Variable is Good: Adaptive Sparse Channel Estimation using VSS-ZA-NLMS Algorithm

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***Abstract*—Broadband wireless communication often requires accurate channel state information (CSI) at the receiver side due to the fact that broadband channel is described well by sparse channel model. To exploit the channel sparsity, *invariable step-size zero-attracting normalized least mean square* (ISS-ZA-NLMS) algorithm was applied in adaptive sparse channel estimation (ASCE). However, ISS-ZA-NLMS cannot trade off the algorithm convergence rate, estimation performance and computational cost. In this paper, we propose a *variable step-size ZA-NLMS* (VSS-ZA-NLMS) algorithm to improve the adaptive sparse channel estimation in terms of bit error rate (BER) and mean square error (MSE) metrics. First, we derive the proposed algorithm and explain the difference between VSS-ZA-NLMS and ISS-ZA-NLMS algorithms. Later, to verify the effectiveness of the proposed algorithm, several selected computer simulation results are shown.**

Keywords—zero-attracting normalized least mean square (ZA-NLMS), invariable step size (ISS), variable step size (VSS), adaptive sparse channel estimation.

I. INTRODUCTION

Broadband signal transmission is becoming one of the mainstream techniques in the next generation wireless communication systems [1][2]. The channel becomes severely frequency-selective and accurate channel state information (CSI) of such a channel is required at the receiver side for coherent detection (or demodulation). One of the effective approaches is the adaptive channel estimation (ACE) using normalized least mean square (NLMS) algorithm [3]. The merit of the LMS algorithm is its low complexity and easy implementation at the receiver. However, the traditional algorithms cannot be used for adaptive sparse channel estimation (ASCE) because broadband channels often exhibit sparse structure [4][5]. Indeed, many channel measurements have verified this fact. Generally speaking, the sparse channel is composed of a very few channel coefficients and most of them are zeros. A typical example of sparse channel is shown in Fig. 1 with the length of finite impulse response (FIR) set to $N = 16$ and the number of dominant coefficients, $K = 3$. Unfortunately, ACE with NLMS algorithm always neglects the inherent sparse structure information. Hence, it may not be able to achieve the estimation performance comparable to sparse ASCE which exploits the channel sparsity. Inspired by least absolute shrinkage and selection operator (LASSO) algorithm [6], an ℓ_1-norm sparse constraint function can be introduced to take advantage of the channel sparsity in ASCE; zero-attracting NLMS (ZA-NLMS) algorithm has been proposed for ASCE [7][8] to improve the estimation performance.

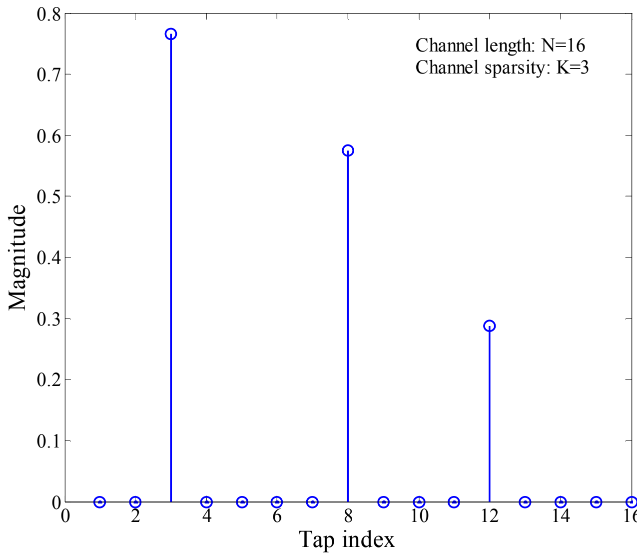

Fig. 1. A typical example of sparse multipath channel with channel length 16 and 3 nonzero taps.

It is well known that in NLMS algorithm, step size is a critical parameter which controls the estimation performance, convergence rate and computational cost. Different from conventional *invariable* step size NLMS (ISS-NLMS) algorithm [3], *variable* step size NLMS (VSS-NLMS) was proposed for ACE to improve the estimation performance [9]. To the best of our knowledge, no paper has reported the use of sparse VSS-NLMS algorithm to exploit the channel sparsity.

Considering two disadvantages of the ISS-ZA-NLMS and VSS-NLMS algorithms, in this paper, we propose an improved ASCE with *variable step size zero-attracting* NLMS (VSS-ZA-NLMS). The main contribution of this paper is to propose the ZA-NLMS algorithm using VSS rather than ISS for ASCE. Computer simulation is conducted to confirm the effectiveness of our proposed algorithms in terms of bit error rate (BER) and mean square error (MSE) metrics.

The remainder of this paper is organized as follows. A system model is described and ISS-ZA-NLMS algorithm is introduced in Section II. In section III, VSS-ZA-NLMS algorithm is proposed. Figure example is also given to clarify the difference between ISS and VSS. Computer simulation is presented in Section IV in order to evaluate and compare the performance of the proposed ASCE method. Finally, we conclude the paper in Section V.

II. ISS-ZA-NLMS Algorithm

Consider a baseband-equivalent frequency-selective fading wireless communication system where FIR sparse channel vector $\mathbf{h} = [h_0, h_1, \dots, h_{N-1}]^T$ is N-length and it is supported only by K nonzero channel taps. Assume that an input training signal $x(t)$ is used to probe the unknown sparse channel. At the receiver side, observed signal $y(t)$ is given by

$$y(t) = \mathbf{h}^T \mathbf{x}(t) + z(t), \quad (1)$$

where $\mathbf{x}(t) = [x(t), x(t-1), \dots, x(t-N+1)]^T$ denotes the vector of training signal $x(t)$ and $z(t)$ is the additive white Gaussian noise (AWGN), which is assumed to be independent with $x(t)$; $(\cdot)^T$ denotes the vector transpose operation. The objective of ASCE is to adaptively estimate the unknown sparse channel vector $\mathbf{h}$ using the training signal vector $\mathbf{x}(t)$ and the observed signal $y(t)$. By defining the square estimation error at the n-th update by $e^2(n)$, we can apply invariable step size zero-attracting least mean square algorithm (ISS-ZA-LMS) to exploit the channel sparsity in time domain. First of all, the cost function of ISS-ZA-LMS is given by

$$G(n) = \frac{1}{2} e^2(n) + \lambda \left\| \tilde{\mathbf{h}}(n) \right\|_1, \quad (2)$$

where λ is the *regularization parameter* to balance the square estimation error $e^2(n)$ and sparse penalty of $\tilde{\mathbf{h}}(n)$; $\|.\|_1$ is the ℓ_1-norm operation. At time t, the corresponding channel update equation of ISS-ZA-LMS is

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) - \mu \frac{\partial G(n)}{\partial \tilde{\mathbf{h}}(n)}$$
$$= \tilde{\mathbf{h}}(n) + \mu e(n) \mathbf{x}(t) - \rho \operatorname{sgn}\left(\tilde{\mathbf{h}}(n)\right), \quad (3)$$

where μ is the ISS, $\rho = \mu\lambda$ and $\operatorname{sgn}(\cdot)$ is a component-wise function which is defined by

$$\operatorname{sgn}(h) = \begin{cases} 1, & h > 0 \\ 0, & h = 0 \\ -1, & h < 0 \end{cases}. \quad (4)$$

From the update equation in Eq. (3), the function of its second term is attracting small channel coefficients as zero in high probability. In other word, most of small channel coefficients can be replaced directly by zeros, which speeds up the convergence of this algorithm and also mitigates the extra noise interference on zero positions. Based on the ISS-ZA-LMS algorithm in (3), the update equation of ISS-ZA-NLMS was proposed as follows[7][8]

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu \frac{e(n)\mathbf{x}(t)}{\mathbf{x}^T(t)\mathbf{x}(t)} - \rho \operatorname{sgn}\left(\tilde{\mathbf{h}}(n)\right). \quad (5)$$

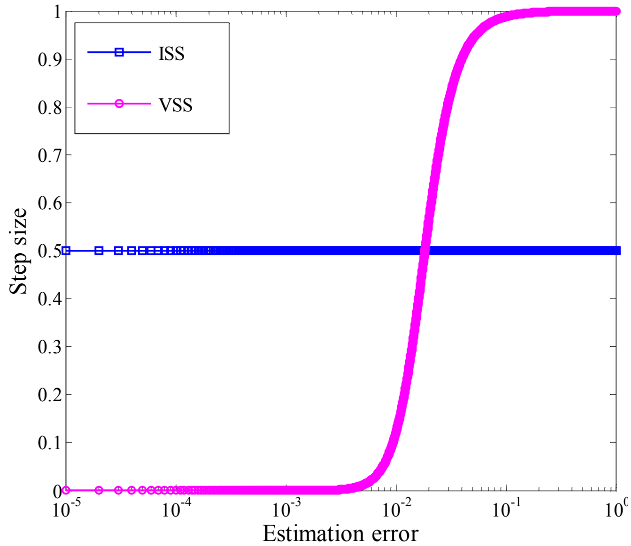

Fig. 2. ISS is invariable but VSS is variable as different estimation error.

III. VSS-ZA-NLMS Algorithm

Recall that the ISS-ZA-NLMS algorithm in (5) does not utilize VSS. The step-size is a critical parameter which controls the estimation accuracy, convergence speed and computational cost. Inspirited by the VSS-NLMS algorithm which has been proposed in [9], to further improve the estimation performance, variable step-size is introduced to adapt to the changes of the estimation error. At time t, based on the previous research on the ISS-ZA-NLMS and VSS-NLMS algorithms, VSS-ZA-NLMS algorithm performs as follows,

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu(n+1) \frac{e(n)\mathbf{x}(t)}{\mathbf{x}^T(t)\mathbf{x}(t)} - \rho \operatorname{sgn}\left(\tilde{\mathbf{h}}(n)\right), (6)$$

where $\mu(n+1)$ is the VSS which is given by

$$\mu(n+1) = \mu_{max} \cdot \frac{\mathbf{p}^T(n+1)\mathbf{p}(n+1)}{\mathbf{p}^T(n+1)\mathbf{p}(n+1) + C}, \quad (7)$$

where C is a positive threshold parameter which is related to $\sigma_n^2 \mathrm{Tr}\{[\mathbf{x}(t)\mathbf{x}^T(t)]^{-1}\}$ and can be written as $C \sim O(1/\mathrm{SNR})$, where SNR is the received signal noise ratio (SNR). According to Eq. (7), the range of VSS is given by $\mu(n+1) \in (0, \mu_{max})$, where μ_{max} is the maximal step-size. To the adaptive algorithm stability, the maximal step-size is less than 2 [3]. It is worth mentioning that $\mathbf{p}(n)$ in Eq. (7) is defined by

$$\mathbf{p}(n+1) = \beta \mathbf{p}(n) + (1-\beta) \frac{\mathbf{x}(t) e(n)}{\mathbf{x}^T(t)\mathbf{x}(t)}, \quad (8)$$

where $\beta \in [0,1)$ is the smoothing factor for controlling the VSS and estimation error. For a better understanding of the difference between ISS and VSS, based on the Eq. (7) and (8), step size μ for ISS-ZA-NLMS is invariable but the step size $\mu(n+1)$ for VSS-ZA-NLMS is variable as depicted in Fig. 2, where the maximal step size and step size are set as $\mu_{max} = 1$ and $\mu = 0.5$, respectively. From the figure, one can easily find that for VSS, $\mu(n)$ decreases as the estimation performance increase and vice versa while ISS is kept invariant.

IV. Computer Simulations

To confirm the effectiveness of the proposed method, two metrics, i.e., MSE and BER, are adopted. Channel estimators are evaluated by average MSE which is defined by

$$\text{Avergae MSE}\{\hat{\mathbf{h}}(n)\} = \mathrm{E}\left\{\left\|\mathbf{h} - \hat{\mathbf{h}}(n)\right\|_2^2\right\}, \quad (9)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator, $\mathbf{h}$ and $\hat{\mathbf{h}}(n)$ are the actual channel vector and its n-th iterative adaptive channel estimator, respectively. $\|\cdot\|_2$ is the Euclidean norm operator and $\|\mathbf{h}\|_2^2 = \sum_{i=1}^{N}|h_i|^2$. System performance is evaluated in terms of BER which adopts different data modulation schemes. The results are averaged over 1000 independent Monte-Carlo runs. The length of channel vector $\mathbf{h}$ is set to $N = 16$ and its number of dominant taps is set to $K = 1$ and 4. Each dominant channel tap follows random Gaussian distribution as $\mathcal{CN}(0, \sigma_{\mathbf{h}}^2)$ which is subject to $\mathrm{E}\{\|\mathbf{h}\|_2^2 = 1\}$ and their positions are randomly decided within the length of $\mathbf{h}$. The received signal-to-noise ratio (SNR) is defined as P_0/σ_n^2, where P_0 is the received power of the pseudo-random noise (PN)-sequence for training signal. Computer simulation parameters are listed in Table. I.

Tab. I. Simulation parameters.

parameters	values
channel length	$N = 16$
no. of nonzero coefficients	$K = 1$ and 4
distribution of nonzero coefficient	random Gaussian $\mathcal{CN}(0,1)$
threshold parameter for VSS-NLMS	$\mathrm{C} \in \{10^{-3}, 10^{-4}\}$
received SNR for channel estimation	$\{5\mathrm{dB}, 15\mathrm{dB}, 25\mathrm{dB}\}$
received SNR E_0/N_0 for symbol	12dB~30dB
step-size	$\mu = 0.2$ and $\mu_{max} = 2$
regularization parameter	$\rho = 0.002\sigma_n^2$
modulation schemes	QPSK, 8PSK, 16PSK and 32PSK 16QAM, 64QAM and 256QAM

In the first example, average MSE performance of the proposed method are evaluated for $K = 1$ and 4 in Figs. 3-8 under three SNR regimes, i.e. 5dB, 15dB and 25dB. To confirm the effectiveness of the proposed method, they are compared with three previous methods, i.e., ISS-NLMS [3], VSS-NLMS [9] and ISS-ZA-NLMS [7] [8] algorithms. In addition, to achieve better steady-state estimation performance, regularization parameter methods for ZA-NLMS-type algorithms are adopted [10][11] and set to $\rho = 0.002\sigma_n^2$. In the case of different SNR regimes, VSS-ZA-NLMS algorithm always achieves a better estimation performance than ISS-ZA-NLMS. In addition, since VSS-ZA-NLMS also taken advantage of the channel sparsity, it obtains a better estimation performance than VSS-NLMS, especially in the extreme sparse channel case, e.g., $K = 1$. This is apparent from Figs. 3 and 4; VSS-ZA-NLMS can exploit more sparse information when $K = 1$ (see Fig. 3) than when $K = 4$ (see Fig. 4).

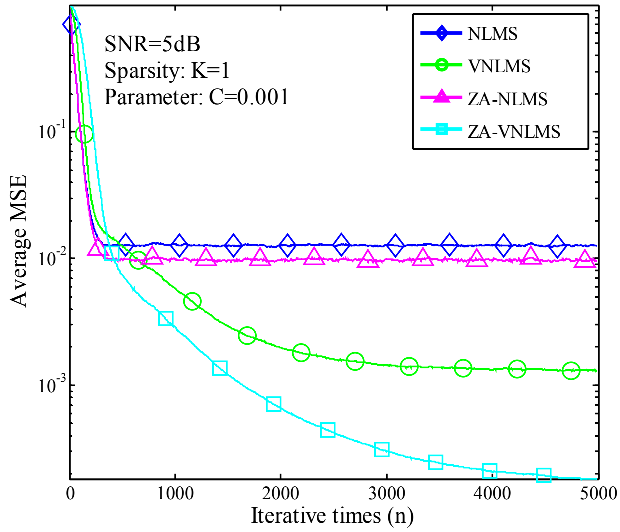

Fig. 3. Average MSE performance versus received iterative times (n).

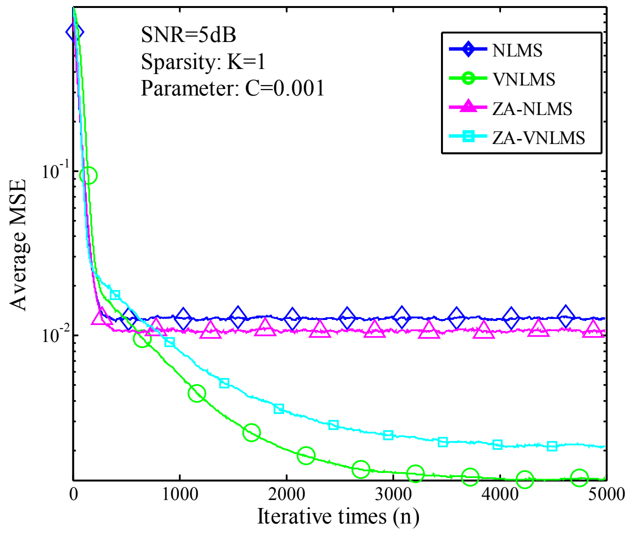

Fig. 4. Average MSE performance versus received iterative times (n).

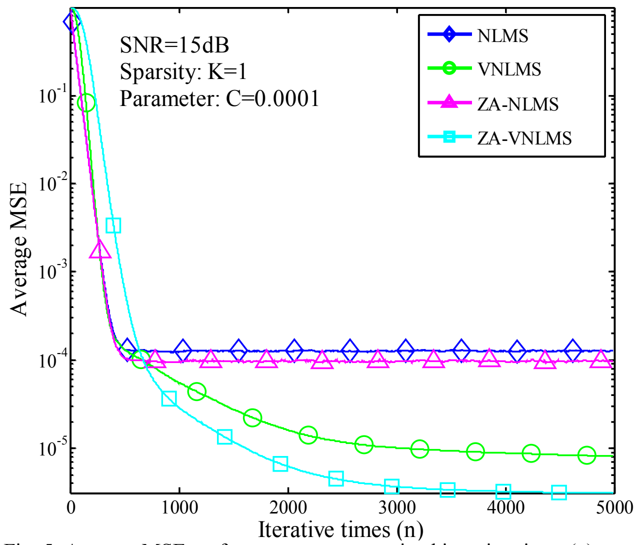

Fig. 5. Average MSE performance versus received iterative times (n).

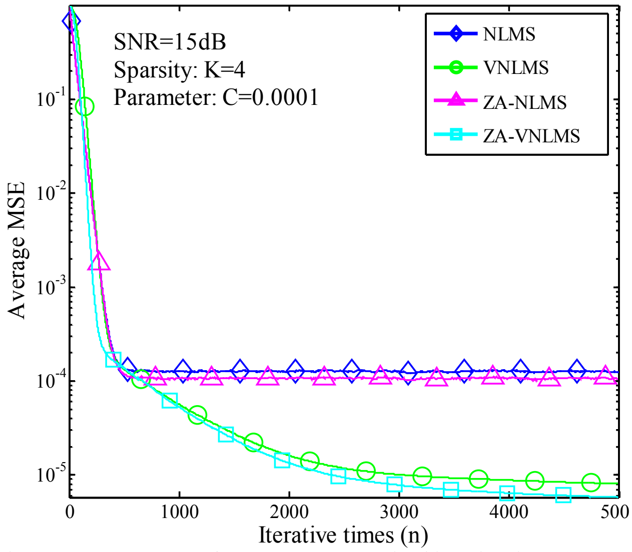

Fig. 6. Average MSE performance versus received iterative times (*n*).

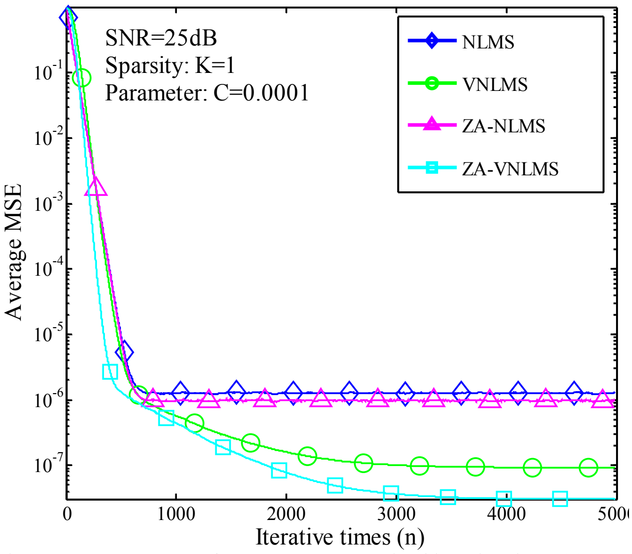

Fig. 7. Average MSE performance versus received iterative times (*n*).

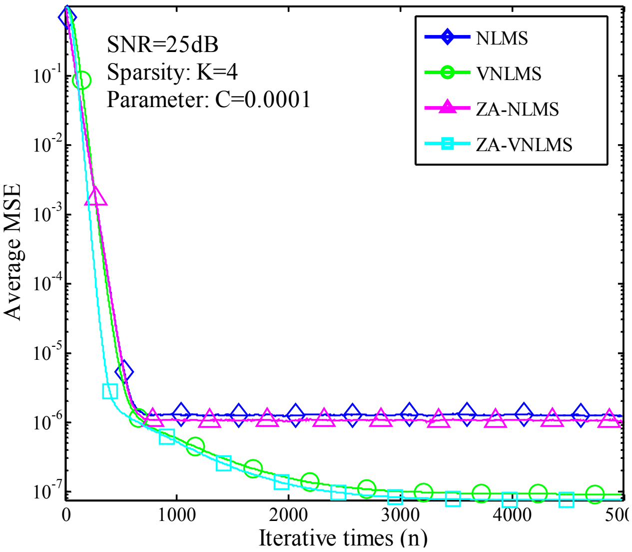

Fig. 8. Average MSE performance versus received iterative times (*n*).

In the second example, system performance using proposed channel estimator is also evaluated with respect to average BER. The channel is assumed to be a steady-state sparse channel with number of nonzero taps of $K = 1$ and SNR = 15dB. Received SNR is defined by E_0/N_0, where E_0 is the received power of symbol and N_0 is the noise power. In Fig. 9, multilevel phase shift keying (PSK) modulation, i.e., QPSK, 8PSK, 16PSK and 32PSK, is considered for data modulation. One can find that there is no big performance difference using QPSK and 8PSK due to high performance. If the higher-level data modulation is adopted, much better BER performance will be achieved when comparing with previous algorithms , i.e., ISS-NLMS, VSS-NLMS and ISS-ZA-NLMS. In Fig. 10, multilevel quadrature amplitude modulation (QAM), i.e., 16QAM, 64QAM and 256QAM, is considered for data modulation. It is observed that the proposed method can achieve a better estimation than previous methods.

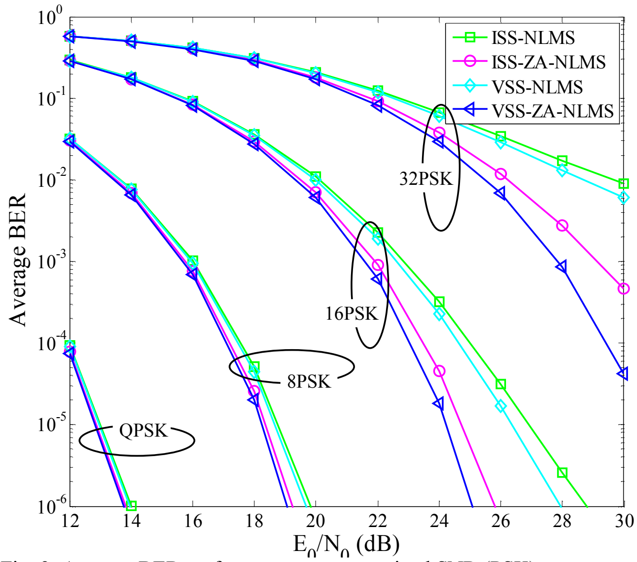

Fig. 9. Average BER performance versus received SNR (PSK).

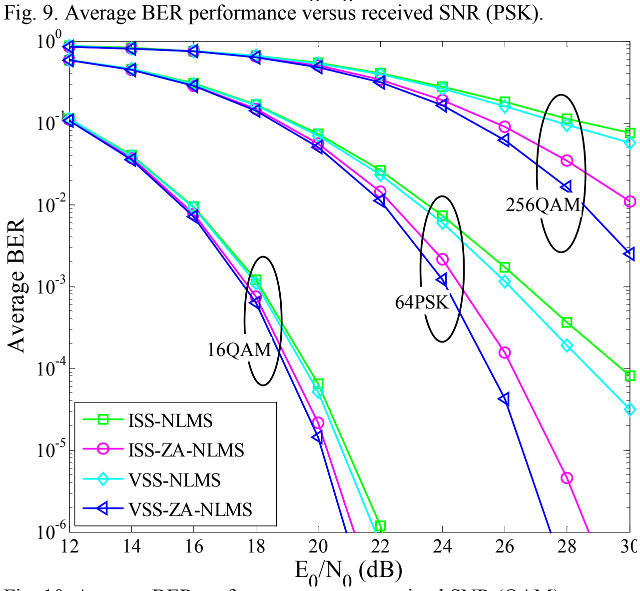

Fig. 10. Average BER performance versus received SNR (QAM).

V. CONCLSION AND FUTURE WORKS

In this paper, an improved VSS-ZA-NLMS algorithm was proposed for adaptive sparse channel estimator. Unlike the traditional ISS-ZA-NLMS algorithm, the proposed algorithm utilized VSS which can change adaptively with the estimation error. Simulation results were provided to confirm the effectiveness of the proposed method via two metrics: MSE and BER.

Since multi input multi output (MIMO) antenna transmission technique is becoming indispensable in the next-generation systems [2]. The proposed adaptive sparse channel estimation can be extended to different communication systems, such as multiple-antenna systems and underwater acoustic communication systems. It is expected that the proposed method can also achieve a better estimation performance than our previous work reported in [12][13].

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