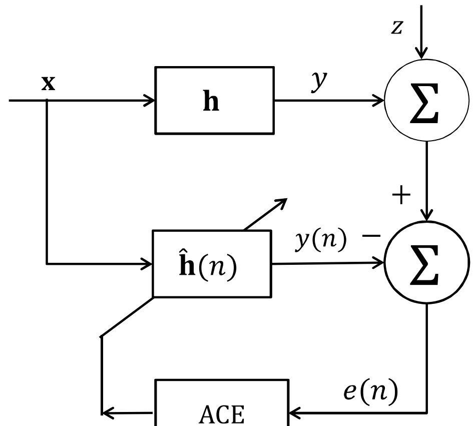

Fig. 1. ACE for broadband communication systems.

Recently, many channel measurements have verified that broadband channels often exhibit sparse structure [6][7]. The sparse channel is composed of a very few channel coefficients and most of them are zeros. A typical example of sparse channel is shown in Fig. 2, where the length of finite impulse response (FIR), N, is 16 and the number of dominant coefficients, K, is 4. Unfortunately, ACE with LMS/F algorithm always neglects this inherent sparse structure information. Hence, it may not be able to achieve the estimation performance comparable to sparse ACE method which exploits the channel sparsity. In this paper, we propose two adaptive sparse channel estimation (ASCE) with *sparse* LMS/F algorithms, namely zero-attracting least mean square/fourth (ZA-LMS/F) and reweighted zero-attracting least mean square/fourth (RZA-LMS/F). Inspired by least absolute shrinkage and selection operator (LASSO) algorithm [8], an ℓ_1-norm sparse constraint function is introduced to exploit channel sparsity.

The main contribution of this paper is to propose the sparse LMS/F algorithms for ASCE. Sparse penalized cost functions are constructed for implementing the sparse LMS/F algorithms. Computer simulations are conducted to confirm the effectiveness of our proposed algorithms. The impacts of channel sparsity, K, and the reweighted factors on the average MSE performance of sparse LMS/F algorithms are evaluated.

The remainder of this paper is organized as follows. A system model is described and standard LMS/F algorithm is introduced in Section II. In section III, sparse ASCE using ZA-LMS/F algorithm is introduced and improved ACSE using RZA-LMS/F algorithm is highlighted. Computer simulations are presented in Section IV in order to evaluate and compare performances of the proposed ASCE methods. Finally, we conclude the paper in Section V.

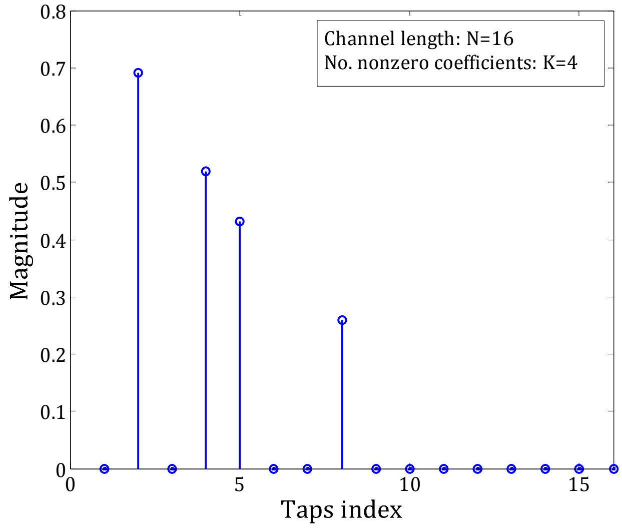

Fig. 2. A typical example of sparse multipath channel.

II. STANDARD LMS/F ALGORITHM

Consider a baseband frequency-selective fading wireless communication system where finite impulse response (FIR) sparse channel vector $\mathbf{h}=[h_0,h_1,\dots,h_{N-1}]^T$ is N-length and it is supported only by K nonzero channel taps. Assume that an input training signal $x(t)$ is used to probe the unknown sparse channel. At the receiver side, observed signal $y(t)$ is given by

$$y(t)=\mathbf{h}^T\mathbf{x}(t)+z(t), \quad (1)$$

where $\mathbf{x}(t)=[x(t),x(t-1),\dots,x(t-N+1)]^T$ denotes the vector of training signal $x(t)$ and $z(t)$ is the additive white Gaussian noise (AWGN), which is assumed to be independent with $x(t)$. The objective of ASCE is to adaptively estimate the unknown sparse channel vector $\mathbf{h}$ using the training signal vector $\mathbf{x}(t)$ and the observed signal $y(t)$. By defining the received error at the n-th update by $e^2(n)$, we can apply standard LMS/F algorithm whose cost function is given as [3]

$$G_{LMSF}(n)=\frac{1}{2}e^2(n)-\frac{1}{2}\lambda\ln(e^2(n)+\lambda), \quad (2)$$

where λ is a positive threshold parameter which controls the convergence speed and stability of the LMS/F algorithm. Here, please note that low convergence speed of the algorithm in (2) means to high computational complexity and vice versa. According to the cost function in (2), corresponding update equation of LMS/F algorithm is given by

$$\hat{\mathbf{h}}(n+1)=\hat{\mathbf{h}}(n)+\mu\frac{\partial G_{LMSF}(n)}{\partial\hat{\mathbf{h}}(n)}$$

$$=\hat{\mathbf{h}}(n)+\mu\frac{e^3(n)\mathbf{x}(t)}{e^2(n)+\lambda}, \quad (3)$$

where μ is the update step-size which controls algorithm stability and gradient descend speed of LMS/F algorithm. The LMS/F algorithm in (3) behaves like the standard LMF with a step size of μ/λ for $\lambda\gg e^2(n)$ and it reduces to the standard LMS algorithm with a step size of μ for $\lambda\ll e^2(n)$. It is necessary to choose λ properly to balance between stability and MSE estimation performance of LMS/F algorithm. For achieving the better steady-state performance without scarifying algorithm stability, we choose optimal λ according to the proposed method in [5]. Setting $e^2(n)=0.1$ for example, λ controls the variable step-size as shown in Fig. 3. If we fix $e^2(n)$, smaller λ achieves smaller step-size μ which ensures LMS/F more stable and better estimation but at the cost of higher computational complexity (larger number of iterations), and vice versa.

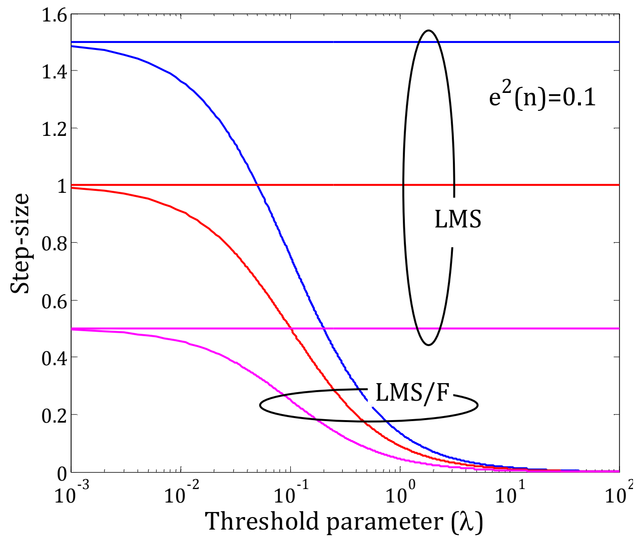

Fig.3. Threshold parameter (λ) controls the variable step-size of LMS/F algorithm.

III. SPARSE LMS/F ALGORITHMS

A. ASCE with ZA-LMS/F algorithm

Recall that the standard LMS/F algorithm in (2) does not make use of the channel sparsity. This is because the original cost function $G_{LMSF}(n)$ utilizes neither sparse constraint nor penalty function. Thus, to exploit the channel sparsity, we introduce an ℓ_1-norm sparse constraint [8] to the cost function in (2) and obtain a new cost function as

$$G_{ZA}(n)=\underbrace{\frac{1}{2}e^2(n)-\frac{1}{2}\lambda\ln(e^2(n)+\lambda)}_{G_{LMSF}(n)}+\rho_{ZA}\|\hat{\mathbf{h}}(n)\|_1, \quad (4)$$

where $\|.\|_1$ is the ℓ_1-norm operation and ρ_{ZA} denotes *a regularization parameter* which balances the error term $G_{LMSF}(n)$ and sparsity of $\hat{\mathbf{h}}(n)$. For better understanding of the difference between $G_{LMSF}(n)$ and $G_{ZA}(n)$, geometrical interpretation is shown in Fig. 4. Whereas $G_{LMSF}(n)$ cannot find sparse solution (convex point) in solution plane, $G_{ZA}(n)$

can find a unique sparse solution (convex point) in solution plane by using sparse constraint. The update equation of ZA-LMS/F algorithm is given by

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu\frac{\partial G_{ZA}(n)}{\partial\hat{\mathbf{h}}(n)} = \hat{\mathbf{h}}(n) + \mu\frac{e^3(n)\mathbf{x}(t)}{e^2(n)+\lambda} + \gamma_{ZA}\mathrm{sgn}\left(\hat{\mathbf{h}}(n)\right), \quad (5)$$

where $\gamma_{ZA} = \mu\rho_{ZA}$ and $\mathrm{sgn}(\cdot)$ denotes the sign function which is given by

$$\mathrm{sgn}\left(\hat{\mathbf{h}}(n)\right) = \frac{\partial\left\|\hat{\mathbf{h}}(n)\right\|_1}{\partial\hat{\mathbf{h}}(n)} = \begin{cases} 1, & \hat{h}_i(n) > 0 \\ 0, & \hat{h}_i(n) = 0, \\ -1, & \hat{h}_i(n) < 0 \end{cases} \quad (6)$$

where $\hat{\mathbf{h}}(n) = \left[\hat{h}_0(n), \ldots, \hat{h}_i(n) \ldots, \hat{h}_{N-1}(n)\right]^T$ and $i \in \{0,1,\ldots,N\}$.

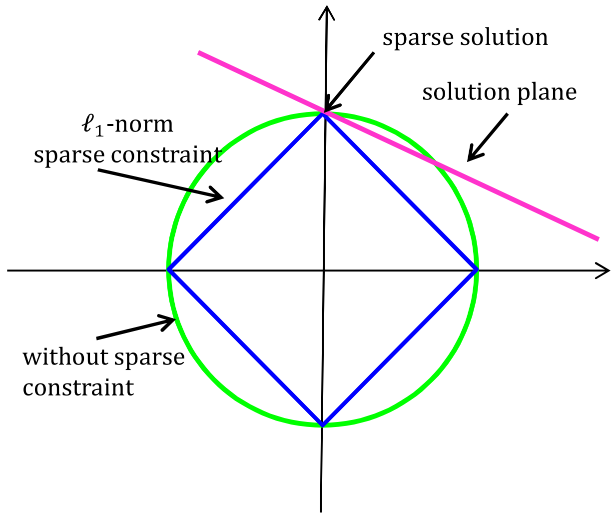

Fig. 4. Sparse channel estimation with ℓ_1-norm sparse constraint.

B. *Improved ASCE method with RZA-LMS/F algorithm*

The ZA-LMS/F algorithm cannot distinguish between zero taps and non-zero taps as all the taps are forced to zero uniformly as show in Fig. 5. Thus, ZA-LMS/F based approach would degrade the estimation performance. Motivated by reweighted ℓ_1-minimization sparse recovery algorithm [9] in compressed sensing (CS) [10], [11], we propose an improved ASCE method with RZA-LMS/F algorithm. The cost function of this method is constructed by

$$G_{RZA}(n) = \frac{1}{2}e^2(n) - \frac{1}{2}\lambda\ln(e^2(n)+\lambda) + \rho_{RZA}\sum_{i=0}^{N-1}\log\left(1+\left|\hat{h}_i(n)\right|/\varepsilon\right), \quad (7)$$

where $\rho_{RZA} > 0$ is *a regularization parameter* which trades off the estimation error and channel sparsity. The corresponding update equation is

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu\frac{\partial G_{RZA}(n)}{\partial\hat{\mathbf{h}}(n)} = \hat{\mathbf{h}}(n) + \mu\frac{e^3(n)\mathbf{x}(t)}{e^2(n)+\lambda} + \gamma_{RZA}\frac{\mathrm{sgn}\left(\hat{\mathbf{h}}(n)\right)}{1+\varepsilon\left|\hat{\mathbf{h}}(n)\right|}, \quad (8)$$

where $\gamma_{RZA} = \mu\rho_{RZA}/\varepsilon$ is a parameter which depends on step-size μ_f, regularization parameter ρ_{RZA} and threshold ε. In the second term of (8), the estimated channel coefficient $\left|\hat{h}_i(n)\right|, i = 0,1,\ldots,N-1$ is replaced by zeros in high probability if it is smaller than $1/\varepsilon$.

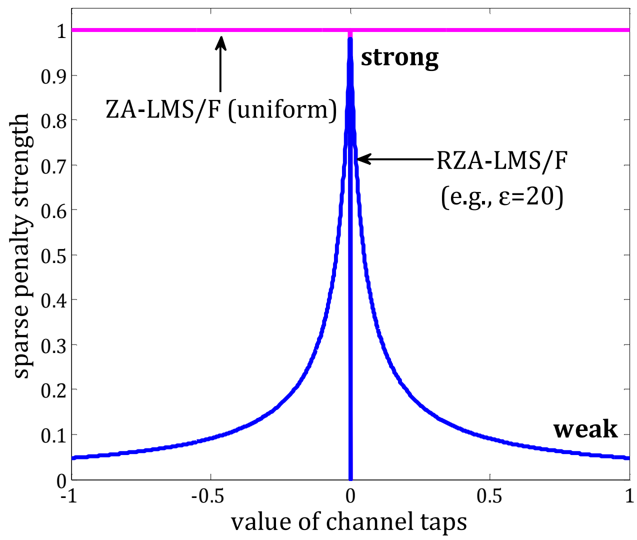

Fig. 5. Sparse penalty strength over different value of channel taps.

C. *Regularization parameter selection for sparse LMS/F algorithms*

It is well known that regularization parameter is very important for LASSO based sparse channel estimation [8]. In [11], a parameter selection method was proposed for LASSO based partial sparse channel estimation. To the best of our knowledge, however, there is no report on regularization parameter selection method for ASCE. Here, we propose an approximate optimal selection method by Monte Carlo simulation which adopts 1000 runs for achieving average performance. Parameters for computer simulation are given in Table. I. The estimation performance is evaluated by average MSE which is defined by

$$\text{Avergae MSE}\{\hat{\mathbf{h}}(n)\} = \mathrm{E}\left\{\left\|\mathbf{h} - \hat{\mathbf{h}}(n)\right\|_2^2\right\}, \quad (9)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator, $\mathbf{h}$ and $\hat{\mathbf{h}}(n)$ are the actual channel vector and its n-th iterative adaptive channel estimator, and $\|\cdot\|_2$ is the Euclidean norm operator and $\|\mathbf{h}\|_2^2 = \sum_{i=1}^N |h_i|^2$, respectively.

Utilizing different regularization parameters, performance curves of ZA-LMS/F and RZA-LMS/F are depicted in Fig. 6 and Fig. 7, respectively. Fig. 6 shows that MSE performance is near optimal with $\rho_{ZA} = 0.0004$ and $\rho_{ZA} = 0.0002$ for $K = 2$ and $K = 4$, respectively. Likewise, in Fig. 7, choosing approximate optimal regularization parameters $\rho_{RZA} = 0.02$

and $\rho_{RZA} = 0.01$ for RZA-LMS/F can achieve near better steady-state estimation performance when $K = 2$ and $K = 4$, respectively. In the following, these parameters will be utilized for performance comparison with sparse LMS algorithms.

TAB. I. SIMULATION PARAMETERS.

parameters	values
channel length	$N = 16$
no. of nonzero coefficients	$K = 2$ and 4
step-size	$\mu = 0.05$
threshold parameter	$\lambda = 0.8$
re-weighted factor for RZA-LMS/F	$\varepsilon = 20$

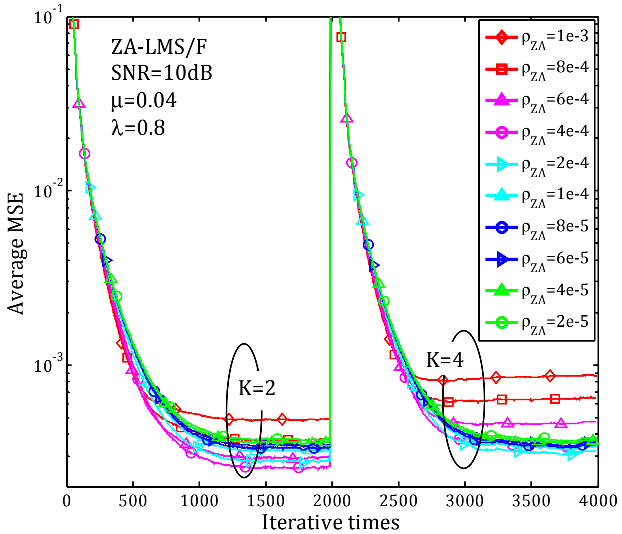

Fig.6. ZA-LMS/F based sparse channel estimation performance depends on regularization parameter ρ_{ZA}.

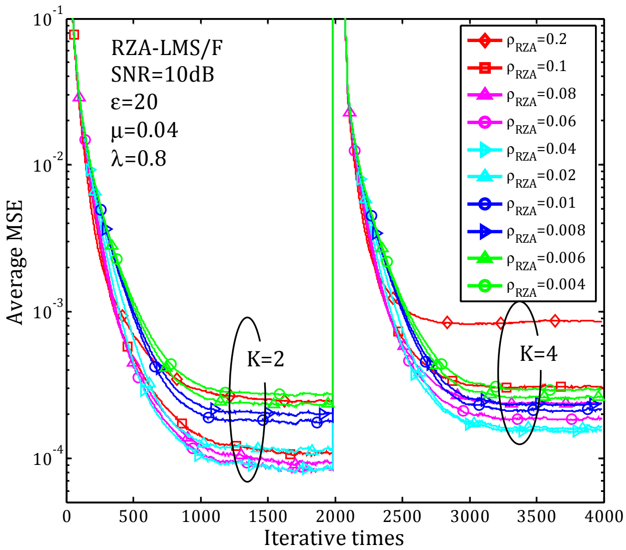

Fig.7. RZA-LMS/F based sparse channel estimation performance depends on regularization parameter ρ_{RZA}.

IV. COMPUTER SIMULATIONS

In this section, the average MSE performance of the proposed ASCE methods using (R)ZA-LMS/F algorithm is evaluated. The results are averaged over 1000 independent Monte-Carlo runs. The length of channel vector $\mathbf{h}$ is set as $N = 16$ and its number of dominant taps is set to $K = 2$ and 4, respectively. Each dominant channel tap follows random Gaussian distribution as $\mathcal{CN}(0, \sigma_{\mathbf{h}}^2)$ which is subject to $E\{||\mathbf{h}||_2^2 = 1\}$ and their positions are randomly decided within the length of $\mathbf{h}$. The received signal-to-noise ratio (SNR) is defined as $10\log(E_0/\sigma_n^2)$, where $E_0 = 1$ is the unit transmission power. Here, we set the SNR as 10dB. All of the step sizes and regularization parameters are listed in Table. II.

TAB. II. SIMULATION PARAMETERS.

parameters	values
channel length	$N = 16$
no. of nonzero coefficients	$K = 2$ and 4
distribution of nonzero coefficient	random Gaussian $\mathcal{CN}(0,1)$
threshold parameter for LMS/F-type	$\lambda = 0.8$
SNR	10dB
step-size	$\mu = 0.04$
regularization parameter for $K = 2$	$\rho_{ZA} = 0.0004$ and $\rho_{RZA} = 0.04$ $\rho_{ZAS} = 0.008$ and $\rho_{RZAS} = 0.8$
regularization parameter for $K = 4$	$\rho_{ZA} = 0.0002$ and $\rho_{RZA} = 0.02$ $\rho_{ZAS} = 0.004$ and $\rho_{RZAS} = 0.4$
re-weighted factor for RZA-LMS(/F)	$\varepsilon = 20$

Firstly, average MSE performance of proposed methods is evaluated for $K = 2$ and 4. To confirm the effectiveness of the proposed methods, we compare them with sparse LMS algorithms, i.e., ZA-LMS and RZA-LMS [12]. For a fair comparison, we utilize the same step-size μ. In addition, to achieve better steady-state estimation performance, regularization parameters for two sparse LMS algorithms are adopted from the paper [13], i.e., $\rho_{ZAS} = 0.008$ and $\rho_{RZAS} = 0.8$ for $K = 2$; $\rho_{ZAS} = 0.004$ and $\rho_{RZAS} = 0.4$ for $K = 4$. Average MSE performance comparison curves are depicted in Fig. 8 and Fig. 9, respectively. Obviously, LMS/F algorithms achieve better estimation performance than LMS algorithms in [12]. Figures clarify that the sparse LMS/F algorithms, i.e., ZA-LMS/F and RZA-LMS/F, achieve better estimation performance than LMS/F due to the fact that sparse LMS/F algorithms utilize ℓ_1-norm sparse constraint function.

Secondly, the estimation performance curves of RZA-LMS/F with different reweighted factors $\varepsilon \in \{1,2,5,\ldots,50\}$ are shown in Fig. 10 for $K = 2$ and 4. Under the simulation setup considered, RZA-LMS/F using $\varepsilon = 20$ or 25 can achieve near optimal estimation performance for $K = 2$. Fig. 10 shows that the performance of the RZA-LMS/F algorithm depends on reweighted factor. Hence, the proper selection of the reweighted factor is important for the RZA-LMS/F algorithm in ASCE.

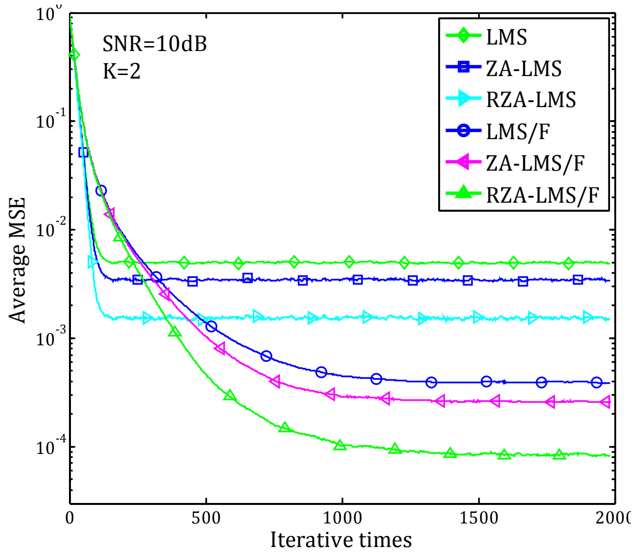

Fig. 8. Performance comparison at $K = 2$.

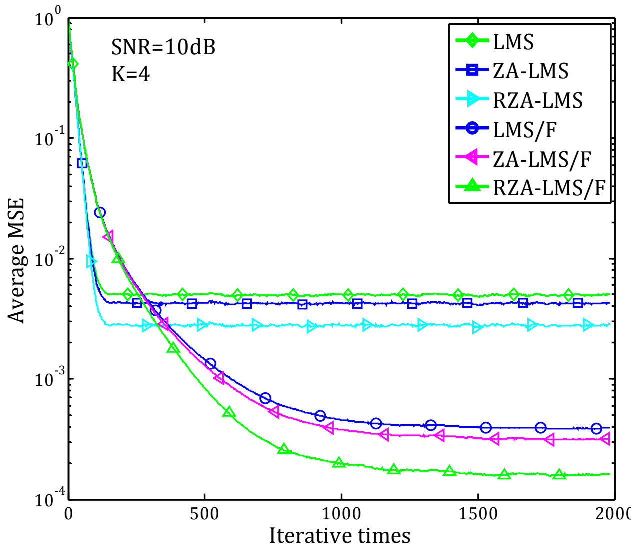

Fig. 9. Performance comparison at $K = 4$.

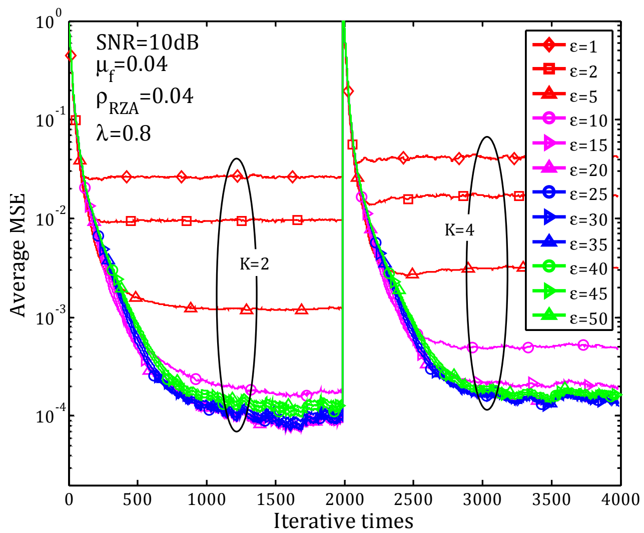

Fig. 10. RZA-LMS/F based sparse channel estimation performance depends on re-weighted factor ε.

V. Conclsion

In this paper, novel sparse LMS/F algorithms were proposed for ASCE. Based on the CS theory, we first proposed an ASCE with ZA-LMS/F algorithm. Inspired by re-weighted ℓ_1 -norm algorithm in CS, an improved ASCE method with RZA-LMS/F algorithm was then proposed. By Monte Carlo simulation, we proposed a simple method for choosing the approximate optimal regularization parameter for ZA-LMS/F and RZA-LMS/F. Simulation results showed that the proposed ASCE methods with ZA-LMS/F and RZA-LMS/F algorithms achieve better performance than any sparse LMS method in point-to-point systems. In addition, the proposed method can also be applied in cooperative systems [15].

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