Pseudo Block Coded Single-Carrier Frequency-Domain Equalization Trasnsmission

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***Abstract*— In this paper, we propose a pseudo block-coded SC transmission with frequency-domain equalization (FDE). We call this scheme as pseudo block-coded SC with FDE (PBCSC-FDE). A concatenation of block encoding matrix, interleaving matrix, DFT matrix, mapping matrix and channel matrix can be viewed as an equivalent MIMO channel and FDE and block decoding can be jointly performed based on the minimum mean square error (MMSE) criterion. We evaluate, by computer simulation, the average bit error rate (BER) performance of PBCSC-FDE and show that its achievable BER performance is superior to a 2-step decoding which performs FDE and hard decision block decoding separately.**

Keywords-component; Frequency-domain equalization, block decoding, single-carrier transmission

I. INTRODUCTION

The bit error rate (BER) performance of broadband single-carrier (SC) transmissions significantly degrades due to the inter-symbol interference (ISI) caused by frequency-selective fading [1]. The minimum mean square error (MMSE) based frequency-domain equalization (FDE) is an effective scheme to overcome the frequency-selective fading [2-4]. SC transmission with MMSE-FDE can take advantage of channel frequency-selectivity and obtain large frequency-diversity gain.

In wireless communications, channel coding, such as block coding and convolutional coding, are necessary to improve BER performance [5]. In block coded SC transmissions in a frequency-selective channel, maximum likelihood (ML) decoding is the optimal decoding scheme. However, its computational complexity is extremely high, and as a consequence, it is quite difficult to apply it to practical systems. On the other hand, hard decision block decoding has low computational complexity; however, its error correction performance is lower compared to ML decoding.

In block coded SC transmission, a concatenation of block encoding, interleaving, discrete Fourier transform (DFT), mapping can be expressed as matrix operation and therefore, can be viewed as an equivalent MIMO channel.

In this paper, we propose a pseudo block-coded SC transmission with frequency-domain equalization. We call this scheme as pseudo block-coded SC with FDE (PBCSC-FDE). The block encoding matrix based on Galois field operation is replaced by pseudo block encoding matrix based on linear operation. Viewing a concatenation of pseudo encoding matrix, interleaving matrix, DFT matrix, mapping matrix and channel matrix as an equivalent MIMO channel, frequency-domain equalization and decoding are jointly performed based on the MMSE criterion. We evaluate, by computer simulation, the average BER performance of PBCSC-FDE and show that the achievable BER performance is superior to 2-step decoding which performs FDE and hard decision decoding separately.

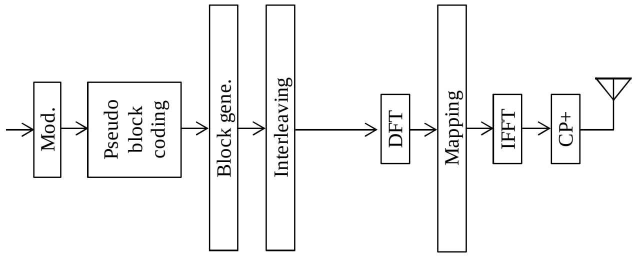
The remainder of this paper is organized as follows. Sect. II presents PBCSC-FDE and derives MMSE frequency-domain equalization and decoding weight. The computer simulation results are presented in Sect. III. Sect. IV offers conclusions.

II. PSEUDO BLOCK CODED SC TRANSMISSION WITH FDE

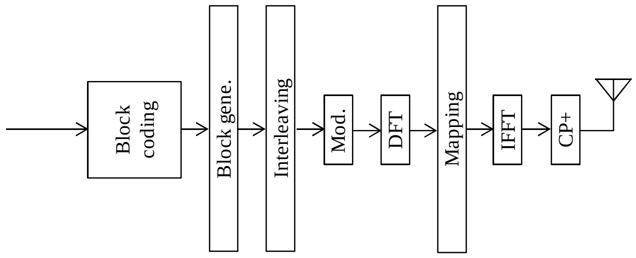
In this paper, we consider PBCSC-FDE. We assume that the transmitter and receiver equip with single antenna, respectively. The transmitter and receiver structures in PBCSC-FDE are illustrated in Fig. 1 and Fig. 2, respectively. For the comparison, the transmitter and receiver structures in block coded SC transmission with 2-step decoding are also shown in Fig. 1 and Fig. 2, respectively. In conventional block coded SC transmission, block encoding is based on Galois field operation and data modulation is also based on nonlinear operation. Therefore, a concatenation of block encoding matrix, interleaving matrix, DFT matrix, mapping matrix and channel matrix cannot be viewed as an equivalent linear MIMO channel. In PBCSC-FDE, to eliminate the above nonlinear operations from block encoding to block decoding, block encoding matrix is replaced by pseudo block encoding matrix based on linear operation. Furthermore, data modulation is performed before pseudo block encoding.

In the transmitter, k pseudo block coded symbols are generated by applying (k,n) pseudo block encoding to n data modulated information symbols. The N_{code} pseudo block coded sequences are generated and then, interleaving is performed to a sequence of kN_{code} symbols. A sequence of MN_{code} symbols after interleaving is transformed into frequency-domain signal by kN_{code}-point DFT and then mapped to N_c subcarriers. The frequency-domain transmit signal after mapping is transformed back to the time-domain transmit signal by N_c-point inverse fast Fourier transform (IFFT). After inserting the cyclic prefix (CP) at the beginning of each block, the transmitter transmits the signal to the receiver.

At the receiver, after CP removal, the received signal is transformed into the frequency-domain signal by N_c-point FFT. A concatenation of pseudo encoding matrix, interleaving matrix, DFT matrix, mapping matrix and channel matrix can be viewed as an equivalent MIMO channel. Then, frequency-domain equalization and decoding are jointly performed based on the minimum mean square error (MMSE) criterion. Finally, data demodulation is performed.

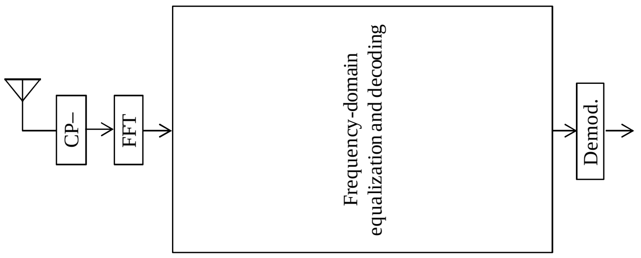


(a) PBCSC-FDE

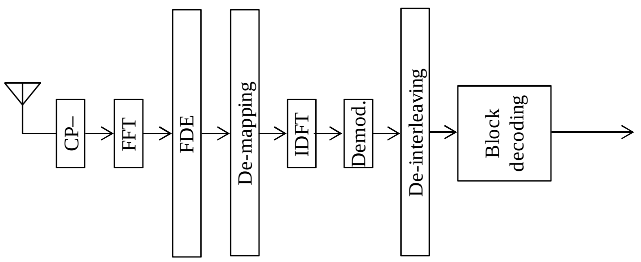


(b) Block coded SC transmission with 2-step decoding

Fig. 1. Transmitter structures.



(a) PBCSC-FDE



(b) Block coded SC transmission with 2-step decoding

Fig. 2. Receiver structures.

A. Signal representation

Below, the symbol-spaced discrete time representation is used.

At the transmitter, the pseudo block coded symbol sequence is generated by applying pseudo block encoding to the data modulated information symbol sequence. The above operation is repeated and N_{code} pseudo block coded symbol sequences are generated. The mth (m=0,...,N_{code}–1) pseudo block coded symbol vector $\mathbf{c}_m=[c_m(0),...,c_m(k-1)]^T$ are given as

$$\mathbf{c}_m=\tilde{\mathbf{C}}\mathbf{d}_m\,, \tag{1}$$

where $\mathbf{d}_m=[d_m(0),...,d_m(n\text{-}1)]^T$ is the mth information symbol vector and $\mathbf{C}$ is $k\times n$ pseudo block encoding matrix based on linear operation. The pseudo block encoding matrix is designed so as to make average transmit power be the same as conventional block encoding and given as

$$\tilde{\mathbf{C}}=\begin{bmatrix}\mathbf{c}(0)\Big/\sqrt{\|\mathbf{c}(0)\|^2}\\ \vdots\\ \mathbf{c}(k-1)\Big/\sqrt{\|\mathbf{c}(k-1)\|^2}\end{bmatrix}. \tag{2}$$

After interleaving, a sequence of kN_{code} symbols is transformed into the frequency-domain transmit signal by kN_{code}-point DFT, and then, mapped to N_c subcarriers. The frequency-domain transmit signal after mapping is transformed back to the time-domain signal by N_c-point IFFT. After CP insertion, the transmitter transmit signal to the receiver.

At the receiver, after CP removal, the received signal is transformed into the frequency-domain received signal by N_c-point FFT. The frequency-domain received signal vector $\mathbf{R}=[R(0),...,R(N_c-1)]^T$ is expressed as

$$\mathbf{R}=\sqrt{2P}\mathbf{HMFB}\overline{\mathbf{C}}\overline{\mathbf{d}}+\mathbf{N}\,, \tag{3}$$

where P is the received signal power. $\mathbf{H}=diag\{H(0),...,H(N_c-1)\}$ denotes the $N_c\times N_c$ channel matrix and $H(j)$ is the jth frequency channel transfer function. $\mathbf{F}$ is the $kN_{code}\times kN_{code}$ DFT matrix and given as

$$\mathbf{F}=\frac{1}{\sqrt{kN_{code}}}\begin{bmatrix} e^{-j\frac{2\pi\cdot 0\cdot 0}{kN_{code}}} & e^{-j\frac{2\pi\cdot 0\cdot 1}{kN_{code}}} & \cdots & e^{-j\frac{2\pi\cdot 0\cdot (kN_{code}-1)}{kN_{code}}}\\ e^{-j\frac{2\pi\cdot 1\cdot 0}{kN_{code}}} & e^{-j\frac{2\pi\cdot 1\cdot 1}{kN_{code}}} & & \vdots\\ \vdots & & \ddots & \vdots\\ e^{-j\frac{2\pi\cdot (kN_{code}-1)\cdot 0}{kN_{code}}} & \cdots & \cdots & e^{-j\frac{2\pi\cdot (kN_{code}-1)\cdot (kN_{code}-1)}{kN_{code}}}\end{bmatrix}. \tag{4}$$

$\mathbf{M}$ denotes the $N_c\times kN_{code}$ mapping matrix and $\mathbf{B}$ is the $kN_{code}\times kN_{code}$ interleaving matrix. $\overline{\mathbf{C}}$ and $\overline{\mathbf{d}}$ are the $kN_{code}\times nN_{code}$ expanded pseudo block encoding matrix and the expanded information symbol vector, respectively. They are expressed as

$$\overline{\mathbf{C}}=\begin{bmatrix}\tilde{\mathbf{C}} & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & \tilde{\mathbf{C}}\end{bmatrix},\qquad \overline{\mathbf{d}}=\begin{bmatrix}\mathbf{d}_0\\ \vdots\\ \mathbf{d}_{M-1}\end{bmatrix}. \tag{5}$$

$\mathbf{N}=[N(0),...,N(N_c-1)]^T$ is the noise vector and $N(j)$ denotes the zero-mean complex valued additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ with N_0 and T_s being the single-sided power spectrum density of AWGN and the symbol duration, respectively.

(3) can be rewritten as

$$\mathbf{R}=\sqrt{2P}\hat{\mathbf{H}}\overline{\mathbf{d}}+\mathbf{N}\,, \tag{6}$$

where $\hat{\mathbf{H}}=\mathbf{HMFB}\overline{\mathbf{C}}$ is $N_c\times nN_{code}$ equivalent channel matrix. It is seen from (6) that the concatenation of the channel matrix, the mapping matrix, DFT matrix, the interleaving matrix and the expanded pseudo block encoding matrix can be viewed as an equivalent MIMO channel in PBCSC-FDE. Therefore, FDE, de-mapping, IDFT, de-interleaving, pseudo block decoding are jointly performed by applying frequency-domain equalization and decoding filtering. The expanded pseudo block decoded information symbol vector $\hat{\mathbf{d}} = [\hat{d}_0(0),...,\hat{d}_0(n-1),...,\hat{d}_{N_{code}-1}(0),...,\hat{d}_{N_{code}-1}(n-1)]^T$ can be expressed as

$$\hat{\mathbf{d}} = \mathbf{W}\mathbf{R}, \tag{7}$$

where $\mathbf{W}$ denotes the $nN_{code}\times N_c$ frequency-domain equalization and decoding weight matrix. Finally, data demodulation is performed.

B. Derivation of MMSE frequency-domain equalization and decoding weight matrix

In this paper, the frequency-domain equalization and decoding weight matrix is designed so as to minimize the mean square error (MSE) between the transmit information symbol vector before pseudo block encoding and the received information symbol vector after frequency-domain equalization and decoding. The MMSE frequency-domain equalization and decoding weight matrix satisfies the optimization problem given as

$$\mathbf{W} = \arg\min e = tr\left[E\left[(\bar{\mathbf{d}} - \hat{\mathbf{d}})(\bar{\mathbf{d}} - \hat{\mathbf{d}})^H\right]\right]. \tag{8}$$

By solving the above optimization problem, the MMSE frequency-domain equalization and decoding weight matrix are obtained as [6]

$$\mathbf{W} = \hat{\mathbf{H}}^H\left[\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \gamma^{-1}\mathbf{I}_{N_c}\right]^{-1}, \tag{9}$$

where γ is the received signal-to-noise power ratio (SNR) and $\mathbf{I}_N$ is $N\times N$ identity matrix. By applying the matrix inversion lemma [7], (9) can be rewritten as

$$\mathbf{W} = \left[\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \gamma^{-1}\mathbf{I}_{nN_{code}}\right]^{-1}\hat{\mathbf{H}}^H. \tag{10}$$

It is seen from (10) that the computational complexity for computing MMSE frequency-domain equalization and decoding weight matrix is proportional to $(nN_{code})^3$.

III. Computer Simulation

We evaluate, by computer simulation, BER performance of PBCSC-FDE. The simulation conditions are summarized in Table I.

In this paper, we consider (7,4) hamming code and (15,7) BCH code as a primary study. BPSK, QPSK and 16QAM data modulation are used. the number of block coded sequences N_{code} is set to $N_{code} = \lfloor N_c/k \rfloor$ in PBCSC-FDE and $N_{code} = \lfloor ZN_c/k \rfloor$ in conventional block coded SC transmission, respectively, so as to keep the number of information bits per block constant, where Z denotes the number of bits per symbol and $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. We use $k\times N_{code}$ block interleaver and consecutive mapping (i.e., $\mathbf{M} = [\mathbf{I}_{kN_{code}}\ \mathbf{0}]^T$). FFT block size and CP length is set to N_c=64 and N_g=16, respectively. The channel is assumed to be a frequency-selective block Rayleigh fading channel having symbol spaced L = 16 path uniform power delay profile. We assume that channel estimation can be perfectly performed at the receiver.

TABLE I. COMPUTER SIMULATION CONDITIONS

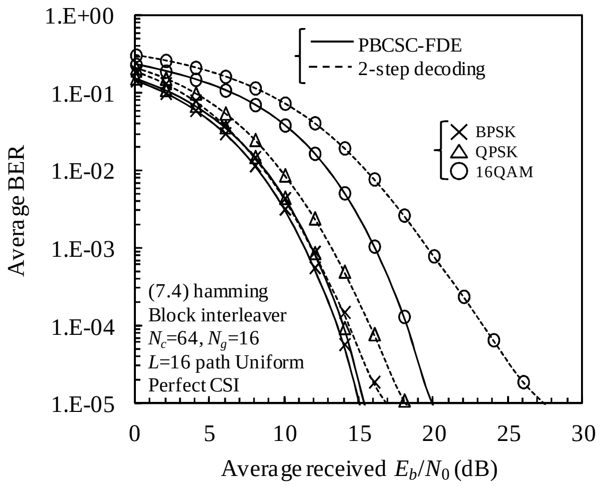
Transmitter /receiver	Channel coding	(7,4) hamming code (15,7) BCH code
	Data modulation	BPSK, QPSK, 16QAM
	Interleaver	Block interleaver
	FFT block size	N_c=64
	GI length	N_g=16
	Channel estimation	Ideal
Channel	Frequency-selective block Rayleigh fading	
	Delay power profile	L=16 path Uniform
	Delay time	Symbol spaced

A. Comparison to 2 step decoding

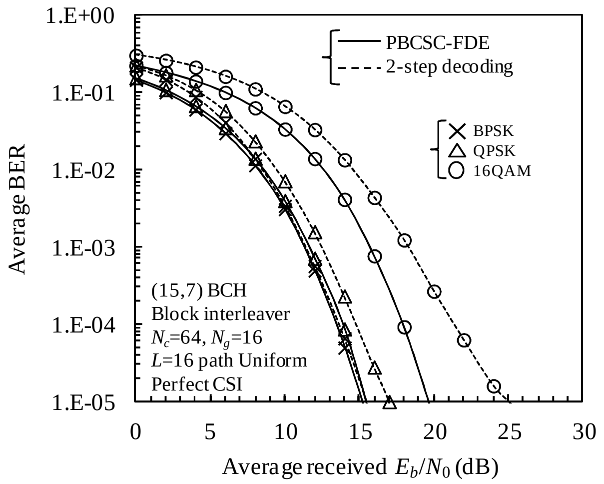
Fig. 3 plots the BER performance of PBCSC-FDE as a function of the average received E_b/N_0. For the comparison, the performance when using the block coded SC transmission with 2-step decoding is also plotted in Fig. 3. It is seen from Fig. 3 that PBCSC-FDE can achieve better BER performance than the block coded SC transmission with 2-step decoding regardless of kinds of block codes. This is because high frequency diversity gain and coding gain can be obtained by jointly performing FDE and block decoding. Furthermore, the performance gap between PBCSC-FDE and the block coded SC transmission with 2-step decoding becomes larger as the modulation level increases. This is because, as the modulation level increases, Euclidian distance between modulated symbols decreases, and as consequence, the effects of frequency diversity gain and coding gain become larger. For example, when using (7,4) hamming code and 16QAM data modulation, PBCSC-FDE can reduce the required E_b/N_0 for BER=10^{-5} by about 7dB compared to the block coded SC transmission with 2-step decoding.

B. Comparison to ML decoding

Fig. 4 shows the BER performances of PBCSC-FDE and the block coded SC transmission with ML decoding as a function of the average received E_b/N_0. QPSK data modulation is used and FFT block size is set to N_c=16 due to the limitation of the computational complexity. For the comparison, the performance of the block coded SC transmission with 2-step decoding is also shown in Fig. 4. It is seen from Fig. 4 that PBCSC-FDE can obtain almost the same diversity order as the block coded SC transmission with ML decoding in high E_b/N_0 region. However, the performance gap between PBCSC-FDE and the block coded SC transmission with ML decoding is still large. For example, when using (7,4) hamming code, PBCSC-FDE requires by about 5dB larger received E_b/N_0 than the block coded SC transmission with ML decoding. This is due to the residual inter-symbol interference after frequency-domain equalization and decoding caused by frequency-selective fading and pseudo block encoding. Therefore, it is conceivable that the performance gap between PBCSC-FDE and the block coded SC transmission with ML decoding can be reduced by mitigating the residual inter-symbol interference after frequency-domain equalization and decoding.



(a) (7,4) hamming code

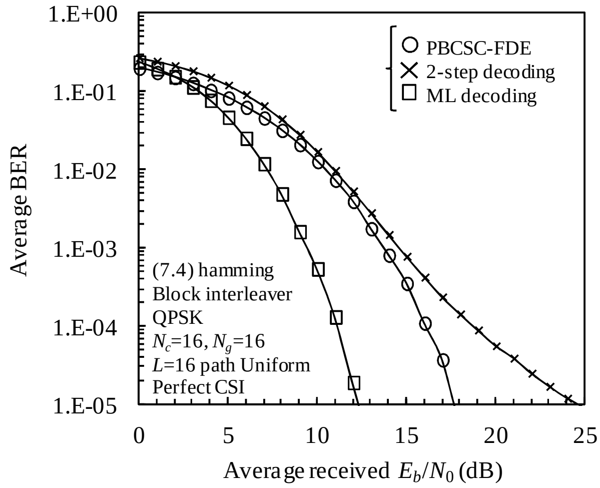


(b) (15,7) BCH code

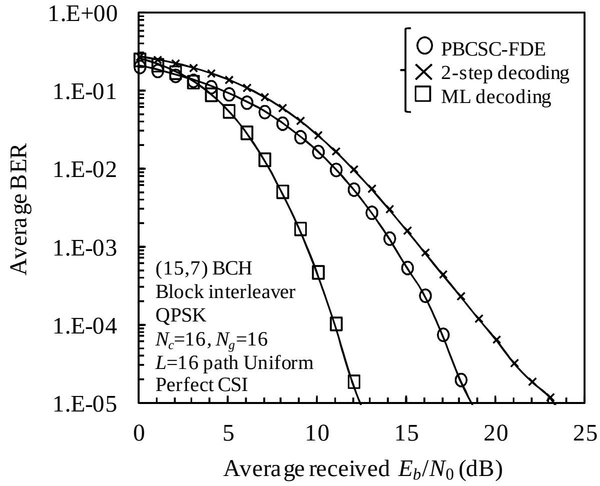
Fig. 3. Comparison to 2-step decoding.

C. *Computational complexity*

The computational complexity when using each transmission scheme at the receiver is summarized in Table II. The computational complexity is defined as the number of real multiplications per block. The computational complexity of PBCSC-FDE is proportional to $(nN_{code})^3$ in order to compute MMSE frequency-domain equalization and decoding weight matrix. However, PBCSC-FDE can cut the computational complexity for de-mapping, IDFT, de-interleaving, that are required in 2 step decoding, because FDE, de-mapping, IDFT, de-interleaving and block decoding are jointly performed. For example, when using (7,4) hamming code and 16QAM data modulation and when setting N_c=64, the computational complexity for the block coded SC transmission with 2-step decoding is 19,758, that for PBCSC-FDE is 867,324 and that for the block coded SC transmission with ML decoding is 18,158×2^{144}, respectively. Therefore, PBCSC-FDE can achieve better BER performance with about 44 times higher computational complexity compared to the block coded SC transmission with 2-step decoding. It is also seen from Table II that PBCSC-FDE can significantly reduce the computational complexity compared to the block coded SC transmission with ML decoding.



(a) (7,4) hamming code



(b) (15,7) BCH code

Fig. 4. Comparison to ML decoding.

TABLE II. COMPUTATINAL COMPLEXITY

2 step decoding		PBCSC-FDE		ML decoding	
FFT	$5N_c\log_2 N_c$	FFT	$5N_c\log_2 N_c$	FFT	$5N_c\log_2 N_c$
FDE weight computation	$8N_c$	Frequency-domain equalization and decoding weight computation	$7nk(N_{code})^2$ $+2nkN_{code}$ $+8n^2k(N_{code})^3$ $+4(nN_{code})^3$	Complexity per a candidate	$12N_{code}+2kN_{code}$ $+4(kN_{code})^2+2N_c$
FDE	$4N_c$				
De-mapping	$2N_c$				
IDFT	$4(kN_{code})^2$	Frequency-domain equalization and decoding	$4nN_{code}N_c$	No. of candidates	2^{nNcode}
De-interleaving	$2kN_{code}$				
Block decoding	$12N_{code}$				
Total	$5N_c\log_2 N_c+10N_c$ $+4(kN_{code})^2$ $+N_{code}(2k+12)$	Total	$5N_c\log_2 N_c$ $+7nk(N_{code})^2$ $+2nkN_{code}$ $+8n^2k(N_{code})^3$ $+4(nN_{code})^3$ $+4nN_{code}N_c$	Total	$5N_c\log_2 N_c$ $+2^{nNcode}\times$ $\{N_{code}(2k+12)$ $+4(kN_{code})^2+2N_c\}$

IV. CONCLUSION

In this paper, we proposed a pseudo block coded SC transmission with FDE (PBCSC-FDE). A concatenation of pseudo block encoding matrix, interleaving matrix, mapping matrix and channel matrix is viewed as an equivalent MIMO channel and frequency-domain equalization and block decoding are jointly performed based on MMSE criterion. It was shown by computer simulation that PBCSC-FDE can achieve BER performance superior to the block coded SC transmission with 2-step decoding.

In this paper, we considered very simple (7,4) hamming codes as a primary study and showed that, when using (7,4) hamming codes and 16QAM data modulation, PBCSC-FDE can reduce the required E_b/N_0 for BER=10^{-5} by about 7dB compared to the block coded SC transmission with 2-step decoding. Application of e.g. BCH codes to PBCSC-FDE is left as our future work.

REFERENCES

[1] J. G. Proakis and M. Salehi, Digital communications, 5th edition, McGraw-Hill, 2008.

[2] H. Sari, G. Karam, and I. Jeanclaude, "Transmission technique for digital terrestrial TV broadcasting," IEEE Commun. Mag., Vol. 33, No. 2, pp. 100-109, Feb. 1995.

[3] D. Falconer, S. L. Ariyavistakul, A. Benyamin-Seeyar, and B. Edison, "Frequency domain equalization for single-carrier broadband wireless systems," IEEE Commun.. Mag., Vol. 40, No. 4, pp. 58-66, Apr. 2002.

[4] F. Adachi, H. Tomeba, and K. Takeda, "Introduction of frequency-domain signal processing to broadband single-carrier transmission in a wireless cheannel," IEICE Trans. Commun., Vol. E92-B, pp. 2789-2808, Sept. 2009.

[5] S. Lin and D. J. Constello, Error control coding: fundamentals and applications, Prentice-Hall, 1983.

[6] S. Haykin, Adaptive Filter Theory, 4th edition, Prentice-Hall, 2002.

[7] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd edition, Johns Hoplins University Press, 1996.