

# Single-Carrier Transmission with Frequency-Domain based Code-Division Multi-Access

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**Abstract**— Single-carrier (SC) transmission is a promising transmission scheme for uplink transmission because of its low peak-to-average power ratio (PAPR). Spread spectrum (SS) transmission achieves higher frequency diversity gain and hence improves the bit error rate (BER) performance. The authors have recently proposed SC with frequency-domain spread spectrum (SC-FDSS) transmission, where spreading and de-spreading are conducted in frequency domain. In this paper, we propose a new code-division multi-access (CDMA) scheme based on SC-FDSS for multi-user environment, called SC with frequency-domain CDMA (SC-FD-CDMA). Theoretical analysis on conditional BER of the uplink SC-FD-CDMA is presented. The BER analysis is confirmed by the computer simulation together with PAPR evaluation. BER and PAPR of SC-FD-CDMA are compared with direct-sequence CDMA (DS-CDMA) and multi-carrier CDMA (MC-CDMA).

**Keywords**— Single-carrier (SC) transmission, spread spectrum, code division multi-access (CDMA), uplink transmission

## I. INTRODUCTION

Broadband wireless channel is characterized as a frequency-selective fading channel, in which inter-symbol interference (ISI) degrades the bit-error rate (BER) performance [1]. Multi-carrier transmission, such as orthogonal frequency division multiplexing (OFDM), is robust against frequency-selective fading but its high peak-to-average power ratio (PAPR) of transmit signal is the main drawback [2]. On the other hand, single-carrier (SC) transmission [3] is more attractive for uplink communication in LTE-Advanced (LTE-A) system because of lower PAPR, while the use of frequency-domain equalization (FDE) can take advantage of the channel frequency selectivity to improve the BER [4].

SC transmission can be combined with multi-access techniques which provide users' orthogonality in different domains [5-7]. Among the various combinations of SC transmission and multi-access, SC with FDMA (called SC-FDMA) [8] and direct-sequence CDMA (DS-CDMA) with FDE [7] are very attractive for uplink transmissions. Multi-carrier CDMA (MC-CDMA) [9] can be possibly considered as a candidate due to its robustness against frequency selectivity.

In SC-FDMA, users are separated in frequency-domain, and hence there is no multi-user interference (MUI) [10]. However, since the number of available subcarriers is limited, a complicated resource allocation algorithm [11] is necessary. On the other hand, CDMA, which is adopted for the third-generation (3G) system [12], allows a user to share the same

bandwidth. This implies that the resource allocation is not needed. Frequency diversity gain is also achievable by either time-domain (for DS-CDMA) or frequency-domain spreading (for MC-CDMA); however, strong MUI occurs since the orthogonality among different spreading codes is severely distorted through the multipath fading, and consequently degrades the BER performance.

Recently, we proposed SC with frequency-domain spread spectrum (SC-FDSS) [13] combined with orthogonal code multiplexing [14]. Spreading and de-spreading are conducted in frequency domain similar to multi-carrier spread spectrum (MC-SS) but with the aid of discrete Fourier transform (DFT). Performance of SC-FDSS was evaluated in [13, 14] to confirm that better BER is achieved compared to SC with time-domain spread spectrum (SC-TDSS). The theoretical results in [14] also showed that the inter-chip interference (ICI) in SC-FDSS is lower than SC-TDSS, implying that SC-FDSS is preferable to be used in strong-ICI environment. Note that the performance evaluation in [13, 14] were done only in single-user environment.

In this paper, we extend the single-user SC-FDSS and propose a novel multi-user SC-FDSS, called SC with frequency-domain CDMA (SC-FD-CDMA). A single-cell multi-user uplink transmission is considered. Performance evaluation of SC-FD-CDMA is done by computer simulation and compared to conventional DS-CDMA [7] and MC-CDMA [9] in terms of BER and PAPR assuming the same number of users and that of subcarriers. It will be shown that the uplink BER performance of SC-FD-CDMA is better than DS-CDMA as a contribution of lower interference.

The rest of this paper is organized as follows. The transceiver model and signal representations for SC-FD-CDMA are presented in Sect. II. Theoretical analysis on conditional BER is presented in Sect. III. Section IV presents the simulation results of BER and PAPR. Section V concludes the paper.

## II. TRANSMISSION SYSTEM MODEL

Chip-spaced discrete-time signal representation is used throughout this paper. A single-cell single-antenna consisting of  $U$  users is considered, where all  $U$  users transmit the data to the base station. The number of available subcarriers is  $N_c$ . Fig. 1 illustrates the baseband transmission system models of (a) transmitter of the  $u$ -th user,  $u=0\sim U-1$ , and (b) receiver at the base station of uplink SC-FD-CDMA. Note that  $U \leq SF$  when  $SF$  represents the spreading factor.



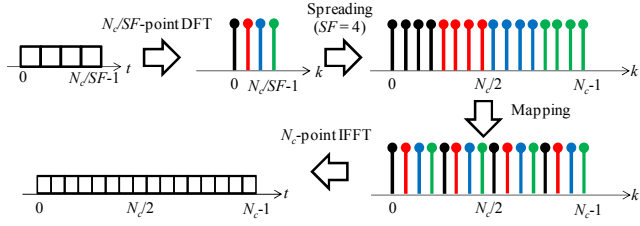


Fig. 2. Transmit signal processing in SC-FD-CDMA.

where  $\tilde{\mathbf{s}}_u$  is obtained from (5), and  $\mathbf{n}_u$  is noise vector in which element is zero-mean additive white Gaussian noise (AWGN) having the variance  $2N_0/T_c$  with  $T_c$  and  $N_0$  being the chip duration and the one-sided noise power spectrum density, respectively. Channel response matrix  $\mathbf{h}_u$  is a circular matrix representing time-domain channel response between the  $u$ -th user and the base station, which is

$$\mathbf{h}_u = \begin{bmatrix} h_{u,0} & & & h_{u,L-1} & \cdots & h_{u,1} \\ h_{u,1} & \ddots & & & & \vdots \\ \vdots & & h_{u,0} & \mathbf{0} & & h_{u,L-1} \\ h_{u,L-1} & & h_{u,1} & \ddots & & \\ \mathbf{0} & \ddots & \vdots & & \ddots & \\ \mathbf{0} & h_{u,L-1} & \cdots & \cdots & \cdots & h_{u,0} \end{bmatrix}. \quad (8)$$

The received signal vector  $\mathbf{r}$  is transformed into frequency domain by  $N_c$ -point FFT, obtaining the frequency-domain received signal  $\mathbf{R}$  as

$$\begin{aligned} \mathbf{R} &= \sum_{u=0}^{U-1} \sqrt{2P_u} \mathbf{F}_{N_c} \mathbf{h}_u \tilde{\mathbf{s}}_u + \mathbf{F}_{N_c} \mathbf{n}_u \\ &= \sum_{u=0}^{U-1} \sqrt{2P_u} \mathbf{F}_{N_c} \mathbf{h}_u \mathbf{F}_{N_c}^H \tilde{\mathbf{S}}_u + \mathbf{F}_{N_c} \mathbf{n}_u, \\ &= \sum_{u=0}^{U-1} \sqrt{2P_u} \mathbf{H}_u \tilde{\mathbf{S}}_u + \mathbf{N}_u \end{aligned} \quad (9)$$

where the frequency-domain channel response  $\mathbf{H}_u$  is  $\mathbf{H}_u \equiv \mathbf{F}_{N_c} \mathbf{h}_u \mathbf{F}_{N_c}^H = \text{diag}[H_u(0), \dots, H_u(N_c-1)]$

FDE based on minimum mean-square error criterion (MMSE-FDE) is introduced for mitigating the ISI occurred by frequency-selective fading channel. The frequency-domain received signal after applying MMSE-FDE of the  $u$ -th user is

$$\hat{\mathbf{R}}_u = \mathbf{W}_u \mathbf{R}, \quad (10)$$

where  $\mathbf{W}_u = \text{diag}\{W_u(0), \dots, W_u(N_c-1)\}$  is a  $N_c \times N_c$  diagonal matrix. In this paper, a conventional MMSE-FDE, which minimize the mean-square error (MSE) between  $\tilde{\mathbf{S}}_u$  and  $\hat{\mathbf{R}}_u$ , is considered for a fair comparison with DS-CDMA and MC-CDMA.  $W_u(k)$  is described in [15] as

$$W_u(k) = \frac{\frac{P_u T_c}{N_0} H_u^*(k)}{\sum_{u=0}^{U-1} \frac{P_u T_c}{N_0} |H_u(k)|^2 + 1}. \quad (11)$$

After that, de-mapping is applied to the received signal after applying MMSE-FDE  $\hat{\mathbf{R}}_u$ , obtaining the frequency-domain signal  $\tilde{\mathbf{R}}_u = [\tilde{R}_u(0), \tilde{R}_u(1), \dots, \tilde{R}_u(N_c-1)]^T$ . De-mapping can be expressed by

$$\tilde{R}_u((p \times SF) + q) = \hat{R}_u(p + (q \times M)), \quad (12)$$

where  $p=0 \sim M-1$  and  $q=0 \sim SF-1$ . It can be seen that de-mapping in (13) is simply an inverse operation of (4).

De-spreading is also applied in frequency-domain approach by simply multiplying  $\tilde{\mathbf{R}}_u$  by an inverse operation of (3), resulting in frequency-domain vector  $\tilde{\mathbf{D}}_u = [\tilde{D}_u(0), \tilde{D}_u(1), \dots, \tilde{D}_u(M-1)]^T$  as

$$\tilde{\mathbf{D}}_u = \mathbf{C}_u^H \tilde{\mathbf{R}}_u. \quad (13)$$

Note that  $\mathbf{C}_u^H$  has dimension of  $M \times N_c$ . Finally,  $\tilde{\mathbf{D}}_u$  is transformed back into time domain by  $M$ -point inverse DFT (IDFT) matrix, obtaining time-domain received vector of the  $u$ -th user  $\tilde{\mathbf{d}}_u = [\tilde{d}_u(0), \tilde{d}_u(1), \dots, \tilde{d}_u(M-1)]^T$  as

$$\tilde{\mathbf{d}}_u = \mathbf{F}_M^H \tilde{\mathbf{D}}_u. \quad (14)$$

### III. BER ANALYSIS

In this section, conditional SINR and BER analysis is derived for the proposed SC-FD-CDMA uplink. SINR is derived by firstly determining a frequency-domain signal after FDE of the  $j$ -th user  $\tilde{\mathbf{R}}_j = [\tilde{R}_j(0), \tilde{R}_j(1), \dots, \tilde{R}_j(N_c-1)]^T$  as

$$\begin{aligned} \tilde{R}_j(k) &= \sqrt{2P_j} \hat{H}_{j,j}(k) C_j(k) D_j\left(\left\lfloor \frac{k}{SF} \right\rfloor\right) \\ &+ \sum_{\substack{u=0 \\ u \neq j}}^{U-1} \sqrt{2P_u} \hat{H}_{u,j}(k) C_u(k) D_u\left(\left\lfloor \frac{k}{SF} \right\rfloor\right) \\ &+ \tilde{W}_j(k) N(k) \end{aligned} \quad (15)$$

Here,  $\hat{H}_{u,j}(k)$ ,  $\tilde{H}_u(k)$  and  $\tilde{W}_j(k)$  are expressed as follows.

$$\begin{cases} \hat{H}_{u,j}(k) = \tilde{H}_u(k) \tilde{W}_j(k) \\ \tilde{H}_u((p \times SF) + q) = H_u(p + (q \times M)), \\ \tilde{W}_j((p \times SF) + q) = W_j(p + (q \times M)) \end{cases} \quad (16)$$

where  $p=0 \sim M-1$  and  $q=0 \sim SF-1$ . Substitute (15) into (13) yields

$$\begin{aligned} \tilde{D}_j(m) &= \sum_{k=mSF}^{(m+1)SF-1} \sqrt{2P_j} \hat{H}_{j,j}(k) C_j(k) C_j^*(k) D_j(m) \\ &+ \sum_{k=mSF}^{(m+1)SF-1} \sum_{\substack{u=0 \\ u \neq j}}^{U-1} \sqrt{2P_u} \hat{H}_{u,j}(k) C_u(k) C_u^*(k) D_u(m), \\ &+ \sum_{k=mSF}^{(m+1)SF-1} \tilde{W}_j(k) N(k) \end{aligned} \quad (17)$$

TABLE I. SIMULATION PARAMETERS

Transmitter	Data modulation	QPSK
	FFT/IFFT block size	$N_c = 256$
	Cyclic prefix length	$N_g = 16$
Multi-access	No. of users	$U = 1-4$
	Multiple access technique	MC-CDMA, DS-CDMA, SC-FD-CDMA
	Spreading factor	$SF = 16$
	Spreading code	Long-PN sequence
	Tx power control (TPC)	Ideal slow TPC
Channel	Fading	Frequency-selective block Rayleigh
	Power delay profile	Chip-spaced 16-path uniform
Receiver	Equalization	MMSE-FDE
	Channel estimation	Ideal

where  $m=0 \sim N_c/SF-1$ . It is observed from (17) that the second term and the third term represent residual MUI  $\mu_{MUI}$  and noise  $\mu_{N,j}$ , respectively. The variance of  $\mu_{MUI}$  is expressed by

$$2\sigma_{MUI}^2 = E[|\mu_{MUI}|^2] = \sum_{\substack{u=0 \\ u \neq j}}^{U-1} P_u \left( \frac{1}{SF} \sum_{k=mSF}^{(m+1)SF-1} \left( \frac{\hat{H}_{u,j}(k)}{SF} \right)^2 - \left| \frac{1}{SF} \sum_{k=mSF}^{(m+1)SF-1} \hat{H}_{u,j}(k) \right|^2 \right), \quad (18)$$

which is similar to MUI in MC-CDMA [16].

However, it is also observed that an additional residual ICI occurs when the frequency-domain component  $\tilde{D}_j(m)$  in (17) is transformed back into time-domain received symbol. The variance of additional residual ICI is expressed by

$$2\sigma_{ICI}^2 = P_j \left( \frac{1}{N_c/SF} \sum_{q=0}^{N_c/SF-1} \left( \frac{|\bar{H}_{j,j}(q)|^2}{N_c/SF} - \left| \frac{1}{N_c/SF} \sum_{q=0}^{N_c/SF-1} \bar{H}_{j,j}(q) \right|^2 \right) \right), \quad (19)$$

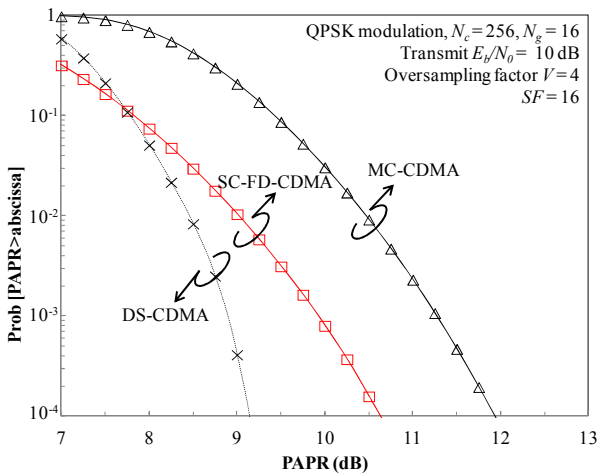


Fig. 3. CCDF of PAPR.

where  $\bar{H}_{j,j}(q)$ ,  $q=0 \sim N_c/SF-1$  represents an equivalent channel gain after de-spreading of the  $j$ -th user, which is

$$\bar{H}_{j,j}(q) = \sum_{k=qSF}^{(q+1)SF-1} \hat{H}_{j,j}(k). \quad (20)$$

Finally, the conditional SINR for the  $j$ -th user with the given  $P_j$  and  $\{\mathbf{H}_u, u=0 \sim U-1\}$  is expressed by

$$\gamma(P_j, \{\mathbf{H}_u\}) = \frac{2P_j \left| \frac{1}{N_c/SF} \sum_{q=0}^{N_c/SF-1} \bar{H}_{j,j}(q) \right|^2}{(2\sigma_{ICI}^2 + 2\sigma_{MUI}^2 + 2\sigma_{N,j}^2)}, \quad (21)$$

where  $2\sigma_{N,j}^2 = (1/N_c) \sum_{k=0}^{N_c-1} |\tilde{W}_j(k)|^2$  represents the noise enhancement of the  $j$ -th user. It is observed in (21) and [16] that  $2\sigma_{ICI}^2$  the additional residual ICI does not appear in MC-CDMA, implying that the SINR, and consequently the BER, of the proposed SC-FD-CDMA slightly degrades compared to MC-CDMA.

For simplicity of analysis, the MUI plus residual ICI plus noise after de-spreading is assumed to be a zero-mean complex-valued Gaussian random variable [13]. The conditional BER of the  $j$ -th user assuming QPSK modulation is given as

$$p_b(P_j, \{\mathbf{H}_u\}) = 0.5 \times \text{erfc} \left( \sqrt{0.25 \times \gamma(P_j, \{\mathbf{H}_u\})} \right), \quad (22)$$

where  $\text{erfc}(\cdot)$  is complementary error function. The theoretical average BER is numerically computed by averaging (22) over all possible  $\{\mathbf{H}_u\}$ . The theoretical BER performance is shown in Section IV together with simulation results.

#### IV. PERFORMANCE EVALUATION

Numerical and simulation parameters are summarized in Table 1. We assume a single-cell with single-antenna base station and  $U$  multi-user uplink transmission environment. A frequency-selective fading channel having chip-spaced  $L=16$  path uniform power delay profile is assumed. 4095-bit long pseudo noise (PN) sequence is used as spreading code  $\{C_u(0), \dots, C_u(SF-1)\}$  for the  $u$ -th user.

##### A. PAPR Performance

PAPR over a block of transmission [17] is defined as

$$PAPR = \frac{\max \{ |\tilde{s}(n)|^2 ; n=0, \frac{1}{V}, \frac{2}{V}, \dots, N_c-1 \}}{E[|\tilde{s}(n)|^2]}, \quad (23)$$

where  $V$  represents oversampling factor. We use complementary cumulative distribution function (CCDF) as an indicator of PAPR performance. Note that Nyquist pulse shaping with roll-off factor  $\alpha=0$  (i.e., frequency-domain ideal rectangular filter) is considered in this paper.

Fig. 3 shows the CCDF of PAPR of single-user transmit signal in DS-CDMA, MC-CDMA, and the proposed SC-FD-CDMA, where the PAPR is evaluated at  $SF=16$ . DS-CDMA provides the lowest transmit PAPR among these transmission schemes. PAPR of the proposed SC-FD-CDMA is higher than DS-CDMA, where PAPR at 0.1% outage probability ( $PAPR_{0.1\%}$ ) is approximately 0.9 dB higher than DS-CDMA. This is because frequency interleaving technique in the proposed SC-FD-CDMA decreases correlation among frequency-domain components. However,  $PAPR_{0.1\%}$  of the proposed SC-FD-CDMA is 1.3 dB lower than MC-CDMA since the waveform of SC-FD-CDMA remains SC waveform property.

### B. BER Performance

Fig. 4 shows the uplink BER performance as a function of average received bit energy-to-noise power spectrum density ratio  $E_b/N_0=0.5(P_u T_c/N_0)(SF)(1+N_g/N_c)$  when  $SF=16$ . Ideal slow transmit power control (TPC) (i.e.,  $P_u=P$  for all  $u$ ) is assumed. BER of SC-FD-CDMA is compared with DS-CDMA and MC-CDMA at the same number of users  $U$ . Also plotted in Fig. 4 are the theoretical BER performance curves obtained using the conditional BER expression derived in Sect. III.

It can be seen from Fig. 4 that the BER performance of SC-FD-CDMA provides better BER than DS-CDMA in every  $U$ . The reasons are well described in [13] as the SC-FD-CDMA can achieve more ICI mitigation inherited from frequency interleaving in (4), and lower phase error after de-spreading since the de-spreading is done in frequency domain. The BER of the proposed SC-FD-CDMA is slightly worse than MC-CDMA due to the additional residual ICI. However, SC-FD-CDMA produces lower PAPR of the transmit signal waveform. In addition, a fairly good agreement is observed between the theoretical and simulation results.

## V. CONCLUSION

In this paper, SC-FDSS with orthogonal code multiplexing was extended to multi-user uplink transmission called SC-FD-CDMA. In SC-FD-CDMA, spreading and de-spreading are done in frequency-domain, providing additional frequency-diversity gain and robustness against ISI and MUI. Simulation results assuming the single-cell environment confirmed that the proposed SC-FD-CDMA improves the BER performance compared to DS-CDMA. It was also clarified that the proposed SC-FD-CDMA provides a similar BER performance with lower transmit PAPR compared to MC-CDMA.

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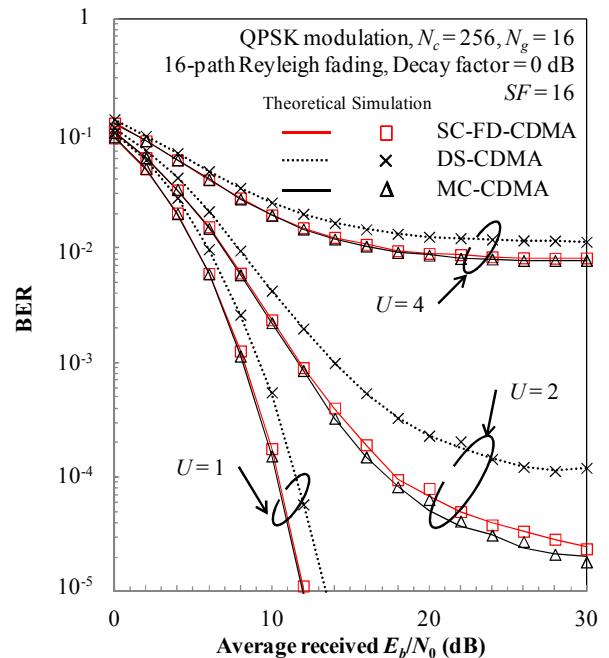


Fig. 4. BER performance of SC as a function of  $SF$ .

