Variable Earns Profit: Improved Adaptive Channel Estimation Using Sparse VSS-NLMS Algorithms

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Abstract—**Accurate channel estimation is essential for broadband wireless communications. Adaptive sparse channel estimation schemes based on normalized least mean square (NLMS) have been proposed to exploit channel sparsity for improved performance. However, their performance bound as derived in this paper indicates that the invariable step size (ISS) usually used for iteration in these schemes would lead to performance loss or/and slow convergence speed as well as high computational cost. To solve this problem, based on the observation that a large step size is preferred for fast convergence while a small step size is preferred for accurate estimation, we then propose to replace the ISS by the variable step size (VSS) to improve the performance of sparse channel estimation. The key idea is that the VSS can be adaptive to the estimation error in each iteration, i.e., a large step size is used in the case of large estimation error to accelerate the convergence speed, while a small step size is used when the estimation error is small to improve the steady-state estimation accuracy. Finally, simulation results verify that better mean square error (MSE) and bit error rate (BER) performance could be achieved by the proposed scheme.**

I. Introduction

Broadband signal transmission is becoming one of the mainstream techniques in the next generation wireless communication systems [1], [2]. The channel in broadband wireless systems becomes severely frequency-selective, and accurate channel state information (CSI) of such a channel is required for coherent detection or demodulation [3].

Various channel estimation methods have been proposed in the last decades [4]–[11]. One of the effective approaches is the adaptive channel estimation (ACE) using the normalized least mean square (NLMS) algorithm [6], which has low complexity and can be easily implemented. On the other hand, many channel measurements have verified that wireless channels often exhibit a large delay spread but a small number of nonzero taps, and this channel sparsity has led to adaptive sparse channel estimation (ASCE) with improved accuracy [7]–[11]. Unfortunately, the NLMS-based ACE neglects the inherent sparse structure of wireless channels, thus it maybe not able to achieve the estimation performance comparable to ASCE. Recently, an ℓ_1-norm sparse constraint function has been considered in the zero-attracting NLMS (ZA-NLMS) [12] and reweighted zero-attracting NLMS (RZA-NLMS) [13] algorithms to take advantage of the channel sparsity to improve the estimation performance. In NLMS-based algorithms including ZA-NLMS and RZA-NLMS, it is well known that the step size is a critical parameter to control the estimation performance, convergence speed and computational cost. However, only the invariable step size (ISS) has been considered [12], [13], which leads to performance loss or/and slow convergence speed as well as high computational cost. Although variable step size NLMS (VSS-NLMS) has been proposed for ACE to improve the estimation accuracy [14], channel sparsity has not been considered in the VSS-NLMS algorithm.

In this paper, inspired by the observation that a large step size is preferred for fast convergence while a small step size is preferred for accurate estimation, we propose two improved ASCE methods named as VSS zero-attracting NLMS (VSS-ZA-NLMS) and VSS reweighted zero-attracting NLMS (VSS-RZA-NLMS) algorithms by jointly exploiting channel sparsity and VSS-NLMS. The key idea of the proposed VSS-ZA-NLMS and VSS-RZA-NLMS algorithms is to replace the ISS by VSS in conventional NLMS-based algorithms to improve the adaptive sparse channel estimation in terms of mean square error (MSE) and bit error rate (BER) metrics. The VSS is adaptive to the estimation error in each iteration, i.e., a large step size is used in the case of large estimation error to accelerate the convergence speed, while a small step size is used when the estimation error is small to improve the steady-state estimation accuracy.

Notation: Throughout the paper, matrices and vectors are represented by boldface upper case letters and boldface lower case letters, respectively; $(\cdot)^T$, $(\cdot)^H$, $\mathrm{Tr}(\cdot)$ and $(\cdot)^{-1}$ denote the transpose, the Hermitian transpose, the trace and the inverse operators, respectively; $\mathrm{E}(\cdot)$ denotes the expectation operator; $\|\mathbf{h}\|_0$ is the ℓ_0-norm operator that counts the number of nonzero taps in $\mathbf{h}$, and $\|\mathbf{h}\|_p$ stands for the ℓ_p-norm operator which is computed as $\|\mathbf{h}\|_p = (\sum_i \|h_i\|^p)^{1/p}$, where $p \in \{1, 2\}$ is considered in this paper; $\mathrm{sgn}(\cdot)$ is a component-wise function which is defined as $\mathrm{sgn}(h) = 1$ for $h > 0$, $\mathrm{sgn}(h) = 0$ for $h = 0$, and $\mathrm{sgn}(h) = -1$ for $h < 0$.

II. Convectional NLMS-Based Algorithms

We consider a baseband equivalent multipath sparse channel $\mathbf{h} = [h_0, h_1, \cdots, h_{N-1}]^T$ of length N with K nonzero channel taps [7], [12], and assume that an input training signal $\boldsymbol{x}(n)$ is used to probe the unknown sparse channel. At the receiver, the corresponding observed signal $y(n)$ is given by

$$y(n) = \mathbf{h}^T \boldsymbol{x}(n) + z(n), \tag{1}$$

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where $\boldsymbol{x}(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T$ denotes the vector of input training signal $x(n)$, $z(n)$ is the additive white Gaussian noise (AWGN) with zero mean and the variance σ^2, which is assumed to be independent of $x(n)$. The objective of ASCE is to adaptively estimate the unknown sparse channel vector $\mathbf{h}$ using the training signal vector $\boldsymbol{x}(n)$ and the observed signal $y(n)$.

A. ISS-ZA-NLMS Algorithm

By defining the square estimation error at the nth iteration as $e^2(n)$, ISS-ZA-NLMS [12] was proposed as

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu \frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\mathrm{ZA}}\mathrm{sgn}(\tilde{\mathbf{h}}(n)), \quad (2)$$

where $\tilde{\mathbf{h}}(n)$ is the channel estimate in the nth iteration, $\mu \in (0, 1/\lambda_{\max})$ is the ISS, $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{R} = \mathrm{E}[\boldsymbol{x}(n)\boldsymbol{x}^T(n)]$, $\rho_{\mathrm{ZA}} = \mu\lambda_{\mathrm{ZA}}$ is a parameter which depends on the ISS μ and the regularization parameter λ_{ZA}, and the term $\rho_{\mathrm{ZA}}\mathrm{sgn}(\tilde{\mathbf{h}}(n))$ is used to attract small channel coefficients as zero in a high probability. In other words, most of small channel coefficients can be directly replaced by zero, which is helpful to speed up the convergence of this algorithm and also to mitigate the extra noise or interference on zero positions.

B. ISS-RZA-NLMS Algorithm

The sparse constraint of ISS-ZA-NLMS always gives the identical penalty to all taps which are forced to be zero with the same probability. Motivated by the reweighted ℓ_1-norm minimization recovery algorithm [15], recently we have proposed an improved algorithm ISS-RZA-NLMS [13] as

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu \frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\mathrm{RZA}} \frac{\mathrm{sgn}(\tilde{\mathbf{h}}(n))}{1 + \epsilon_{\mathrm{RZA}}|\tilde{\mathbf{h}}|}, \quad (3)$$

where $\rho_{\mathrm{RZA}} = \mu\lambda_{\mathrm{RZA}}$, λ_{RZA} is the regularization parameter, and ϵ_{RZA} is the reweighted factor which can be set as $\epsilon_{\mathrm{RZA}} = 20$ as suggested in [13]. Note that the third term on the right side of (3) attracts the channel coefficients $\{\tilde{h}_i(n), i = 0, 1, \cdots, N-1\}$ whose magnitudes are comparable to $1/\epsilon_{\mathrm{RZA}}$ to zero.

C. Drawbacks of Conventional Sparse ISS-NLMS Algorithms

In this subsection, we derive the steady-state MSE performance of ISS-ZA-NLMS as the typical example to mathematically illustrate the drawbacks of conventional sparse ISS-NLMS algorithms [12], [13].

Under the assumption of independent channel realization, the steady-state MSE of ISS-NLMS estimator has been derived in [16] as

$$\begin{aligned}\xi_t(\infty) &= \lim_{n\to\infty} \mathrm{E}\left\{\left[(\tilde{\mathbf{h}}(n) - \mathbf{h})^T \boldsymbol{x}(n)\right]^2\right\} \\ &= \frac{\mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]\sigma^2}{2 - \mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]} \\ &\geq \frac{\lambda_{\max}\sigma^2}{2 - 3\mu\lambda_{\max}},\end{aligned} \quad (4)$$

where σ^2 is noise power (or variance) of the AWGN $z(n)$. Similarly, we can derive the steady-state MSE of the ISS-ZA-NLMS estimator as

$$\begin{aligned}\xi_s(\infty) &= \lim_{n\to\infty} \mathrm{E}\left\{\left[(\tilde{\mathbf{h}}(n) - \mathbf{h})^T \boldsymbol{x}(n)\right]^2\right\} \\ &= \frac{\mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]\sigma^2}{2 - \mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]} + \frac{\gamma_1\rho_{\mathrm{ZA}}(\rho_{\mathrm{ZA}} - 2\gamma_2/\gamma_1)}{(2 - \mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right])\mu} \\ &< \frac{\mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]\sigma^2}{2 - \mathrm{Tr}\left[\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right]} \\ &\leq \frac{\lambda_{\max}\sigma^2}{2 - 3\mu\lambda_{\max}} \\ &\leq \xi_t(\infty),\end{aligned} \quad (5)$$

where $\gamma_1 = \mathrm{E}[\mathrm{sgn}(\tilde{\mathbf{h}}^T(n))(\mathbf{I} - \mu\mathbf{R})^{-1}\mathrm{sgn}(\tilde{\mathbf{h}}(n))] > 0$, and $\gamma_2 = E\{\|\tilde{\mathbf{h}}(\infty)\|_1 - \|\mathbf{h}\|_1\}$. To exploit the channel sparsity, ρ_{ZA} should be selected in the range $(0, 2\gamma_2/\gamma_1]$ so that $(\rho_{\mathrm{ZA}} - 2\gamma_2/\gamma_1) \leq 0$. Hence, $\xi_s(\infty)$ in (5) is lower than $\xi_t(\infty)$ in (4). According to (5), the upper bound of $\xi_s(\infty)$ depends on three factors: $\{\lambda_{\max},\ \sigma^2,\ \mu\}$. Since $\lambda_{\max}$ and σ^2 are determined by the input signal $x(n)$ and the AWGN $z(n)$, respectively, the only configurable parameter is the step size μ. In the extreme case when $\mu \to 0$, the upper bound of steady-state MSE of sparse ISS-NLMS algorithm can be derived as

$$\lim_{\mu\to 0} \xi_s(\infty) \leq \lim_{\mu\to 0} \frac{\lambda_{\max}\sigma^2}{2 - 3\mu\lambda_{\max}} = \frac{\lambda_{\max}\sigma^2}{2}. \quad (6)$$

It is clear from (4) and (5) that a smaller step size μ leads to a better MSE performance. However, if a small step size is adopted, it will incur slow convergence speed (i.e., high computational cost) of the adaptive channel estimation in the iteration process. Hence, it is expected that a large step size is used in the case of large estimation error to accelerate the convergence speed, while a small step size is used in the case of small estimation error to improve the steady-state MSE performance. To simultaneously achieve fast convergence speed and low steady-state MSE performance, we propose sparse VSS-NLMS algorithms in the next section.

III. Proposed Adaptive Channel Estimation Using Sparse VSS-NLMS Algorithms

Recall that sparse ISS-NLMS algorithms in (2) and (3) do not utilize VSS. Inspired by the VSS-NLMS algorithm which has been proposed in [14], we propose sparse VSS-NLMS algorithms called as VSS-ZA-NLMS and VSS-RZA-NLMS by simultaneously exploiting VSS and channel sparsity to further improve the estimation performance.

For the given observed signal $y(n)$, based on the conventional ISS-ZA-NLMS algorithm (2), the proposed VSS-ZA-NLMS algorithm performs as follows

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu(n+1)\frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\mathrm{ZA}}\mathrm{sgn}(\tilde{\mathbf{h}}(n)), \quad (7)$$

where $\mu(n+1)$ is the VSS, which is updated as

$$\mu(n+1) = \mu_{\max}\frac{\boldsymbol{p}^T(n+1)\boldsymbol{p}(n+1)}{\boldsymbol{p}^T(n+1)\boldsymbol{p}(n+1) + c}, \quad (8)$$

Input: 1) The maximal step size $\mu_{\max}$, sparse regularization parameter λ_{RZA}, positive threshold parameter c, smoothing factor β, ρ_{ZA} for VSS-ZA-NLMS and $\{\rho_{\text{RZA}}, \epsilon_{\text{RZA}}\}$ for VSS-RZA-NLMS;
2) Training signal vector $\boldsymbol{x}(n)$;
3) Observed signal $y(n)$.

Output: Final channel estimate $\tilde{\mathbf{h}}$.

$n \leftarrow 0$;
$\tilde{\mathbf{h}}(n) \leftarrow \mathbf{0}$;
$\boldsymbol{p}(n) \leftarrow \mathbf{0}$.

while $\|\tilde{\mathbf{h}}(n+1) - \tilde{\mathbf{h}}\|_2^2 \leq 10^{-5}$ or $n \geq 5000$ **do**

$n \leftarrow n+1$;

$e(n) \leftarrow y(n) - \tilde{\mathbf{h}}^T(n-1)\boldsymbol{x}(n)$;

$\boldsymbol{p}(n+1) \leftarrow \beta \boldsymbol{p}(n) + (1-\beta)\frac{\boldsymbol{x}(n)e(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)}$;

$\mu(n+1) \leftarrow \mu_{\max}\frac{\boldsymbol{p}^T(n+1)\boldsymbol{p}(n+1)}{\boldsymbol{p}^T(n+1)\boldsymbol{p}(n+1)+c}$;

$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu(n+1)\frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\text{ZA}}\text{sgn}(\tilde{\mathbf{h}}(n))$

for VSS-ZA-NLMS in (7) or

$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu(n+1)\frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\text{RZA}}\frac{\text{sgn}(\tilde{\mathbf{h}}(n))}{1+\epsilon_{\text{RZA}}|\tilde{\mathbf{h}}|}$

for VSS-RZA-NLMS in (10);

end

$\tilde{\mathbf{h}} \leftarrow \tilde{\mathbf{h}}(n+1)$;

Algorithm 1: Proposed sparse VSS-NLMS algorithms for adaptive sparse channel estimation.

where $\mu_{\max}$ is the maximal step size, c is a positive threshold parameter related to $\sigma^2\text{Tr}\{[\boldsymbol{x}(n)\boldsymbol{x}^T(n)]^{-1}\}$ and can be set as $c \sim \mathcal{O}(1/\text{SNR})$, where SNR is the signal-to-noise ratio (SNR) at the receiver, and $\boldsymbol{p}(n)$ is defined by

$$\boldsymbol{p}(n+1) = \beta\boldsymbol{p}(n) + (1-\beta)\frac{\boldsymbol{x}(n)e(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)}, \tag{9}$$

where $\beta \in [0,1]$ is the smoothing factor for controlling the VSS and estimation error. According to (8), the VSS range is given by $\mu(n+1) \in (0, \mu_{\max})$. To ensure the stability of the adaptive estimation algorithm (7), the maximal step size $\mu_{\max}$ should be less than 2 [14].

Similarly, based on the conventional ISS-RZA-NLMS algorithm (3), the VSS-RZA-NLMS algorithm is proposed as

$$\tilde{\mathbf{h}}(n+1) = \tilde{\mathbf{h}}(n) + \mu(n+1)\frac{e(n)\boldsymbol{x}(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} - \rho_{\text{RZA}}\frac{\text{sgn}(\tilde{\mathbf{h}}(n))}{1+\epsilon_{\text{RZA}}|\tilde{\mathbf{h}}|}. \tag{10}$$

Based on the sparse VSS-NLMS algorithms (7) and (10), two improved adaptive sparse channel estimation methods are summarized in **Algorithm 1**.

For a better understanding of the difference between ISS and VSS, Fig. 1 depicts the step size μ for sparse ISS-NLMS and VSS-NLMS algorithms when the maximal step size $\mu_{\max} = 1$ and the step size $\mu \in \{0.5, 1.0\}$ are considered, respectively. It is clear from Fig. 1 that the VSS $\mu(n)$ for VSS-NLMS decreases as the estimation error becomes smaller and vice versa, while the step size μ for ISS-NLMS remains invariant no matter how small the estimation error is.

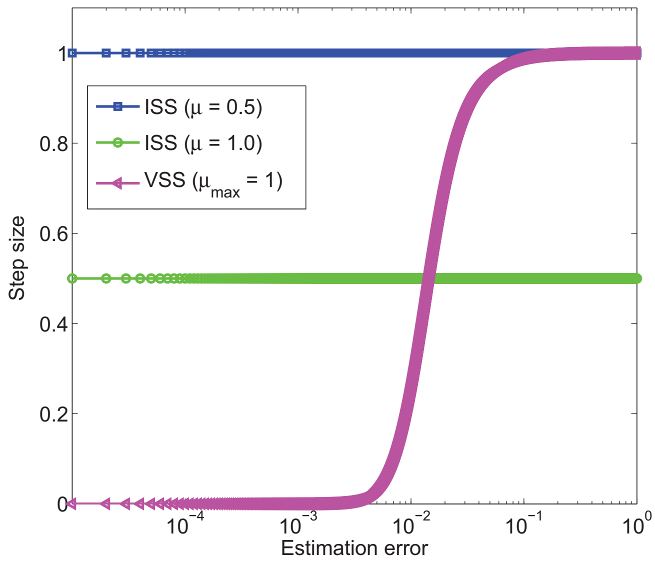

Fig. 1. Step size vs. estimation error for sparse ISS-NLMS and VSS-NLMS algorithms.

TABLE I
SIMULATION PARAMETERS.

Parameters	**Values**
Channel length	$N = 60$
Channel sparsity level	$K = 3$ and 6
Distribution of nonzero coefficient	Random Gaussian $\mathcal{CN}(0,1)$
Threshold parameter	$c = 10^{-4}$ for 5 dB and $c = 10^{-5}$ for 10 dB
Step size for iteration	$\mu = 0.2$ and $\mu_{\max} = 2$
Regularization parameters	$\rho_{\text{ZA}} = 0.0002\sigma^2$ and $\rho_{\text{RZA}} = 0.002\sigma^2$
Modulation schemes	8 PSK, 16 PSK, 16 QAM, 64 QAM

IV. SIMULATIONS RESULTS

In this section, simulation results in terms of MSE and BER are provided to validate the effectiveness of the proposed scheme. The MSE of channel estimation is defined as

$$\text{MSE}\{\tilde{\mathbf{h}}(n)\} = \text{E}\{\|\mathbf{h} - \tilde{\mathbf{h}}(n)\|_2^2\}. \tag{11}$$

The BER performance is obtained by averaging the results of 1000 independent Monte-Carlo runs. Parameters for simulations are provided in Tab. I. Note that the reweighted factor of sparse RZA-NLMS algorithms (for both ISS and VSS) is set as $\epsilon_{\text{RZA}} = 20$ to achieve better steady-state estimation performance [13].

Figures 2-5 present the MSE performance of the proposed VSS-ZA-NLMS and VSS-RZA-NLMS algorithms for the

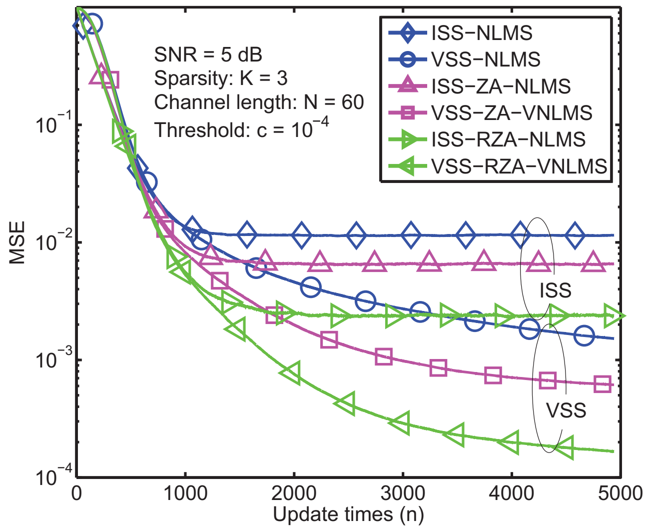

Fig. 2. MSE performance comparison versus algorithm update times when SNR = 5 dB and $K = 3$.

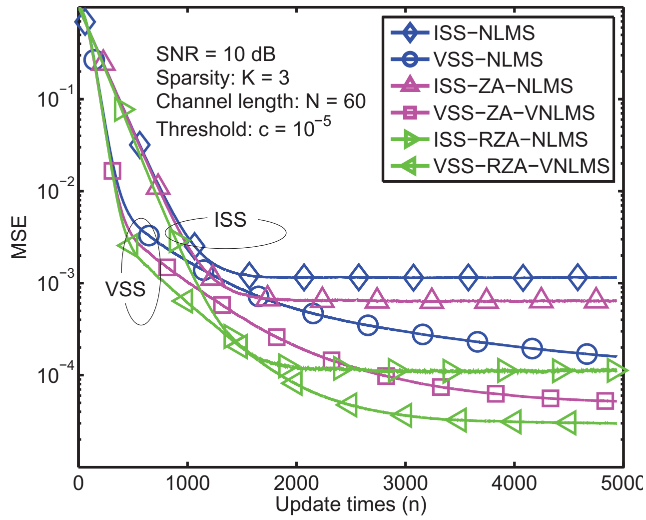

Fig. 4. MSE performance comparison versus algorithm update times when SNR = 10 dB and $K = 3$.

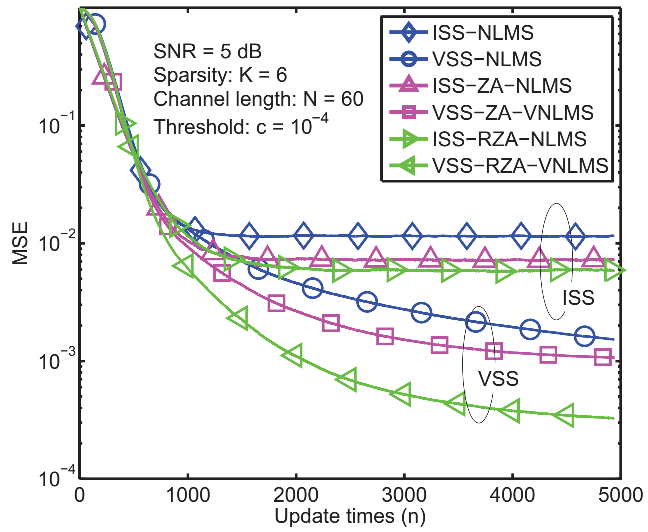

Fig. 3. MSE performance comparison versus algorithm update times when SNR = 5 dB and $K = 6$.

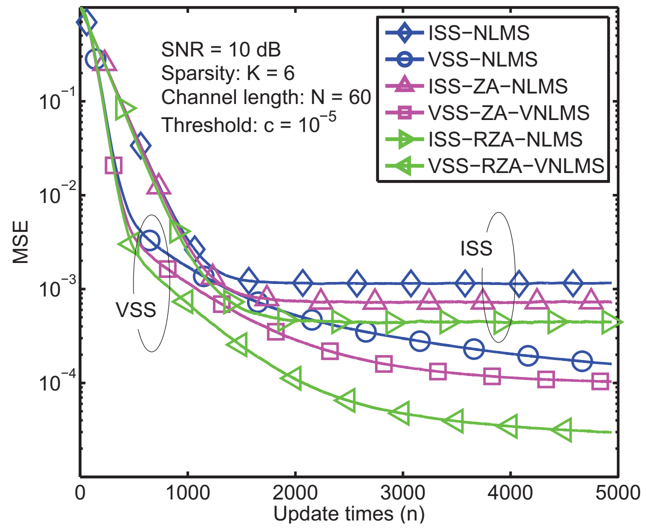

Fig. 5. MSE performance comparison versus algorithm update times when SNR = 10 dB and $K = 6$.

channel sparsity level $K = 3$ and $K = 6$ under the SNR of 5 dB and 10 dB, respectively. To confirm the effectiveness of the proposed scheme, the MSE performance of three conventional methods, i.e., ISS-NLMS [6], VSS-NLMS [14] and sparse ISS-NLMS algorithms [13], is also included for comparison. It can be observed that for different considered SNRs, the proposed sparse VSS-NLMS algorithms always achieve better MSE performance as well as faster convergence speed than conventional sparse ISS-NLMS ones. This is due to the fact that sparse VSS-NLMS algorithms utilize the VSS which is adaptive to estimation error in the iteration process. In other words, sparse VSS-NLMS algorithm adopts a large step size to accelerate the convergence speed while a small step size is adaptively used to improve the estimation accuracy. In addition, sparse VSS-NLMS algorithms also take advantage of the channel sparsity, so they can achieve better estimation performance than sparse ISS-NLMS ones, especially when the channel is very sparse with low sparsity level.

Figures 6 and 7 show the system BER performance when the proposed channel estimation scheme is adopted. Here, the channel sparsity level $K = 3$ is considered. Fig. 6 plots the BER performance of multilevel phase shift keying (PSK) modulation schemes, i.e., 8 PSK and 16 PSK. One can find that both of the proposed VSS-ZA-NLMS and VSS-RZA-NLMS algorithms can achieve better BER performance than conventional ISS-ZA-NLMS and ISS-RZA-NLMS algorithms. In addition, the BER performance when VSS-RZA-NLMS is adopted is better than that when VSS-ZA-NLMS is used, be-

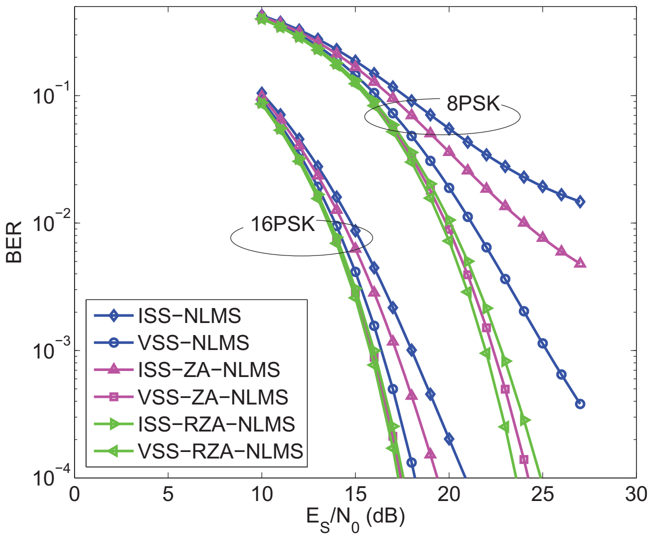

Fig. 6. BER performance comparison for PSK modulations.

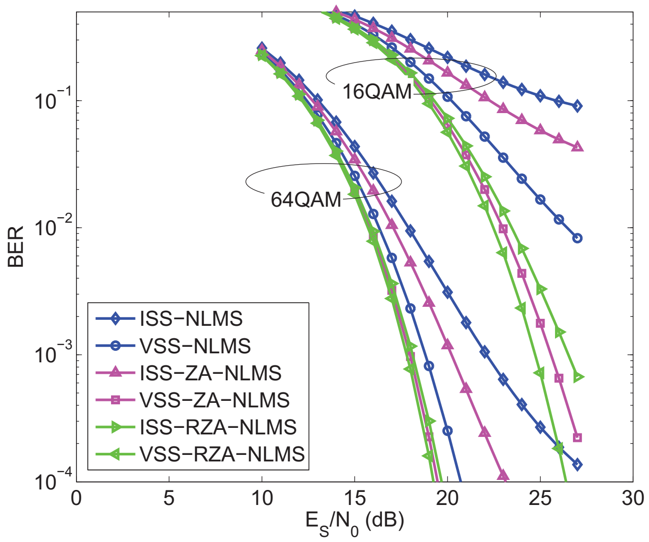

Fig. 7. BER performance comparison for QAM modulations.

cause the former algorithm takes more channel sparse information into consideration than the latter one. In Fig. 7, the BER performance of multilevel quadrature amplitude modulation (QAM) schemes, i.e., 16 QAM and 64 QAM, are considered. It can be observed that the proposed schemes could also achieve better BER performance than conventional methods.

V. Conclusions

The key drawback of conventional sparse ISS-NLMS algorithms is that they cannot balance the convergence speed and steady-state performance for adaptive sparse channel estimation. In this paper, we propose two sparse VSS-NLMS algorithms, i.e., VSS-ZA-NLMS and VSS-RZA-NLMS, to improve channel estimation accuracy. Instead of the ISS, the proposed algorithms utilize the VSS which can change adaptively according to the estimation error in the iteration process, i.e., the step size becomes smaller as the estimation accuracy improves and vice versa. Simulation results validate that better performance in terms of MSE and BER could be achieved by the proposed algorithms.

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