

Transmit FDE Weight Design for Single-Carrier Space-Time Block Coded Joint Transmit/Receive Diversity

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Abstract— Single-carrier (SC) transmission using frequency-domain space-time block coded joint transmit/receive diversity (FD-STBC-JTRD) combined with transmit frequency-domain equalization (FDE) obtains full spatial diversity gain and frequency diversity gain. Channel state information (CSI) is required only at transmitter for transmit FDE. In our previous study of SC FD-STBC-JTRD, single transmit FDE weight matrix was used. In this paper, noting that a sequence of data blocks is STBC encoded into a code-word composed of a new sequence of coded data blocks, we derive the optimal transmit FDE weight design. Multiple transmit FDE weight matrices, each associated with each coded block in a STBC code-word, are used unlike our previous transmit FDE weight design. Transmit FDE weight matrices are jointly optimized based on the minimization of the mean square error (MSE) between the transmitted signal before STBC encoding and the received signal after STBC decoding. We show by theoretical analysis that the optimal transmit FDE weight design can achieve $1/R_{STBC}$ times higher signal-to-interference plus noise power ratio (SINR) than our previous design (single transmit FDE weight matrix), where R_{STBC} denotes the STBC coding rate. And then, we show, by computer simulation, the optimal transmit FDE weight design achieves better bit error rate (BER) performance than our previous design.

Keywords— component; Frequency-domain space-time coded joint transmit/receive diversity, transmit frequency-domain equalization, single-carrier transmission

I. INTRODUCTION

The bit error rate (BER) performance of broadband single-carrier (SC) transmissions significantly degrades due to the inter-symbol interference (ISI) caused by frequency-selective fading [1]. The minimum mean square error (MMSE) based frequency-domain equalization (FDE) can be used to improve the bit error rate (BER) performance [2,3]. An additional use of antenna diversity further improves the BER performance. Our previously proposed frequency-domain space-time block coded joint transmit/receive diversity (FD-STBC-JTRD) is a combination of STBC [4,5] and transmit FDE [6,7] to obtain both spatial diversity and frequency diversity gains [8-10]. FD-STBC-JTRD requires transmit FDE at the transmitter while requiring simple addition and subtraction at the receiver. Hence, it requires the channel state information (CSI) only at the transmitter. Furthermore, it allows an arbitrary number of transmit antennas while limiting the number of receive antennas. Therefore, FD-STBC-JTRD is suitable for the downlink (base-to-mobile link) application to alleviate the complexity problem of mobile terminals.

In our previous study of SC FD-STBC-JTRD [9], the single transmit FDE weight matrix was used while the STBC code-word is composed of sequence of coded data blocks. Multiple transmit FDE weight matrices, each associated with each coded block in a STBC code-word, can be used unlike our previous transmit FDE weight design. Our previous transmit FDE weight design cannot minimize the MSE and therefore, is a suboptimal solution in the MMSE sense.

In this paper, noting that a sequence of data blocks is STBC encoded into a code-word composed of a new sequence of coded data blocks, we derive the optimal transmit FDE weight design. Multiple transmit FDE weight matrices, each associated with each block in a STBC code-word, are used unlike our previous transmit FDE weight design. The transmit FDE weight matrices are jointly optimized based on the minimization of the mean square error (MSE) between the transmitted signal before STBC encoding at the transmitter and the received signal after STBC decoding at the receiver.

In this paper, we show by theoretical analysis that the optimal transmit FDE weight design can achieve $1/R_{STBC}$ times higher signal-to-interference plus noise power ratio (SINR) than our previous transmit FDE weight design, where R_{STBC} denotes the STBC coding rate. Then, we show, by computer simulation, that the optimal FDE weight design improves the BER performance compared to the previous design.

The remainder of this paper is organized as follows. Sect. II presents FD-STBC-JTRD for SC transmissions. Sect. III proposes the transmit FDE weight design and Sect. IV derives the analysis of SINR and BER. The computer simulation results are presented in Sect. V. Sect. VI offers conclusions.

II. FD-STBC-JTRD FOR SC TRANSMISSION

In this paper, we consider the SC transmission using FD-STBC-JTRD. Fig. 1 illustrates the transmitter/receiver structure considering in this paper. We assume that the transmitter equips with N_t antennas and the receiver has N_r antennas. It is assumed that the perfect CSI is available at the transmitter. Throughout the paper, the symbol-spaced discrete time representation is used.

A. Encoding

At the transmitter, the $J \times N_c$ data modulated symbols are divided into the a sequence of J block of N_c symbol each. The

J transmit signal blocks $\{d_j(t); t=0, \dots, N_c-1, j=0, \dots, J-1\}$ are transformed into the frequency-domain signal by the N_c -point fast Fourier transform (FFT). The frequency-domain signal $\{D_j(k); k=0, \dots, N_c-1, j=0, \dots, J-1\}$ is given as

$$D_j(k) = \frac{1}{\sqrt{N_c}} \sum_{t=0}^{N_c-1} d_j(t) \exp(-j2\pi kt/N_c). \quad (1)$$

A sequence of J frequency-domain signals is encoded into N_r streams of Q coded frequency-domain single blocks each. A combination of J and Q and STBC coding rate R_{STBC} is summarized in table I.

TABLE I. RELATIONSHIP OF J , Q AND CODING RATE R_{STBC}

N_r	J	Q	R_{STBC}
2	2	2	1
3	3	4	3/4
4	3	4	3/4
5	10	15	2/3

The encoded frequency-domain signal block $\{X_q(n_r, k); k=0, \dots, N_c-1, n_r=0, \dots, N_r-1, q=0, \dots, Q-1\}$ can be expressed as

$$\begin{pmatrix} \mathbf{X}_0^T(k) \\ \mathbf{X}_1^T(k) \end{pmatrix} = \begin{pmatrix} D_0(k) & D_1(k) \\ -D_1^*(k) & D_0^*(k) \end{pmatrix}, \quad \dots \text{ for } N_r=2, \quad (2a)$$

$$\begin{pmatrix} \mathbf{X}_0^T(k) \\ \mathbf{X}_1^T(k) \\ \mathbf{X}_2^T(k) \\ \mathbf{X}_3^T(k) \end{pmatrix} = \begin{pmatrix} D_0(k) & D_1(k) & D_2(k) \\ -D_1^*(k) & D_0^*(k) & 0 \\ -D_2^*(k) & 0 & D_0^*(k) \\ 0 & D_2(k) & -D_1(k) \end{pmatrix}, \quad \dots \text{ for } N_r=3, \quad (2b)$$

$$\begin{pmatrix} \mathbf{X}_0^T(k) \\ \mathbf{X}_1^T(k) \\ \mathbf{X}_2^T(k) \\ \mathbf{X}_3^T(k) \end{pmatrix} = \begin{pmatrix} D_0(k) & D_1(k) & D_2(k) & 0 \\ -D_1^*(k) & D_0^*(k) & 0 & D_2(k) \\ -D_2^*(k) & 0 & D_0^*(k) & D_1^*(k) \\ 0 & D_2(k) & -D_1(k) & D_0(k) \end{pmatrix}, \quad \dots \text{ for } N_r=4, \quad (2c)$$

$$\begin{pmatrix} \mathbf{X}_0^T(k) \\ \mathbf{X}_1^T(k) \\ \mathbf{X}_2^T(k) \\ \mathbf{X}_3^T(k) \\ \mathbf{X}_4^T(k) \\ \mathbf{X}_5^T(k) \\ \mathbf{X}_6^T(k) \\ \mathbf{X}_7^T(k) \\ \mathbf{X}_8^T(k) \\ \mathbf{X}_9^T(k) \\ \mathbf{X}_{10}^T(k) \\ \mathbf{X}_{11}^T(k) \\ \mathbf{X}_{12}^T(k) \\ \mathbf{X}_{13}^T(k) \\ \mathbf{X}_{14}^T(k) \end{pmatrix} = \begin{pmatrix} D_0(k) & D_1^*(k) & D_2^*(k) & D_3^*(k) & 0 \\ D_1(k) & -D_0^*(k) & 0 & 0 & D_4^*(k) \\ D_2(k) & 0 & -D_0^*(k) & 0 & -D_5^*(k) \\ 0 & D_2(k) & -D_1(k) & 0 & D_6(k) \\ D_3(k) & 0 & 0 & -D_0^*(k) & D_7^*(k) \\ 0 & -D_3(k) & 0 & D_1(k) & -D_8(k) \\ 0 & 0 & -D_3(k) & D_2(k) & D_9(k) \\ D_4(k) & 0 & -D_6^*(k) & -D_8^*(k) & -D_1^*(k) \\ 0 & D_4(k) & -D_5(k) & D_7(k) & D_0(k) \\ D_5(k) & -D_6^*(k) & 0 & -D_9^*(k) & D_2^*(k) \\ D_6(k) & D_5^*(k) & D_4^*(k) & 0 & 0 \\ D_7(k) & D_8^*(k) & -D_9^*(k) & 0 & -D_3^*(k) \\ D_8(k) & -D_7^*(k) & 0 & D_4^*(k) & 0 \\ D_9(k) & 0 & D_7^*(k) & D_5^*(k) & 0 \\ 0 & -D_9(k) & -D_8(k) & D_6(k) & 0 \end{pmatrix}, \quad \dots \text{ for } N_r=5, \quad (2d)$$

where $\mathbf{X}_q(k)=[X_q(0,k), \dots, X_q(N_r-1,k)]^T$ is the q th transmit signal block vector which is transmitted in the q th time-slot. After STBC encoding, the transmit FDE is performed for transmission of each block. The q th transmit signal block vector after the transmit FDE, $\mathbf{S}_q(k)=[S_q(0,k), \dots, S_q(N_r-1,k)]^T$ is given as

$$\mathbf{S}_q(k) = A_{N_r} \mathbf{W}_q(k) \mathbf{X}_q(k), \quad (3)$$

where $\mathbf{W}_q(k) = [\mathbf{W}_q(0,k), \dots, \mathbf{W}_q(N_r-1,k)]$ with $\mathbf{W}_q(n_r, k) = [W_q(0, n_r, k), \dots, W_q(N_r-1, n_r, k)]^T$ is the $N_r \times N_r$ transmit FDE weight matrix for the q th transmit signal block. A_{N_r} is the power normalization factor to keep average transmit power constant given as

$$A_{N_r} = \frac{1}{\sqrt{\frac{1}{N_c} \frac{1}{Q} \sum_{q=0}^{Q-1} \sum_{n_r=0}^{N_r-1} \sum_{k=0}^{N_c-1} \|\mathbf{W}_q(n_r, k)\|^2}}. \quad (4)$$

The transmit signal after the transmit FDE is transformed back to the time-domain signal by applying N_c -point inverse FFT (IFFT). After insertion of cyclic prefix (CP) into the beginning of each transmit signal block, the transmitter transmits signal to the receiver during Q time-slot.

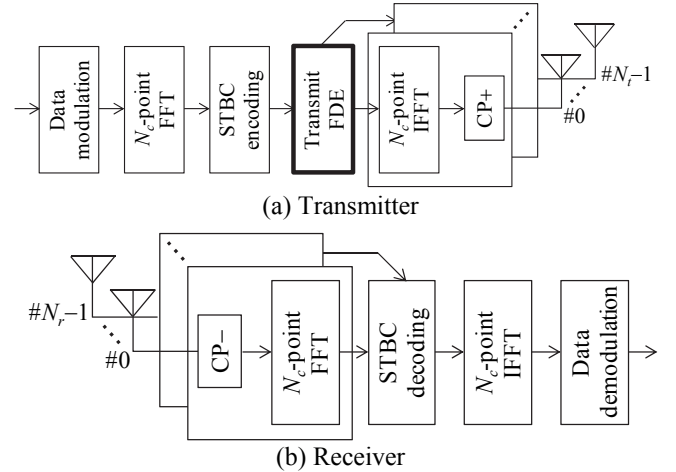


Fig. 1. Transmitter/receiver structure

B. Decoding

At the receiver, a super position of N_r transmitted signal is received by N_r antennas. After CP removal, the received signal is transformed into the frequency-domain signal by N_c -point FFT. The frequency-domain received signal, $\{R_q(n_r, k); k=0, \dots, N_c-1, n_r=0, \dots, N_r-1\}$ in the q th time-slot is expressed as

$$\mathbf{R}_q(k) = \sqrt{2P} \mathbf{H}(k) \mathbf{S}_q(k) + \mathbf{N}_q(k), \quad (5)$$

where $\mathbf{R}_q(k) = [R_q(0,k), \dots, R_q(N_r-1,k)]^T$ is the frequency-domain received signal vector in q th time-slot. $\mathbf{H}(k)=[\mathbf{H}(0,k), \dots, \mathbf{H}(N_r-1,k)]^T$ with $\mathbf{H}(n_r, k)=[H(n_r, 0, k), \dots, H(n_r, N_r-1, k)]$ is the $N_r \times N_r$ channel transfer function matrix. P denotes the transmit power. $\mathbf{N}_q(k)=[N_q(0,k), \dots, N_q(N_r-1,k)]^T$ is the zero mean complex-valued additive white Gaussian noise (AWGN) vector having variance $2N_0/T_s$ with N_0 and T_s being the single-sided power

spectrum density of AWGN and the symbol duration, respectively.

The STBC decoding is performed to obtain the decoded frequency-domain signal. The decoded frequency-domain signal $\{\hat{D}_j(k); k=0, \dots, N_c-1, j=0, \dots, J-1\}$ as

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \end{pmatrix} = \begin{pmatrix} R_0(0, k) + R_1^*(1, k) \\ R_0(1, k) - R_1^*(0, k) \end{pmatrix}, \quad \dots \text{ for } N_r=2, \quad (6a)$$

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \end{pmatrix} = \begin{pmatrix} R_0(0, k) + R_1^*(1, k) + R_2^*(2, k) \\ R_0(1, k) - R_1^*(0, k) + R_3^*(2, k) \\ R_0(2, k) - R_2^*(0, k) - R_3^*(1, k) \end{pmatrix}, \quad \dots \text{ for } N_r=3, \quad (6b)$$

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \end{pmatrix} = \begin{pmatrix} R_0(0, k) + R_1^*(1, k) + R_2^*(2, k) + R_3^*(3, k) \\ R_0(1, k) - R_1^*(0, k) - R_2^*(3, k) + R_3^*(2, k) \\ R_0(2, k) + R_1(3, k) - R_2^*(0, k) - R_3^*(1, k) \end{pmatrix}, \quad \dots \text{ for } N_r=4, \quad (6c)$$

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \\ \hat{D}_3(k) \\ \hat{D}_4(k) \\ \hat{D}_5(k) \\ \hat{D}_6(k) \\ \hat{D}_7(k) \\ \hat{D}_8(k) \\ \hat{D}_9(k) \end{pmatrix} = \begin{pmatrix} R_0(0, k) - R_1^*(1, k) - R_2^*(2, k) - R_3^*(3, k) + R_8(4, k) \\ R_1(0, k) + R_0^*(1, k) - R_3(2, k) + R_5(3, k) - R_7^*(4, k) \\ R_2(0, k) + R_3(1, k) + R_0^*(2, k) + R_6(3, k) + R_9^*(4, k) \\ R_4(0, k) - R_5(1, k) - R_6(2, k) + R_0^*(3, k) - R_{11}^*(4, k) \\ R_7(0, k) + R_8(1, k) + R_{10}^*(2, k) + R_{12}^*(3, k) + R_1^*(4, k) \\ R_9(0, k) + R_{10}^*(1, k) - R_8^*(2, k) + R_{13}^*(3, k) - R_2^*(4, k) \\ R_{10}(0, k) - R_9^*(1, k) - R_7^*(2, k) + R_{14}(3, k) + R_3(4, k) \\ R_{11}(0, k) - R_{12}^*(1, k) + R_{13}^*(2, k) + R_8(3, k) + R_4(4, k) \\ R_{12}(0, k) + R_{11}^*(1, k) - R_{14}(2, k) - R_7^*(3, k) - R_5(4, k) \\ R_{13}(0, k) - R_{14}(1, k) - R_{11}^*(2, k) - R_9^*(3, k) + R_6(4, k) \end{pmatrix}, \quad \dots \text{ for } N_r=5, \quad (6d)$$

As understood from (6), addition/subtraction and conjugate operations are only required and the CSI is not required at the receiver. The decoded frequency-domain signal is transformed back to the time-domain signal by N_c -point IFFT, and finally, the data demodulation is carried out.

III. TRANSMIT FDE WEIGHT DESIGN

In this paper, we will derive the transmit FDE weight matrices which minimize the MSE between the transmitted signal and the received signal. Since the transmit FDE alters the transmitted signal spectrum shape, the signal-to-noise power ratio (SNR) is unproportional to the MSE. Therefore, in this paper, we introduce the relative MSE, e , defined as [9]

$$e = \sum_{j=0}^{J-1} \sum_{k=0}^{N_c-1} E \left[\frac{|\hat{D}_j(k) - \sqrt{2P} A_{N_r} D_j(k)|^2}{\sqrt{2P} A_{N_r} \sqrt{E[|D_j(k)|^2]}} \right]. \quad (7)$$

A. Previous FDE weight design (single weight matrix)

The previous FDE weight is derived under a condition that the single transmit FDE weight matrix is used over a STBC codeword [9]. Assuming $\mathbf{W}_0(n_r, k) = \mathbf{W}_1(n_r, k) = \dots = \mathbf{W}(n_r, k)$ for

$n_r=0, \dots, N_r-1$, the previous transmit FDE weight is derived from $\partial e / \partial \mathbf{W}(0, k) = 0, \dots, \partial e / \partial \mathbf{W}(N_r-1, k) = 0$ as

$$\mathbf{W}(n_r, k) = \mathbf{H}^H(n_r, k) C_{subopt.}^{-1}(k), \quad (8)$$

where

$$C_{subopt.}(k) = \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 + N_r \left(\frac{P}{N} \right)^{-1}, \quad (9)$$

and $N=N_0/T_s$ is the noise power. This FDE weight is a suboptimal solution in the MMSE sense.

B. Optimal FDE weight design (multiple weight matrices)

The optimal FDE weight is derived so as to minimize MSE considering that multiple transmit FDE weight matrices, each associated with each coded block in a STBC code-word, can be used. By solving $\partial e / \partial \mathbf{W}_0(0, k) = 0, \dots, \partial e / \partial \mathbf{W}_{Q-1}(N_r-1, k) = 0$, the optimal transmit FDE weight is derived as

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) \end{pmatrix} C_{opt.}^{-1}(k), \quad \dots \text{ for } N_r=2, \quad (10a)$$

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \\ \mathbf{W}_2(k) \\ \mathbf{W}_3(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & 0 \\ \mathbf{H}_0^H(k) & 0 & \mathbf{H}_2^H(k) \\ 0 & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) \end{pmatrix} C_{opt.}^{-1}(k), \quad \dots \text{ for } N_r=3, \quad (10b)$$

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \\ \mathbf{W}_2(k) \\ \mathbf{W}_3(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & 0 \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & 0 & \mathbf{H}_3^H(k) \\ \mathbf{H}_0^H(k) & 0 & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) \\ 0 & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) \end{pmatrix} C_{opt.}^{-1}(k), \quad \dots \text{ for } N_r=4, \quad (10c)$$

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \\ \mathbf{W}_2(k) \\ \mathbf{W}_3(k) \\ \mathbf{W}_4(k) \\ \mathbf{W}_5(k) \\ \mathbf{W}_6(k) \\ \mathbf{W}_7(k) \\ \mathbf{W}_8(k) \\ \mathbf{W}_9(k) \\ \mathbf{W}_{10}(k) \\ \mathbf{W}_{11}(k) \\ \mathbf{W}_{12}(k) \\ \mathbf{W}_{13}(k) \\ \mathbf{W}_{14}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & 0 \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & 0 & 0 & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & 0 & \mathbf{H}_2^H(k) & 0 & \mathbf{H}_4^H(k) \\ 0 & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & 0 & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & 0 & 0 & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ 0 & \mathbf{H}_1^H(k) & 0 & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ 0 & 0 & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & 0 & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ 0 & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & 0 & \mathbf{H}_3^H(k) & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & 0 & 0 \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & 0 & \mathbf{H}_4^H(k) \\ \mathbf{H}_0^H(k) & \mathbf{H}_1^H(k) & 0 & \mathbf{H}_3^H(k) & 0 \\ \mathbf{H}_0^H(k) & 0 & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & 0 \\ 0 & \mathbf{H}_1^H(k) & \mathbf{H}_2^H(k) & \mathbf{H}_3^H(k) & 0 \end{pmatrix} C_{opt.}^{-1}(k), \quad \dots \text{ for } N_r=5, \quad (10d)$$

where

$$C_{opt.}^{-1}(k) = \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 + N_r \left(\frac{J}{Q}\right) \left(\frac{P}{N}\right)^{-1}. \quad (11)$$

It is seen from (2), (8) and (10) that the optimal transmit FDE weight matrices are sparse according to the STBC encoding matrix while the previous transmit FDE weight matrix is dense. Therefore, the optimal transmit FDE can reduce the norm of the transmit weight matrix and can improve the received SNR compared to the previous transmit FDE.

IV. ANALYSIS OF SINR AND BER

In this section, we derive theoretically the received SINRs when using the optimal and previous transmit FDE designs and then, the conditional BERs. Assuming the sum of the residual inter-symbol interference (ISI) and noise as a new zero-mean complex-valued Gaussian variable, the received SINRs, $\gamma_{opt.}$ and $\gamma_{subopt.}$, when using the optimal and previous transmit FDE designs can be respectively derived as

$$\left\{ \begin{array}{l} \gamma_{opt.} = \frac{2\left(\frac{P}{N}\right)\left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{opt.}(k)\right)^2}{\left(\frac{P}{N}\right)\left\{\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{opt.}^2(k) - \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{opt.}(k)\right)^2\right\} + N_r \frac{J}{Q} \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{opt.}(k)} \\ \gamma_{subopt.} = \frac{2\left(\frac{P}{N}\right)\left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{subopt.}(k)\right)^2}{\left(\frac{P}{N}\right)\left\{\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{subopt.}^2(k) - \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{subopt.}(k)\right)^2\right\} + N_r \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{subopt.}(k)} \end{array} \right. , \quad (12)$$

where

$$\left\{ \begin{array}{l} \tilde{H}_{opt.}(k) = \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 C_{opt.}^{-1}(k) \\ \tilde{H}_{subopt.}(k) = \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 C_{subopt.}^{-1}(k) \end{array} \right. . \quad (13)$$

When the sufficiently high SNR is obtained by FD-STBC-JTRD, i.e., $\frac{1}{N_r} \left(\frac{P}{N}\right) \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 \gg 1$, (12) can be approximated as

$$\left\{ \begin{array}{l} \gamma_{opt.} \approx \frac{2}{N_r} \frac{1}{J/Q} \left(\frac{P}{N}\right) = \frac{1}{R_{STBC}} \gamma_{subopt.} \\ \gamma_{subopt.} \approx \frac{2}{N_r} \left(\frac{P}{N}\right) \end{array} \right. . \quad (14)$$

Therefore, the optimal transmit FDE design can obtain $1/R_{STBC}$ times higher SINR than the previous transmit FDE design.

Furthermore, assuming QPSK data modulation, the conditional BER, $p_{e,opt.}$ and $p_{e,subopt.}$, for the given channel transfer function when using the optimal and previous transmit FDE can be respectively given as [1]

$$\left\{ \begin{array}{l} p_{e,opt.} = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{\gamma_{opt.}}{4}} \right] \\ p_{e,subopt.} = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{\gamma_{subopt.}}{4}} \right] \end{array} \right. . \quad (15)$$

The average BER can be numerically evaluated by averaging (15) over possible channel transfer function.

V. COMPUTER SIMULATION

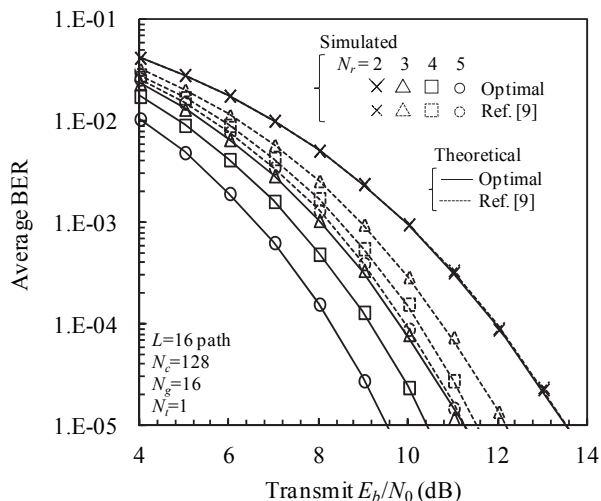
We evaluate, by computer simulation, the BER performance when using FD-STBC-JTRD with the optimal transmit FDE design. Computer simulation conditions are summarized in Table II. We consider QPSK data modulation. FFT block size N_c and CP length N_g are set to $N_c = 128$ symbols and $N_g = 16$ samples, respectively. The number of transmitter antennas N_t is set to $N_t = 1$ as an example. The channel is assumed to be a frequency-selective block Rayleigh fading channel having symbol spaced $L = 16$ path exponential power delay profile with decay factor α . Ideal channel estimation is assumed.

TABLE II. SIMULATION CONDITIONS

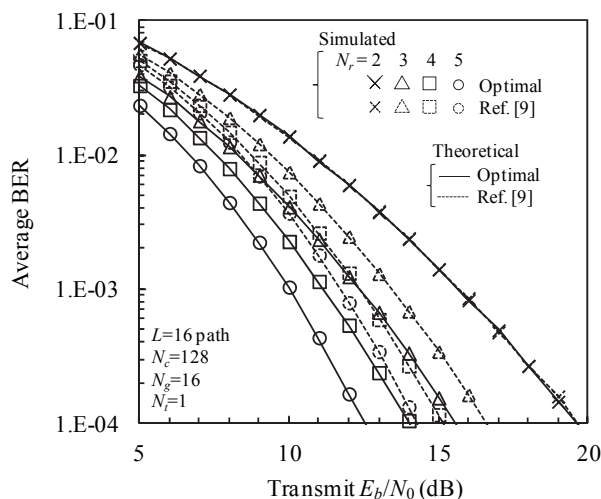
Transmitter	Data modulation	QPSK
	FFT block size	$N_c=128$
	CP length	$N_g=16$
	The number of transmit antennas	$N_t=1$
Channel	Channel estimation	Ideal
	Frequency-selective block Rayleigh fading	
	Power delay profile	$L=16$ -path exponential
	Decay factor	$\alpha=0, 6\text{dB}$
Receiver	Delay time	$\tau_f=l$
	The number of receive antennas	$N_r=2,3,4,5$

BER performance

Fig. 2 shows the BER performances when using the optimal transmit FDE design as a function of transmit E_b/N_0 . For the comparison, the performance when using the previous transmit FDE design [9] is also plotted in Fig. 2. It is seen from Fig. 2 that when $N_r = 2$, the optimal transmit FDE obtains the same BER performance as the previous transmit FDE. This is because the STBC encoding rate R_{STBC} is $R_{STBC} = 1$ and the both previous and optimal transmit FDE matrix are dense when $N_r = 2$. On the other hand, when $N_r > 2$, the optimal transmit FDE can improve the BER performance compared to the previous transmit FDE. This is because the optimal transmit FDE matrices are sparse and the norm of the optimal transmit FDE matrix is smaller than that of the previous transmit FDE matrix. When the allowable BER is $\text{BER}=10^{-4}$ and the number of receiver antennas $N_r = 3, 4$ ($N_r = 5$), the optimal transmit FDE can reduce the required transmit E_b/N_0 by 1.2 dB (1.6 dB) compared to the previous transmit FDE. This result corresponds to the theoretical analysis presented in the previous section.



(a) $\alpha=0\text{dB}$



(b) $\alpha=6\text{dB}$

Fig. 2. Average BER performance.

VI. CONCLUSION

In this paper, we derived the optimal transmit FDE weight design for SC FD-STBC-JTRD. The optimal transmit FDE weight design uses multiple transmit FDE weight matrices, each associated with each coded block in a STBC code-word, unlike our previous transmit FDE weight design. We showed by theoretical analysis and computer simulation that the optimal transmit FDE weight design achieves $1/R_{STBC}$ times higher SINR than the previous transmit FDE and that when $N_r = 3$ and 4 ($N_r = 5$), the transmit E_b/N_0 required for achieving $\text{BER}=10^{-4}$ can be reduced by about 1.2 dB (1.6 dB) compared to our previous transmit FDE weight design.

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