Performance Analysis of Space-Time Block Coded Joint Tx/Rx Diversity Using Optimal Transmit FDE in Presence of Channel Estimation Error

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Abstract— **Single-carrier (SC) transmission using frequency-domain space-time block coded joint transmit/receive diversity (FD-STBC-JTRD) combined with transmit frequency-domain equalization (FDE) obtains full spatial diversity gain and frequency diversity gain. Recently, we derived the optimal transmit frequency-domain equalization (FDE) for SC FD-STBC-JTRD and showed that the optimal transmit FDE, which uses multiple weight matrices corresponding to multiple coded blocks in a STBC code-word, achieves the received signal-to-interference plus noise power ratio (SINR) $1/R_{STBC}$ times higher than the conventional transmit FDE which uses the single weight matrix over a STBC code-word, where R_{STBC} denotes the STBC code rate. However, our past study of SC FD-STBC-JTRD with the optimal transmit FDE assumed ideal channel estimation. In this paper, we provide theoretical analysis of the received SINR and the conditional bit error rate (BER) of SC FD-STBC JTRD with cyclic delay pilot channel estimation (CDP-CE) and discuss the impact of channel estimation error. Then, we evaluate, by computer simulation, the average BER performance when using SC FD-STBC-JTRD with the optimal transmit FDE and show that the optimal transmit FDE can achieve the received SINR $1/R_{STBC}$ times higher than the conventional transmit FDE even in the presence of the channel estimation error.**

Keywords-component; Frequency-domain space-time block coded joint transmit/receive diversity, transmit frequency-domain equalization, channel estimation error

I. INTRODUCTION

The bit error rate (BER) performance of broadband single-carrier (SC) transmissions significantly degrades due to the inter-symbol interference (ISI) caused by frequency-selective fading [1]. The minimum mean square error (MMSE) based frequency-domain equalization (FDE) can be used to improve bit error rate (BER) performance [2-4]. An additional use of antenna diversity further improves BER performance [5]. Our proposed frequency-domain space-time block coded joint transmit/receive diversity (FD-STBC-JTRD) is a combination of STBC [6,7] and transmit FDE [8,9] to obtain both spatial diversity and frequency diversity gains [10,11]. FD-STBC-JTRD requires transmit FDE at the transmitter while requiring simple addition/subtraction at the receiver. Hence, it requires the channel state information (CSI) only at the transmitter.

Recently, we pointed out that the use of single weight matrix over a STBC code-word does not necessarily minimize the mean square error (MSE) between the transmit signal before STBC encoding and the received signal after STBC decoding [12]. Later, we derived the optimal transmit FDE for SC FD-STBC-JTRD based on the MMSE criterion [12]. Instead of using the single weight matrix, the multiple transmit FDE matrices are used corresponding to multiple coded blocks in a STBC code-word. The optimal transmit FDE achieves the received signal-to-interference plus noise power ratio (SINR) $1/R_{STBC}$ times higher than the conventional transmit FDE which uses the single weight matrix over a STBC code-word, where R_{STBC} denotes the STBC code rate [12].

FD-STBC-JTRD requires accurate CSI of each pair of transmit and receive antennas. The CSI error significantly affects the FD-STBC-JTRD performance. Thus, how the CSI error affects the FD-STBC-JTRD performance is a practically important problem. However, in our study in [12], ideal channel estimation was assumed. Least square channel estimation (LSCE) [13] and code-multiplexed pilot based channel estimation (CMP-CE) [14] can simultaneously estimate all transmit/receive antenna pairs. However, the former requires high computational complexity and the latter suffers from the code orthogonality distortion due to the frequency-selective fading. The cyclic delay pilot aided channel estimation (CDP-CE) [15] can simultaneously estimate all transmit/receive antenna pairs with low complexity, and hence, is suitable for SC FD-STBC-JTRD. In [16], the impact of CSI error on SC FD-STBC-JTRD with CDP-CE is discussed. However, in [16], the conventional transmit FDE was assumed and the impact of CSI error was not theoretically analyzed.

In this paper, we theoretically analyze the impact of CSI error on SC FD-STBC-JTRD with the optimal transmit FDE and CDP-CE. At first, we derive the variance of CSI error when using CDP-CE and then, derive the conditional BER in the presence of CSI error by assuming the CSI error as a zero-mean complex-valued Gaussian variable. It is shown by theoretical analysis that when using practical CDP-CE, the orthogonality of STBC code-word is lost due to CSI error. However, the optimal transmit FDE achieves the received SINR $1/R_{STBC}$ times higher than the conventional transmit FDE even in the presence of CSI error. The theoretical analysis is confirmed by computer simulation.

II. FD-STBC-JTRD AND CDP-CE FOR SC TRANSMISSION

In this paper, we consider the SC transmission using FD-STBC-JTRD and CDP-CE. We assume that the transmitter equips with N_t antennas and the receiver has N_r antennas. Fig. 1 illustrates the frame structure considered in this paper. The channel estimation and the transmit FDE are performed at the transmitter to make the receiver structure simple and to reduce the amount of CSI feedback. The pilot stage consists of N_p pilot blocks and is inserted per N_B data symbol blocks. In the pilot stage, the receiver transmits the pilot blocks to the transmitter and the transmitter estimates CSI by using CDP-CE. Then, the transmitter performs SC transmission with FD-STBC-JTRD. In this paper, we consider a quasi-static fading channel (i.e., Doppler frequency $f_D \to 0$). Throughout the paper, the symbol-spaced discrete time representation is used.

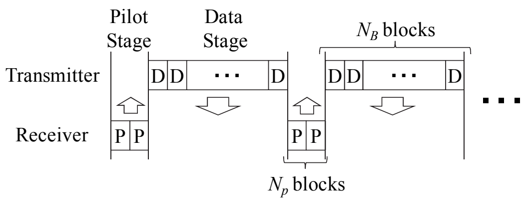


Fig. 1. Frame structure.

A. CDP-CE

In CDP-CE, a different cyclic delay is added to a difference antenna [15]. Denoting the mth (m=0,…,N_p–1) pilot signal as { $p_0^{(m)}(t)$:t=0,…,N_c–1}, the mth pilot signal, { $p^{(m)}(n_r,t)$:t=0,…,N_c–1}, at the n_rth receiver antenna is given as $p^{(m)}(n_r,t)=p_0^{(m)}((t-n_r\theta) \bmod N_c)$, where θ is the amount of cyclic delays. After inserting the cyclic prefix (CP) at the beginning of each pilot block, the receiver transmits the pilot signal to the transmitter.

At the transmitter, after CP removal, the received pilot signal is transformed into the frequency-domain signal by N_c-point fast Fourier transform (FFT). The mth frequency-domain received pilot signal, { $R^{(m)}(n_r,k)$:k=0,…,N_c–1}, at the n_tth transmitter antenna can be expressed as

$$R^{(m)}(n_t,k)=\sqrt{\frac{2P_t}{N_r}}\sum_{n_r=0}^{N_r-1}H(n_r,n_t,k)\exp\left(-\frac{j2\pi k n_r\theta}{N_c}\right)P_0^{(m)}(k) + N(n_t,k), \quad (1)$$

where P_t denotes the transmit power. $H(n_r,n_t,k)$ is the kth frequency channel transfer function between the n_rth receiver antenna and the n_tth transmitter antenna. $N(n_t.k)$ is the zero-mean complex valued additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ with N_0 and T_s being the single-sided power spectrum density of AWGN and the symbol duration, respectively. $P_0^{(m)}(k)$ is the kth frequency component of the mth pilot signal $p_0^{(m)}(t)$. The pilot is removed by the reverse modulation as $\overline{H}^{(m)}(n_t,k)=R^{(m)}(n_t,k)\cdot\left(P_0^{(m)}(k)\right)^*/\left|P_0^{(m)}(k)\right|^2$. Then, the total channel impulse response { $\overline{h}^{(m)}(n_t,\tau)$:τ=0,…,N_c–1} is obtained by applying N_c-point inverse FFT (IFFT) to the received pilot signal after the reverse modulation $\overline{H}^{(m)}(n_t,k)$. The total channel impulse response, $\overline{h}^{(m)}(n_t,\tau)$, can be expressed as

$$\overline{h}^{(m)}(n_t,\tau)=\sum_{n_r=0}^{N_r-1}\widetilde{h}(n_r,n_t,(\tau-n_r\theta)\bmod N_c)+\widetilde{n}(n_t,\tau), \quad (2)$$

where $\widetilde{h}(n_r,n_t,\tau)$ is the channel impulse response of the link between the n_rth receiver antenna and the n_tth transmitter antenna and given as

$$\widetilde{h}(n_r,n_t,\tau)=\frac{1}{\sqrt{N_c}}\sum_{k=0}^{N_c-1}\sqrt{\frac{2P_t}{N_r}}H(n_r,n_t,k)\exp(j2\pi k\tau/N_c). \quad (3)$$

The channel estimates of each pair of transmit and receive antennas are obtained by applying the delay time-domain windowing [15] and N_c-point FFT to the total channel impulse response. The mth channel estimate of the link between the n_rth receiver antenna and the n_tth transmitter antenna, $\widetilde{H}^{(m)}(n_r,n_t,k)$, can be expressed as

$$\widetilde{H}^{(m)}(n_r,n_t,k)=\frac{1}{\sqrt{N_c}}\sum_{\tau=0}^{N_g-1}\overline{h}^{(m)}(n_t,(\tau+n_r\theta))\exp\left(-\frac{j2\pi k\tau}{N_c}\right). \quad (4)$$

Finally, the accurate channel estimate, $\widetilde{H}(n_r,n_t,k)$, is obtained by averaging N_p channel estimates as

$$\widetilde{H}(n_r,n_t,k)=\frac{1}{N_p}\sum_{m=0}^{N_p-1}\widetilde{H}^{(m)}(n_r,n_t,k). \quad (5)$$

In addition, the noise power, σ_n^2, is estimated as

$$\sigma_n^2=\frac{1}{2N_p}\frac{1}{N_c-N_gN_r}\sum_{m=0}^{N_p-1}\sum_{\tau=0}^{N_c-1}\left|\overline{h}^{(m)}(n_t,\tau)\right|^2. \quad (6)$$

B. FD-STBC-JTRD

Fig. 2 shows the transmitter/receiver structures at the data stage. At the transmitter, the $J\times N_c$ data modulated symbols are divided into the a sequence of J blocks of N_c symbol each. The J transmit signal blocks are transformed into the frequency-domain signal by N_c-point fast Fourier transform (FFT). A sequence of J frequency-domain signals is encoded into N_r streams of Q coded frequency-domain single blocks each. In FD-STBC-JTRD, A combination of J and Q and the STBC code rate R_{STBC}=J/Q is determined by the number of receiver antennas [10]. Denoting the jth frequency-domain transmit signal block as {$D_j(k)$;k=0,…,N_c–1, j=0,…,J–1}, the encoded frequency-domain signal block {$X_q(n_r,k)$;k=0,...,N_c–1, n_r=0,…,N_r–1, q=0,…,Q–1} can be expressed as

$$\begin{pmatrix}\mathbf{X}_0^T(k)\\ \mathbf{X}_1^T(k)\end{pmatrix}=\begin{pmatrix}D_0(k) & D_1(k)\\ -D_1^*(k) & D_0^*(k)\end{pmatrix},$$

... for N_r=2, (7a)

$$\begin{pmatrix}\mathbf{X}_0^T(k)\\ \mathbf{X}_1^T(k)\\ \mathbf{X}_2^T(k)\\ \mathbf{X}_3^T(k)\end{pmatrix}=\begin{pmatrix}D_0(k) & D_1(k) & D_2(k)\\ -D_1^*(k) & D_0^*(k) & 0\\ -D_2^*(k) & 0 & D_0^*(k)\\ 0 & D_2(k) & -D_1(k)\end{pmatrix},$$

... for N_r=3, (7b)

$$\begin{pmatrix}\mathbf{X}_0^T(k)\\ \mathbf{X}_1^T(k)\\ \mathbf{X}_2^T(k)\\ \mathbf{X}_3^T(k)\end{pmatrix}=\begin{pmatrix}D_0(k) & D_1(k) & D_2(k) & 0\\ -D_1^*(k) & D_0^*(k) & 0 & D_2(k)\\ -D_2^*(k) & 0 & D_0^*(k) & D_1^*(k)\\ 0 & D_2(k) & -D_1(k) & D_0(k)\end{pmatrix},$$

… for N_r=4, (7c)

where $\mathbf{X}_q(k)$=$[X_q(0,k)$,…, $X_q(N_r–1,k)]^T$ is the qth transmit signal block vector which is transmitted in the qth time-slot. After STBC encoding, the transmit FDE is performed for transmission of each block. The qth transmit signal block vector after the transmit FDE, $\mathbf{S}_q(k)$=$[S_q(0,k)$,…, $S_q(N_t–1,k)]^T$, is given as $\mathbf{S}_q(k)$=$A\mathbf{W}_q(k)\mathbf{X}_q(k)$, where $\mathbf{W}_q(k)$ = [$\mathbf{W}_q(0,k)$,…, $\mathbf{W}_q(N_r–1,k)$] with $\mathbf{W}_q(n_r,k)$ = $[W_q(0,n_r,k)$,…., $W_q(N_t–1,n_r,k)]^T$ is the $N_t\times N_r$ transmit FDE weight matrix for the qth transmit signal block vector. A is the power normalization factor to keep average transmit power constant given as

$$A=\frac{1}{\sqrt{\frac{1}{N_c}\frac{1}{Q}\sum_{q=0}^{Q-1}\sum_{n_r=0}^{N_r-1}\sum_{k=0}^{N_c-1}\left\|\mathbf{W}_q(n_r,k)\right\|^2}}. \quad (8)$$

The optimal transmit FDE is derived so as to minimize the mean square error (MSE) between the transmit signal before STBC encoding and the received signal after STBC decoding considering that multiple weight matrices can be used corresponding to multiple coded blocks in a STBC code-word. Denoting the channel estimates vector as $\mathbf{H}(n_r,k)=\left[\widetilde{H}(n_r,0,k),\ldots,\widetilde{H}(n_r,N_t-1,k)\right]$ for n_r=0,…,N_r–1, the optimal transmit FDE weight is given as [12]

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) \\ \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) \end{pmatrix} C_{opt.}^{-1}(k),$$

... for N_r=2, (9a)

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \\ \mathbf{W}_2(k) \\ \mathbf{W}_3(k) \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) & \widetilde{\mathbf{H}}^H(2,k) \\ \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) & 0 \\ \widetilde{\mathbf{H}}^H(0,k) & 0 & \widetilde{\mathbf{H}}^H(2,k) \\ 0 & \widetilde{\mathbf{H}}^H(1,k) & \widetilde{\mathbf{H}}^H(2,k) \end{pmatrix} C_{opt.}^{-1}(k),$$

... for N_r=3, (9b)

$$\begin{pmatrix} \mathbf{W}_0(k) \\ \mathbf{W}_1(k) \\ \mathbf{W}_2(k) \\ \mathbf{W}_3(k) \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) & \widetilde{\mathbf{H}}^H(2,k) & 0 \\ \widetilde{\mathbf{H}}^H(0,k) & \widetilde{\mathbf{H}}^H(1,k) & 0 & \widetilde{\mathbf{H}}^H(3,k) \\ \widetilde{\mathbf{H}}^H(0,k) & 0 & \widetilde{\mathbf{H}}^H(2,k) & \widetilde{\mathbf{H}}^H(3,k) \\ 0 & \widetilde{\mathbf{H}}^H(1,k) & \widetilde{\mathbf{H}}^H(2,k) & \widetilde{\mathbf{H}}^H(3,k) \end{pmatrix} C_{opt.}^{-1}(k)$$

, … for N_r=4, (9c)

where

$$C_{opt.}(k) = \sum_{n_r=0}^{N_r-1} \left\| \widetilde{\mathbf{H}}(n_r,k) \right\|^2 + 2\sigma_n^2 N_r \left(\frac{J}{Q} \right). \tag{10}$$

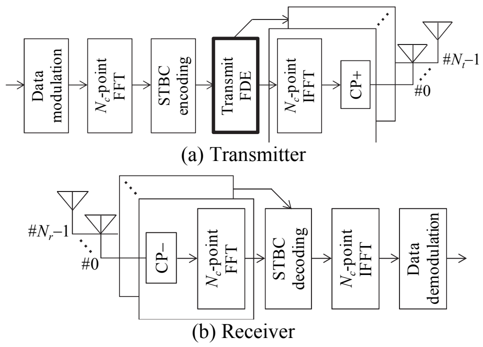
On the other hand, the conventional transmit FDE is derived under the condition that the single weight matrix is used over a STBC code-word ($\mathbf{W}_0(k)=\mathbf{W}_1(k)=...=\mathbf{W}(k)$). The conventional transmit FDE weight is given as [10]

$$\mathbf{W}(k) = \left(\widetilde{\mathbf{H}}^H(0,k) \ \cdots \ \widetilde{\mathbf{H}}^H(N_r-1,k) \right) C_{conv.}^{-1}(k), \tag{11}$$

where

$$C_{conv.}(k) = \sum_{n_r=0}^{N_r-1} \left\| \widetilde{\mathbf{H}}(n_r,k) \right\|^2 + 2\sigma_n^2 N_r . \tag{12}$$

The transmit signal after the transmit FDE is transformed back to the time-domain transmit signal by N_c-point IFFT. After CP insertion, the transmitter transmits signal to the receiver during Q time-slot.



(a) Transmitter

(b) Receiver

Fig. 2. Transmitter/receiver structures.

At the receiver, after CP removal, the received signal is transformed into the frequency-domain signal by N_c-point FFT. The frequency-domain received signal, $\{R_q(n_r,k)$; $k=0,\ldots,N_c-1$, $n_r=0,\ldots,N_r-1\}$ in the qth time-slot is expressed as

$$\mathbf{R}_q(k) = \sqrt{2P_t}\,\mathbf{H}(k)\mathbf{S}_q(k) + \mathbf{N}_q(k), \tag{13}$$

where $\mathbf{R}_q(k) = [R_q(0,k),\ldots, R_q(N_r-1,k)]^T$ is the frequency-domain received signal vector in qth time-slot. $\mathbf{H}(k)=[\mathbf{H}^T(0,k),\ldots,\mathbf{H}^T(N_r-1,k)]^T$ with $\mathbf{H}(n_r,k)=[H(n_r,0,k),\ldots,H(n_r,N_t-1,k)]$ is the $N_r \times N_t$ channel transfer function matrix. $\mathbf{N}_q(k)=[N_q(0,k),\ldots,N_q(N_r-1,k)]^T$ is the zero mean complex-valued AWGN vector having variance $2N_0/T_s$. The STBC decoding is performed to obtain the decoded frequency-domain signal. The decoded frequency-domain signal $\{\hat{D}_j(k)$; $k=0,\ldots,N_c-1$, $j=0,\ldots,J-1\}$ as

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \end{pmatrix} = \begin{pmatrix} R_0(0,k)+R_1^*(1,k) \\ R_0(1,k)-R_1^*(0,k) \end{pmatrix},$$

… for N_r=2, (14a)

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \end{pmatrix} = \begin{pmatrix} R_0(0,k)+R_1^*(1,k)+R_2^*(2,k) \\ R_0(1,k)-R_1^*(0,k)+R_3^*(2,k) \\ R_0(2,k)-R_2^*(0.k)-R_3^*(1,k) \end{pmatrix},$$

… for N_r=3, (14b)

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \end{pmatrix} = \begin{pmatrix} R_0(0,k)+R_1^*(1,k)+R_2^*(2,k)+R_3^*(3,k) \\ R_0(1,k)-R_1^*(0,k)-R_2(3,k)+R_3^*(2,k) \\ R_0(2,k)+R_1(3,k)-R_2^*(0.k)-R_3^*(1,k) \end{pmatrix}.$$

… for N_r=4, (14c)

The decoded frequency-domain signal is transformed back to the time-domain signal by N_c-point IFFT, and finally, the data demodulation is carried out.

III. THEORETICAL ANALYSIS

A. Variance of CSI error

The variance of CSI error when using CDP-CE $2\sigma_e^2$ is given as

$$2\sigma_e^2 = \frac{1}{N_r N_t} \sum_{n_r=0}^{N_r-1} \sum_{n_t=0}^{N_t-1} E\left[\left| H(n_r n_t,k) - \widetilde{H}(n_r n_t,k) \Big/ \sqrt{2P_t/N_r} \right|^2 \right]. \tag{15}$$

From (1), (2), (3) and (4), (15) can be rewritten as

$$2\sigma_e^2 = \frac{N_r}{N_p} \frac{N_g}{N_c} \left(\frac{P_t}{\sigma_n^2} \right)^{-1}. \tag{16}$$

It is seen from (16) that the CSI error increases as the number of the receiver antenna increases. This is because the transmit power per antenna in pilot stage decreases. However, CSI error can be reduced as the number of pilot blocks increases by the averaging operation.

B. Received SINR and conditional BER

Assuming the sum of the residual inter-symbol interference (ISI), the noise and CSI error as a new zero-mean complex-valued Gaussian variable, the received SINRs, $\gamma_{opt.}$ and $\gamma_{conv.}$, when using the optimal and conventional transmit FDE can be respectively derived as

$$\gamma_{opt.} = \frac{2\left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \sum_{n_r=0}^{N_r-1} \mathbf{H}(n_r,k)\overline{\mathbf{H}}^*(n_r,k)C_{opt.}^{-1}(k) \right)^2}{\left[\begin{gathered} \left\{ \begin{gathered} \frac{1}{N_c} \sum_{k=0}^{N_c-1} \left(\sum_{n_r=0}^{N_r-1} \mathbf{H}(n_r,k)\overline{\mathbf{H}}^*(n_r,k)C_{opt.}^{-1}(k) \right)^2 \\ -\left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \sum_{n_r=0}^{N_r-1} \mathbf{H}(n_r,k)\overline{\mathbf{H}}^*(n_r,k)C_{opt.}^{-1}(k) \right)^2 \end{gathered} \right\} \\ +\left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} I(k)C_{opt.}^{-1}(k) \right) 2\sigma_e^2 \\ +\frac{J}{Q}\frac{N_r}{N_c} \left(\frac{P_t}{\sigma_n^2} \right)^{-1} \sum_{k=0}^{N_c-1} \sum_{n_r=0}^{N_r-1} \left\| \overline{\mathbf{H}}(n_r,k)C_{opt.}^{-1}(k) \right\|^2 \end{gathered} \right]}, \tag{17a}$$

$$\gamma_{conv.}=\frac{2\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\sum_{n_r=0}^{N_r-1}\mathbf{H}(n_r,k)\overline{\mathbf{H}}(n_r,k)C_{conv.}^{-1}(k)\right)^2}{\left[\begin{array}{l}\left\{\begin{array}{l}\left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}\left(\sum_{n_r=0}^{N_r-1}\mathbf{H}(n_r,k)\overline{\mathbf{H}}^*(n_r,k)C_{conv.}^{-1}(k)\right)^2\right| \\ -\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\sum_{n_r=0}^{N_r-1}\mathbf{H}(n_r,k)\overline{\mathbf{H}}^*(n_r,k)C_{conv.}^{-1}(k)\right)^2\end{array}\right\} \\ +\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}I(k)C_{conv.}^{-1}(k)\right)2\sigma_e^2 \\ +\frac{N_r}{N_c}\left(\frac{P_t}{\sigma_n^2}\right)^{-1}\sum_{k=0}^{N_c-1}\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)C_{conv.}^{-1}(k)\right\|^2\end{array}\right]}, \quad (17b)$$

where $\overline{\mathbf{H}}(n_r,k)=\sqrt{2P_t/N_r}\,\mathbf{H}(n_r,k)$ and $I(k)$ is given as

$$I(k)=\begin{cases}\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)\right\|^2 & \text{if } N_r=2\\ \left\{\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)\right\|^2+\left\|\overline{\mathbf{H}}(0,k)\right\|^2\right\} & \text{if } N_r=3\\ 2\cdot\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)\right\|^2 & \text{if } N_r=4\end{cases}. \quad (18)$$

In (17), the first term in the denominator is the contribution of the residual ISI after STBC decoding and the second term is the contribution of the residual STBC code-word interference due to the STBC code-word orthogonality distortion caused by CSI error. The third term is the contribution of the noise. When the sufficiently high SNR is obtained by FD-STBC-JTRD, i.e., $\frac{1}{N_r}\left(\frac{P}{N}\right)\sum_{n_r=0}^{N_r-1}\left\|\mathbf{H}(n_r,k)\right\|^2>>1$, the ratio of the received SINRs, $\gamma_{opt.}/\gamma_{conv.}$, can be approximated as

$$\frac{\gamma_{opt.}}{\gamma_{conv.}}\approx\frac{2\sigma_e^2 I_{STBC}+1}{2\sigma_e^2 I_{STBC}+(J/Q)}\leq\frac{1}{R_{STBC}}, \quad (19)$$

where

$$I_{STBC}=\sum_{k=0}^{N_c-1}\frac{I(k)}{\left(\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)\right\|^2\right)^2}\Bigg/\sum_{k=0}^{N_c-1}\frac{1}{\left(\sum_{n_r=0}^{N_r-1}\left\|\overline{\mathbf{H}}(n_r,k)\right\|^2\right)^2}. \quad (20)$$

It is seen from (19) that the ratio of the received SINRs, $\gamma_{opt.}/\gamma_{conv.}$, becomes smaller than $1/R_{STBC}$ due to the residual STBC code-word interference caused by CSI error. Substituting (16) to (19), (19) can be rewritten as

$$\frac{\gamma_{opt.}}{\gamma_{conv.}}\approx\frac{\frac{N_r}{N_p}\frac{N_g}{N_c}\left(\frac{P_t}{\sigma_n^2}\right)^{-1}I_{STBC}+1}{\frac{N_r}{N_p}\frac{N_g}{N_c}\left(\frac{P_t}{\sigma_n^2}\right)^{-1}I_{STBC}+(J/Q)}\leq\frac{1}{R_{STBC}}. \quad (21)$$

It is seen from (21) that the optimal transmit FDE can achieve the received SINR about $1/R_{STBC}$ times higher than the conventional transmit FDE if the number of pilot blocks is sufficiently large. However, as the number of pilot blocks increases, the power loss due to pilot insertion increases, and as a consequence, the BER performance degrades. Therefore, there exists the best number of pilot blocks which minimizes the BER.

Furthermore, assuming QPSK data modulation, the conditional BER, $p_{e,opt.}$ and $p_{e,conv.}$, for the given channel transfer function when using the optimal and conventional transmit FDE can be respectively given as [1]

$$\begin{cases}p_{e,opt.}=\frac{1}{2}erfc\left[\sqrt{\frac{\gamma_{opt.}}{4}}\right]\\ p_{e,conv.}=\frac{1}{2}erfc\left[\sqrt{\frac{\gamma_{conv.}}{4}}\right]\end{cases}. \quad (22)$$

The average BER can be numerically evaluated by averaging (22) over possible channel transfer function.

IV. COMPUTER SIMULATION

We evaluate, by computer simulation, the average BER performance when using FD-STBC-JTRD with the optimal transmit FDE and CDP-CE. We consider QPSK data modulation. FFT block size N_c and CP length N_g are set to N_c = 128 symbols and N_g = 16 samples, respectively. The number of transmitter antennas N_t is set to N_t = 1 as an example. The channel is assumed to be a frequency-selective block Rayleigh fading channel having symbol spaced L = 16 path uniform power delay profile. In this paper, we consider a quasi-static fading channel (i.e., Doppler frequency $f_D\rightarrow 0$). We use a Chu-sequence [17] for pilot signal. The pilot insertion interval N_B is set to N_B=16.

A. BER performance

Fig. 3 shows the BER performances when using SC FD-STBC-JTRD with the optimal transmit FDE and CDP-CE as a function of transmit E_b/N_0. The number of pilot blocks is set to N_p=2. For the comparison, the performances when using the conventional transmit FDE [10] with CDP-CE and when using the optimal transmit FDE with perfect CSI are also plotted in Fig. 3(a) and Fig. 3(b), respectively. It is seen from Fig. 3(a) that the optimal transmit FDE can achieve better BER performance than the conventional transmit FDE even in the presence of CSI error when N_r>2. This is because the optimal transmit FDE matrices are sparse and the norm of the optimal transmit FDE matrix is smaller than that of the conventional transmit FDE [12]. When N_r=4, the optimal transmit FDE reduces the transmit E_b/N_0 required for BER=10^{-4} by about 1.2 dB compared to the conventional transmit FDE. This amount of E_b/N_0 reduction is equal to $1/R_{STBC}$. It is seen from Fig. 3(b) that the performance difference between CDP-CE and perfect CSI increases as the number of receiver antenna increases. This is because CSI error increases proportional to the number of receiver antenna. It is also seen from Fig. 3 that the simulation results and the theoretical results match well.

B. The impact of the number of pilot blocks

Fig. 4 shows that the required transmit E_b/N_0 for achieving BER=10^{-4} as a function of the number of pilot blocks. It is seen from Fig. 4 that the required transmit E_b/N_0 difference between the optimal transmit FDE and the conventional transmit FDE increases as the number of pilot blocks increases. This is because, as the number of pilot blocks increases, CSI error decreases and as a consequence, the residual STBC code-word interference caused by CSI error decreases. However, when N_p>3, the required E_b/N_0 increases as the number of pilot blocks increases because the power loss due to pilot insertion increases. Therefore, the use of N_p=2 pilot blocks is sufficient to improve the required transmit E_b/N_0. It is also seen from Fig.

4 that, when N_p =2, the required transmit E_b/N_0 difference between the optimal transmit FDE and the conventional transmit FDE is about 1.2 dB and the same as $1/R_{STBC}$ (=1.2dB). Therefore, by letting the number of pilot blocks be N_p=2, the optimal transmit FDE can achieve the received SINR $1/R_{STBC}$ times higher than the conventional transmit FDE even in the presence of CSI error.

V. Conclusion

In this paper, we theoretically analyzed the received SINR and conditional BER of SC FD-STBC-JTRD using CDP-CE and discussed the impact of CSI error. We showed that the use of N_p=2 pilot blocks minimizes the required transmit E_b/N_0 and the optimal transmit FDE can achieve the received SINR $1/R_{STBC}$ times higher than the conventional transmit FDE even in the presence of CSI error. The theoretical analysis was confirmed by the computer simulation.

In this paper, we assumed a quasi-static fading channel. The impact of CSI error caused by the time selectivity of channel is left as our future work. We considered CDP-CE in this paper. Performance analysis of SC FD-STBC-JTRD using other CE schemes is also an interesting future work.

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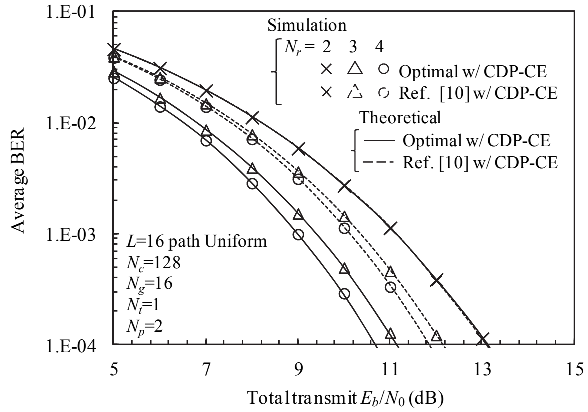
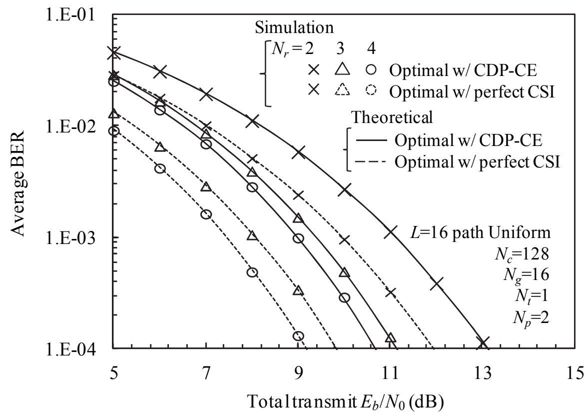


(a) Comparison to the conventional transmit FDE



(b) Comparison to perfect CSI

Fig. 3. Average BER performance.

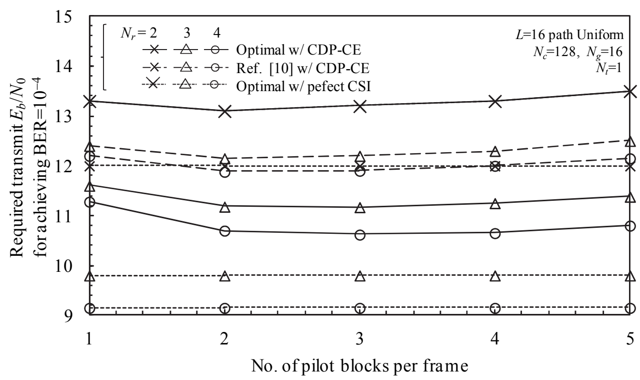


Fig. 4. Impact of the number fo pilot blocks.