Joint Iterative Tx/Rx MMSE Filtering & Interference Cancellation for SC-MIMO Spatial Multiplexing

Shinya Kumagai and Fumiyuki Adachi

Dept. of Communications Engineering, Graduate School of Engineering, Tohoku University
6-6-05, Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579, JAPAN
E-mail: kumagai@mobile.ecei.tohoku.ac.jp, adachi@ecei.tohoku.ac.jp

Abstract—**Recently, we proposed a joint transmit/receive spatial/frequency filtering based on minimum mean square error criterion (joint Tx/Rx MMSE filtering) for broadband single-carrier (SC) multiple-input multiple-output (MIMO) spatial multiplexing (SM). The residual inter-symbol interference (ISI) after joint Tx/Rx MMSE filtering limits the improvement of transmission performance. The introduction of iterative interference cancellation (IC) is effective to suppress the residual ISI and further improves the transmission performance. In this paper, we propose a joint iterative Tx/Rx MMSE filtering & IC for SC-MIMO-SM, and derive the optimal Tx/Rx filters. We show, by computer simulation, that joint iterative Tx/Rx MMSE filtering & IC can significantly improve the average bit error rate (BER) performance of turbo coded SC-MIMO-SM.**

Keywords—Single-carrier; MIMO; spatial/frequency filtering; iterative interference cancellation

I. INTRODUCTION

Multiple-input multiple-output (MIMO) spatial multiplexing (SM) [1] is a powerful technique to increase the transmission data rate without signal bandwidth expansion. MIMO-SM with orthogonal frequency-division multiplexing (OFDM) [2] has been attracting much attention because of its robustness against the frequency-selective fading [3]. However, the OFDM signal has a disadvantage of its high peak-to-average power ratio (PAPR) property.

Single-carrier (SC) block transmission with MIMO-SM has been attracting much attention as an alternative technique because of its lower PAPR property [4]. SC-MIMO-SM suffers not only from the inter-antenna interference (IAI) but also from the inter-symbol interference (ISI) caused by severe frequency-selectivity of the channel. The minimum mean square error based linear receive spatial/frequency filtering (Rx MMSE filtering) [4] for SC-MIMO-SM can achieve a good transmission performance with low-complexity. However, its performance improvement is limited due to the residual IAI and residual ISI after the Rx MMSE filtering.

Recently, we proposed a linear joint transmit/receive spatial/frequency filtering based MMSE criterion (joint Tx/Rx MMSE filtering) for SC-MIMO-SM [5]. The channel state information (CSI) is required at both transmitter and receiver. Joint Tx/Rx MMSE filtering transforms the MIMO channel to multiple orthogonal channels (i.e., eigenmodes) to avoid the IAI. Joint Tx/Rx MMSE filtering also jointly performs MMSE based frequency-domain transmit power allocation and receive frequency-domain equalization (FDE) on each eigenmode to suppress the ISI. As a result, joint Tx/Rx MMSE filtering achieves a better transmission performance than Rx MMSE filtering.

However, SC-MIMO-SM with joint Tx/Rx MMSE filtering still suffers from the residual ISI on each eigenmode. Iterative interference cancellation (IC) is effective to reduce the residual interference and its introduction to SC-MIMO-SM with Rx MMSE filtering was studied in [4]. The residual IAI and residual ISI replicas are generated using the bit log-likelihood ratio (LLR) of the received signal after channel decoding and are subtracted from the received signal after Rx MMSE filtering. A sequence of Rx MMSE filtering, IC, and channel decoding is repeated for a sufficient number of times. As a result, iterative Rx MMSE filtering & IC can well suppress the residual IAI and residual ISI, and improves the transmission performance of SC-MIMO-SM.

Precoding technique for SC-MIMO-SM with iterative IC was studied and the optimal precoding matrix which minimizes the total transmit power with bit error ratio (BER) requirement was proposed [6]. However, an iterative algorithm, such as interior-point method, is required to obtain the optimal matrix.

In this paper, we propose an iterative joint Tx/Rx MMSE filtering & IC for SC-MIMO-SM and derive the optimal Tx/Rx filters in closed-forms. The iterative joint Tx/Rx MMSE filtering & IC transforms the MIMO channel to eigenmodes to avoid the IAI, and at the same time, carries out the MMSE based power allocation and iterative FDE & IC to each eigenmode to suppress the ISI. We show, by computer simulation, that joint iterative Tx/Rx MMSE filtering & IC can significantly improve the average BER performance of turbo coded SC-MIMO-SM.

The remainder of this paper is organized as follows. Sect. II presents the system model and signal representation for SC-MIMO-SM with iterative joint Tx/Rx MMSE filtering & IC. Sect. III derives the Tx/Rx filters based on MMSE criterion and discusses their behavior. In Sect. IV, we evaluate the average BER performance achievable with the proposed method by computer simulation. Sect. V gives the concluding remarks.

Notations: $E[.]$, tr[.], $[.]^T$, and $[.]^H$ denote the ensemble average operation, the trace operation, the transpose operation, and the Hermitian transpose operation, respectively. $(x)^+$ denotes $\max(0,x)$. $\mathbf{I}_N$ is the $N\times N$ identity matrix and $\mathbf{0}_{N\times M}$ is the $N\times M$ zero-matrix.

II. SC-MIMO-SM WITH ITERATIVE JOINT TX/RX MMSE FILTERING & IC

In this paper, we use a symbol-spaced discrete-time signal representation. Fig. 1 shows the system model of SC-MIMO-SM with iterative joint Tx/Rx MMSE filtering & IC. Transmitter and receiver have N_t and N_r antennas, respectively.

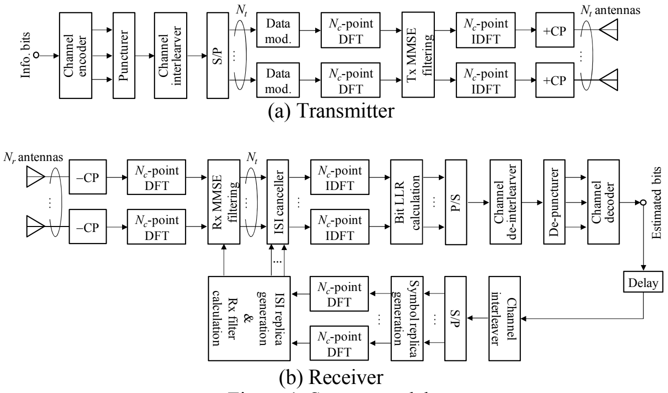

Figure 1. System model.

A. Transmit signal

At the transmitter, two parity bit sequences generated by the channel encoder are punctured to form the codeword of length K. The codeword is interleaved, serial-to-parallel (S/P) converted to N_t parallel bit sequences, and then each sequence is data-modulated. Each symbol sequence is divided to N_c-symbol blocks, where N_c is the size of discrete Fourier transform (DFT) and inverse DFT (IDFT), and each symbol block is transformed into a frequency-domain symbol block by N_c-point DFT. The N_t×1 transmit symbol vector $\mathbf{S}(k)$ at the k(=0~N_c−1)-th frequency is obtained by applying the Tx MMSE filtering to the N_t×1 frequency-domain symbol vector $\mathbf{D}(k)=[D_0(k),...,D_n(k),...,D_{N_t-1}(k)]^T$ at the k-th frequency, which is expressed as

$$\mathbf{S}(k)=[S_0(k),...,S_n(k),...,S_{N_t-1}(k)]^T=\mathbf{W}_t(k)\mathbf{D}(k)\,, \quad (1)$$

where $\mathbf{W}_t(k)$ is the N_t×N_t Tx filter matrix. N_c-point IDFT is applied to each transmit symbol block $\{S_n(k);\ k=0\sim N_c-1\}$, $n=0\sim N_t-1$, to transform back to time-domain transmit blocks. Finally, the last N_g symbols of each transmit block are copied as a cyclic prefix (CP) and inserted into the guard interval (GI) at the beginning of each transmit block and then transmitted from N_t antennas.

B. Received signal

At the receiver, each CP is removed from the signal blocks received by N_r antennas and then, each block is transformed into the frequency-domain signal block by N_c-point DFT. The N_r×1 frequency-domain received signal vector $\mathbf{R}(k)$ at the k-th frequency after N_c-point DFT is expressed as

$$\begin{aligned}\mathbf{R}(k)&=[R_0(k),...,R_m(k),...,R_{N_r-1}(k)]^T\\&=\sqrt{2E_s/T_s}\,\mathbf{H}(k)\mathbf{S}(k)+\mathbf{Z}(k),\end{aligned} \quad (2)$$

where E_s and T_s are the average transmit symbol energy and symbol duration, respectively. $\mathbf{H}(k)$ is the N_r×N_t MIMO channel matrix and $\mathbf{Z}(k)=[Z_0(k),\ldots,Z_m(k),\ldots,Z_{N_r-1}(k)]^T$ is the noise vector whose elements are zero-mean complex-valued random variables having variance $2N_0/T_s$ with N_0 being the one-sided power spectrum density of additive white Gaussian noise (AWGN).

Iterative Rx filtering & IC is applied to $\mathbf{R}(k)$. Let us consider the i(=0~I)-th iteration stage, where I denotes the number of outer loop (between Rx filter and channel decoder) iterations. The N_t×1 frequency-domain soft-output vector $\hat{\mathbf{R}}^{(i)}(k)$ is obtained by performing the Rx MMSE filtering on $\mathbf{R}(k)$ as

$$\begin{aligned}\hat{\mathbf{R}}^{(i)}(k)&=[\hat{R}_0(k),...,\hat{R}_n(k),...,\hat{R}_{N_t-1}(k)]^T\\&=\mathbf{W}_r^{(i)}(k)\mathbf{R}(k)\\&=\sqrt{\frac{2E_s}{T_s}}\mathbf{D}(k)\\&+\sqrt{\frac{2E_s}{T_s}}\left(\mathbf{W}_r^{(i)}(k)\mathbf{H}(k)\mathbf{W}_t(k)-\mathbf{I}_{N_t}\right)\mathbf{D}(k)\\&+\mathbf{W}_r^{(i)}(k)\mathbf{Z}(k),\end{aligned} \quad (3)$$

where $\mathbf{W}_r^{(i)}(k)$ is the N_t×N_r Rx filter matrix at the i-th iteration. In the right side of Eq. (3), the first, second, and third terms are the desired signal, residual ISI after joint Tx/Rx filtering, and noise, respectively.

In order to suppress the residual ISI expressed as the second term of the right side of Eq. (3), the residual ISI replica is generated using the N_t×1 frequency-domain soft-output data symbol replica vector $\widetilde{\mathbf{D}}^{(i-1)}(k)=[\widetilde{D}_0^{(i-1)}(k),...,\widetilde{D}_n^{(i-1)}(k),...,\widetilde{D}_{N_t-1}^{(i-1)}(k)]^T$ at the previous (i.e., (i−1)-th) iteration stage, and is subtracted from $\hat{\mathbf{R}}^{(i)}(k)$. For the first iteration (i.e., i=0), $\widetilde{\mathbf{D}}^{(-1)}(k)=\mathbf{0}_{N_t\times1}$. The N_t×1 frequency-domain soft-output vector $\hat{\mathbf{D}}^{(i)}(k)$ after IC is expressed as

$$\begin{aligned}\hat{\mathbf{D}}^{(i)}(k)&=[\hat{D}_0(k),...,\hat{D}_n(k),...,\hat{D}_{N_t-1}(k)]^T\\&=\hat{\mathbf{R}}^{(i)}(k)-\sqrt{\frac{2E_s}{T_s}}\left(\mathbf{W}_r^{(i)}(k)\mathbf{H}(k)\mathbf{W}_t(k)-\mathbf{I}_{N_t}\right)\widetilde{\mathbf{D}}^{(i-1)}(k)\\&=\sqrt{\frac{2E_s}{T_s}}\mathbf{D}(k)\\&+\sqrt{\frac{2E_s}{T_s}}\left(\mathbf{W}_r^{(i)}(k)\mathbf{H}(k)\mathbf{W}_t(k)-\mathbf{I}_{N_t}\right)\left(\mathbf{D}(k)-\widetilde{\mathbf{D}}^{(i-1)}(k)\right)\\&+\mathbf{W}_r^{(i)}(k)\mathbf{Z}(k).\end{aligned} \quad (4)$$

N_c-point IDFT is applied to each frequency-domain soft-output block $\{\hat{D}_n^{(i)}(k);\ k=0\sim N_c-1\}$, $n=0\sim N_t-1$. Then, the time-domain soft-output vector $\{\hat{\mathbf{d}}^{(i)}(t)=[\hat{d}_0^{(i)}(t),...,\hat{d}_n^{(i)}(t),...,\hat{d}_{N_t-1}^{(i)}(t)]^T;\ t=0\sim N_c-1\}$ is obtained.

Channel decoding is carried out using the bit LLRs obtained from $\hat{\mathbf{d}}^{(i)}(t)$. The time-domain soft-output data symbol replica vector $\{\widetilde{\mathbf{d}}^{(i)}(t)=[\widetilde{d}_0^{(i)}(t),...,\widetilde{d}_n^{(i)}(t),...,\widetilde{d}_{N_t-1}^{(i)}(t)]^T;\ t=0\sim N_c-1\}$ is calculated using the resultant bit LLRs after the channel decoding and transformed into the frequency-domain soft-output data symbol replica vector $\{\widetilde{\mathbf{D}}^{(i)}(k);\ k=0\sim N_c-1\}$ by N_c-point DFT. Then, the residual ISI replica for the (i+1)-th iteration stage is generated using $\widetilde{\mathbf{D}}^{(i)}(k)$. The calculation of $\widetilde{\mathbf{d}}^{(i)}(t)$ is based on the expectation using *a posteriori* probability [7]. The above steps are iterated for I times and data decision is carried out using the resultant bit LLRs after channel decoding.

III. Derivation of Optimal Tx/Rx Filters

In this section, we derive the optimal Tx and Rx filters based on MMSE criterion. The total MSE of the blocks between the transmit symbol vector $\mathbf{D}(k)$ and the soft-output vector $\hat{\mathbf{D}}^{(i)}(k)$ after the i-th IC is defined as

$$\varepsilon^{(i)} \equiv E\left[\sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\left(\mathbf{D}(k)-\frac{\hat{\mathbf{D}}^{(i)}(k)}{\sqrt{2E_s/T_s}}\right)\left(\mathbf{D}(k)-\frac{\hat{\mathbf{D}}^{(i)}(k)}{\sqrt{2E_s/T_s}}\right)^H\right\}\right]. \quad (5)$$

From Eqs. (4) and (5), the total MSE can be rewritten as

$$\begin{aligned}\varepsilon^{(i)} &= \sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\begin{matrix}\left(\mathbf{I}_{N_t}-\mathbf{W}_r^{(i)}(k)\mathbf{H}(k)\mathbf{W}_t(k)\right)\boldsymbol{\rho}^{(i-1)}\\ \times\left(\mathbf{I}_{N_t}-\mathbf{W}_r^{(i)}(k)\mathbf{H}(k)\mathbf{W}_t(k)\right)^H\end{matrix}\right\}\\ &\quad+\gamma^{-1}\sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\mathbf{W}_r^{(i)}(k)\left(\mathbf{W}_r^{(i)}(k)\right)^H\right\},\end{aligned} \quad (6)$$

where $\gamma=E_s/N_0$, and we use $E[\mathbf{D}(k)\mathbf{D}^H(k)]=\mathbf{I}_{N_t}$ and $E[\mathbf{Z}(k)\mathbf{Z}^H(k)]=(2N_0/T_s)\mathbf{I}_{N_r}$. $\boldsymbol{\rho}^{(i-1)}$ is the $N_t\times N_t$ diagonal matrix whose (n,n)-th component $\rho_n^{(i-1)}$ is a coefficient which indicates the accuracy of the symbol replicas generated at the $(i-1)$-th iteration and is expressed as

$$\rho_n^{(i-1)} = E\left[\left(D_n(k)-\widetilde{D}_n^{(i-1)}(k)\right)\left(D_n(k)-\widetilde{D}_n^{(i-1)}(k)\right)^*\right]. \quad (7)$$

For the first iteration (i.e., i=0), $\boldsymbol{\rho}^{(-1)}=\mathbf{I}_{N_t}$. As the number of iterations increases, the residual ISI is reduced (i.e., accuracy of the symbol replicas becomes high) and $\boldsymbol{\rho}^{(i-1)}$ gets close to $\mathbf{0}_{N_t\times N_t}$.

The minimization of the total MSE given by Eq. (6) under the total transmit power constraint is formulated as

$$\text{(P0)}\quad \min_{\left\{\mathbf{W}_t(k),\mathbf{W}_r^{(i)}(k);k=0\sim N_c-1\right\}}\varepsilon^{(i)} \quad (8a)$$

$$\text{s.t.}\ \sum_{k=0}^{N_c-1}\mathrm{tr}\left(\mathbf{W}_t(k)\mathbf{W}_t^H(k)\right)=N_c. \quad (8b)$$

The Tx and Rx filters which satisfy the problem (P0) are the optimal solution of MMSE filters. However, it is quite difficult to derive a set of MMSE matrices {$\mathbf{W}_t(k)$, $\mathbf{W}_r^{(i)}(k)$ } at the same time since $\mathbf{W}_t(k)$ (or $\mathbf{W}_r^{(i)}(k)$) is the function of $\mathbf{W}_r^{(i)}(k)$ (or $\mathbf{W}_t(k)$). Therefore, in this paper, as is the case in [5], we first derive the Rx filter matrix $\mathbf{W}_{r,opt}^{(i)}(k)$ considering the concatenation of Tx filter and channel as the equivalent channel $\overline{\mathbf{H}}(k)=\mathbf{H}(k)\mathbf{W}_t(k)$. Then, we derive the Tx filter matrix $\mathbf{W}_{t,opt}(k)$ by solving the problem (P0) for the given $\mathbf{W}_{r,opt}^{(i)}(k)$.

A. Rx filter

By considering $\overline{\mathbf{H}}(k)=\mathbf{H}(k)\mathbf{W}_t(k)$ as the equivalent channel matrix, the cost function $\varepsilon^{(i)}$ becomes a convex function because the Hessian matrix $\nabla^2\varepsilon^{(i)}$ is positive semidefinite [8]. Therefore, $\varepsilon^{(i)}$ is minimized when $\partial\varepsilon^{(i)}/\partial\mathbf{W}_r^{(i)}(k)=0$. The optimal Rx filter matrix is given as

$$\mathbf{W}_{r,opt}^{(i)}(k)=\overline{\mathbf{H}}^H(k)\left(\overline{\mathbf{H}}(k)\boldsymbol{\rho}^{(i-1)}\overline{\mathbf{H}}^H(k)+\gamma^{-1}\mathbf{I}_{N_r}\right)^{-1}. \quad (9)$$

B. Tx filter

From the previous subsection, it can be understood that the Rx filter is updated in each iteration stage since it includes $\boldsymbol{\rho}^{(i-1)}$ as seen in Eq. (9). As the number of iterations increases, $\boldsymbol{\rho}^{(i-1)}$ gets close to $\mathbf{0}_{N_t\times N_t}$ and $\mathbf{W}_{r,opt}^{(i)}(k)$ approaches the maximal-ratio combining (MRC) based filter $\gamma\overline{\mathbf{H}}^H(k)$. This Rx filter maximizes the frequency diversity gain.

In contrast to the Rx filter, Tx filter cannot be updated. In [5], we derived the Tx filter matrix for a given Rx filter matrix. However, the previously proposed Tx filter matrix does not consider the iterative IC at the receiver. If the iterative IC at the receiver is considered at the transmitter, Tx filtering can provide a much higher frequency diversity gain than the previous one.

In this paper, in order to derive the Tx filter matrix $\mathbf{W}_{t,opt}(k)$, we assume a $N_t\times N_r$ virtual Rx filter matrix $\mathbf{W}_r^{\mathrm{Tx}}(k)$ as

$$\mathbf{W}_r^{\mathrm{Tx}}(k)=\overline{\mathbf{H}}^H(k)\left(\overline{\mathbf{H}}(k)\boldsymbol{\rho}^{\mathrm{Tx}}\overline{\mathbf{H}}^H(k)+\gamma^{-1}\mathbf{I}_{N_r}\right)^{-1}, \quad (10)$$

where $\boldsymbol{\rho}^{\mathrm{Tx}}$ is the $N_t\times N_t$ diagonal matrix whose (n,n)-th component ρ_n^{Tx} is a virtual coefficient which is a parameter between 0 to 1. The virtual total MSE $\varepsilon^{\mathrm{Tx}}$ is given as

$$\begin{aligned}\varepsilon^{\mathrm{Tx}} &= \sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\begin{matrix}\left(\mathbf{I}_{N_t}-\mathbf{W}_r^{\mathrm{Tx}}(k)\mathbf{H}(k)\mathbf{W}_t(k)\right)\boldsymbol{\rho}^{\mathrm{Tx}}\\ \times\left(\mathbf{I}_{N_t}-\mathbf{W}_r^{\mathrm{Tx}}(k)\mathbf{H}(k)\mathbf{W}_t(k)\right)^H\end{matrix}\right\}\\ &\quad+\gamma^{-1}\sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\mathbf{W}_r^{\mathrm{Tx}}(k)\left(\mathbf{W}_r^{\mathrm{Tx}}(k)\right)^H\right\},\end{aligned} \quad (11)$$

and we solve the virtual optimization problem as

$$\text{(P1)}\quad \min_{\left\{\mathbf{W}_t(k),k=0\sim N_c-1\right\}}\varepsilon^{\mathrm{Tx}} \quad (12)$$

s.t. (8b)

The cost function $\varepsilon^{\mathrm{Tx}}$ is expressed as the function only of the Tx filter matrix $\mathbf{W}_t(k)$ by substituting $\mathbf{W}_r^{\mathrm{Tx}}(k)$ into $\varepsilon^{\mathrm{Tx}}$. $\varepsilon^{\mathrm{Tx}}$ is rewritten by substituting Eq. (10) into Eq. (11) and using the matrix inversion lemma [9] as

$$\varepsilon^{\mathrm{Tx}}=\sum_{k=0}^{N_c-1}\mathrm{tr}\left(\gamma\mathbf{W}_t^H(k)\mathbf{H}^H(k)\mathbf{H}(k)\mathbf{W}_t(k)\boldsymbol{\rho}^{\mathrm{Tx}}+\mathbf{I}_{N_t}\right)^{-1}. \quad (13)$$

In general, tr[$\mathbf{A}^{-1}$] is minimized when $\mathbf{A}$ is a diagonal matrix [9]. Therefore, we diagonalize the matrix $\gamma\mathbf{W}_t^H(k)\mathbf{H}^H(k)\mathbf{H}(k)\mathbf{W}_t(k)\boldsymbol{\rho}^{\mathrm{Tx}}+\mathbf{I}_{N_t}$ in Eq. (13). At the first step, we transpose $\mathbf{H}(k)$ and $\mathbf{W}_t(k)$ by singular value decomposition [9] as

$$\begin{cases}\mathbf{H}(k)=\mathbf{U}_h(k)\sqrt{\boldsymbol{\Lambda}(k)}\mathbf{V}_h^H(k)\\ \mathbf{W}_t(k)=\mathbf{U}_t(k)\sqrt{\mathbf{P}(k)}\mathbf{V}_t^H(k)\end{cases}, \quad (14)$$

where $\mathbf{V}_h(k)$, $\mathbf{U}_t(k)$, and $\mathbf{V}_t(k)$ are respectively the $N_t\times N_t$ unitary matrices and $\mathbf{U}_h(k)$ is the $N_r\times N_r$ unitary matrix. $\boldsymbol{\Lambda}(k)$ is the $N_r\times N_t$ matrix whose (g,g)-th element has the g-th eigenvalue of $\mathbf{H}(k)\mathbf{H}^H(k)$; g=0~rank[$\mathbf{H}(k)\mathbf{H}^H(k)$], and any other elements are zero. $\mathbf{P}(k)$ is the $N_t\times N_t$ matrix whose (n,n)-th element has the n-

th eigenvalue of $\mathbf{W}_t(k)\mathbf{W}_t^H(k)$. Since tr[**AB**]=tr[**BA**], where **A** and **B** are respectively a $A\times B$ and $B\times A$ matrices, Eq. (13) can be rewritten by substituting Eq. (14) as

$$\varepsilon^{\mathrm{Tx}}=\sum_{k=0}^{N_c-1}\mathrm{tr}\left\{\begin{array}{l}\gamma\boldsymbol{\rho}^{\mathrm{Tx}}\sqrt{\mathbf{P}(k)}\mathbf{U}_t^H(k)\mathbf{V}_h(k)\sqrt{\boldsymbol{\Lambda}^T(k)}\\ \times\sqrt{\boldsymbol{\Lambda}(k)}\mathbf{V}_h^H(k)\mathbf{U}_t(k)\sqrt{\mathbf{P}(k)}+\mathbf{I}_{N_t}\end{array}\right\}^{-1}. \quad (15)$$

It can be seen from Eq. (15) that the cost function does not depend on $\mathbf{V}_t(k)$ (i.e., $\mathbf{V}_t(k)$ can be set to arbitrary $N_t\times N_t$ unitary matrix). In this paper, we set $\mathbf{V}_t(k)=\mathbf{I}_{N_t}$ for the sake of brevity. The cost function expressed as Eq. (15) is minimized when $\mathbf{U}_t(k)=\mathbf{V}_h(k)$ because tr[$\mathbf{A}^{-1}$] is minimized when **A** is a diagonal matrix. From the above, $\mathbf{W}_{t,opt}(k)$ is expressed as

$$\mathbf{W}_{t,opt}(k)=\mathbf{V}_h(k)\sqrt{\mathbf{P}_{opt}(k)}. \quad (16)$$

The optimization problem (P1) is rewritten by substituting $\mathbf{U}_t(k)=\mathbf{V}_h(k)$ into Eq. (8b) and (15) as

$$\text{(P2)}\quad \min_{\{P_n(k);n=0\sim N_t-1,k=0\sim N_c-1\}}\sum_{k=0}^{N_c-1}\sum_{n=0}^{N_t-1}\left(\gamma\rho_n^{\mathrm{Tx}}P_n(k)\Lambda_n(k)+1\right)^{-1} \quad (17a)$$

$$\text{s.t.}\sum_{k=0}^{N_c-1}\sum_{n=0}^{N_t-1}P_n(k)=N_c, \quad (17b)$$

where $P_n(k)$ and $\Lambda_n(k)$ are respectively the n-th diagonal elements of $\mathbf{P}(k)$ and $\boldsymbol{\Lambda}(k)$. Following [8], the optimal solution is given as (for the sake of brevity, the derivation is omitted)

$$P_{n,opt}(k)=\left(\frac{1}{\sqrt{\mu}}\frac{1}{\sqrt{\gamma\Lambda_n(k)}}-\frac{1}{\gamma\rho_n^{\mathrm{Tx}}\Lambda_n(k)}\right)^+, \quad (18)$$

where μ is chosen to satisfy the total transmit power constraint. Eq. (18) indicates that the Tx filter matrix depends on $\boldsymbol{\rho}^{\mathrm{Tx}}$ which indicates the reliability of the iterative IC at the receiver predicted by the transmitter. If the transmitter assumes that the receiver can sufficiently suppress the residual ISI, each diagonal element of $\boldsymbol{\rho}^{\mathrm{Tx}}$ is set to be small. On the other hand, if the transmitter assumes that the receiver cannot sufficiently suppress the residual ISI, each diagonal element of $\boldsymbol{\rho}^{\mathrm{Tx}}$ is set to be close to 1. The optimal $\boldsymbol{\rho}^{\mathrm{Tx}}$ depends on the coding gain, instantaneous channel condition, the average transmit E_s/N_0, data modulation scheme, the number of iterations of Rx filtering & IC, and so on. Therefore, it is quite difficult to analytically find the optimal $\boldsymbol{\rho}^{\mathrm{Tx}}$ and hence, in this paper, we find a $\boldsymbol{\rho}^{\mathrm{Tx}}$ such that the average BER is minimized for a given average transmit bit energy to noise power spectrum density ratio (E_b/N_0) by computer simulation.

C. Behavior of joint iterative Tx/Rx MMSE filtering & IC

In this subsection, we discuss the behavior of joint iterative Tx/Rx MMSE filtering & IC. The equivalent channel matrix $\hat{\mathbf{H}}^{(i)}(k)$ after the i-th Rx filtering is expressed as

$$\hat{\mathbf{H}}^{(i)}(k)=\mathbf{W}_{r,opt}^{(i)}(k)\mathbf{H}(k)\mathbf{W}_{t,opt}(k)$$
$$=\mathrm{diag}\left[\frac{P_{0,opt}(k)\Lambda_0(k)}{\rho_0^{(i-1)}P_{0,opt}(k)\Lambda_0(k)+\gamma^{-1}},\ldots,\frac{P_{N_t-1,opt}(k)\Lambda_0(k)}{\rho_{N_t-1}^{(i-1)}P_{N_t-1,opt}(k)\Lambda_0(k)+\gamma^{-1}}\right]. \quad (19)$$

TABLE I. COMPUTER SIMULATION CONDITION.

Channel coding	Turbo encoding with coding rate = 1/2 Log-MAP decoding w/ 1 iteration	
Transmitter & Receiver	No. of coded bits	K=1024
	Data modulation	QPSK, 16QAM
	No. of DFT points	N_c=128
	Guard interval length	N_g=16
	Channel estimation	Ideal
	No. of Tx/Rx antennas	(N_t,N_r)=(2,2)
	Antenna correlation	Uncorrelated
Channel	Fading	Frequency-selective block Rayleigh
	Power delay profile	16-path uniform

It can be seen from Eq. (19) that the MIMO channel matrix $\mathbf{H}(k)$ is diagonalized (i.e., the IAI is avoided) by joint Tx/Rx MMSE filtering. In addition, the ISI can be significantly suppressed by carrying out the MMSE based power allocation expressed in Eq. (18) and iterative FDE & ISI cancellation to each eigenmode.

IV. COMPUTER SIMULATION

A. Computer simulation condition

Computer simulation condition is summarized in Table I. A turbo encoder with the original coding rate 1/3 using two (13,15) recursive systematic convolutional (RSC) encoders, a random interleaver/deinterleaver, and log maximum *a posteriori* probability (MAP) turbo decoding with only 1 iteration (i.e., only 1 iteration is carried out in the inner loop (between two MAP component decoders in turbo decoder)) are used. The codeword length is K=1024 bits with coding rate equals to 1/2. The channel is assumed to be a 16-path frequency-selective block Rayleigh fading having uniform power delay profile. Uncorrelated fading and ideal channel estimation at both transmitter and receiver are assumed.

B. Average BER performance

Fig. 2 shows the average BER performance of SC-MIMO-SM with joint iterative Tx/Rx filtering & IC when (N_t,N_r)=(2,2). Fig. 2 (a) and (b) show that without turbo coding and with turbo coding, respectively. The average BER performance of SC-MIMO-SM with iterative Rx filtering & IC is also shown for comparison.

It can be seen form Fig. 2 (a) that the proposed joint iterative Tx/Rx filtering & IC has a worse average BER performance than the conventional iterative Rx filtering & IC assuming the same number I of iterations except for the case when I=0 (i.e., without iteration). The reason for this degraded performance in the uncoded case is discussed below. All of received soft-output symbols have almost the same reliability when iterative Rx filtering & IC is used. However, there exists a large received quality gap among eigenmodes when joint iterative Tx/Rx filtering & IC is used. This is because the eigenvalue of each eigenmode (i.e., equivalent channel gain) is quite different from each other. Therefore, the accuracy of the symbol replicas whose symbols are transmitted through the eigenmode having low eigenvalue is low and cannot be improved even using iterative IC. This problem can be regarded as a kind of burst error.

On the other hand, it can be seen from Fig. 2 (b) that the proposed joint iterative Tx/Rx filtering & IC has a better average BER performance than the conventional iterative Rx filtering & IC when turbo coding is used. This is because the burst error occurred on the eigenmode having low received quality is randomized by randomizing the systematic bits and parity bits, which are generated at the turbo encoder, at the channel interleaver. Therefore, the accuracy of the symbol replicas can be improved by the iterative IC and turbo decoding and the average BER performance is significantly improved.

It can also be seen from Fig. 2 that the proposed joint iterative Tx/Rx filtering & IC produces more improvement when the modulation level becomes high. This is because the average BER improvement by the received signal-to-interference plus noise power ratio (SINR) improvement becomes large when the modulation level is high (i.e., the distance between symbols is small).

V. CONCLUSION

In this paper, we proposed iterative joint Tx/Rx MMSE filtering & IC for SC-MIMO-SM and derived the optimal Tx/Rx filters in closed-forms. The proposed iterative joint Tx/Rx MMSE filtering & IC transforms the MIMO channel to eigenmodes to avoid the IAI, and at the same time, carries out the MMSE based power allocation and iterative FDE & IC to each eigenmode to suppress the ISI. We showed, by computer simulation, that joint iterative Tx/Rx MMSE filtering & IC can significantly improve the average BER performance of turbo coded SC-MIMO-SM.

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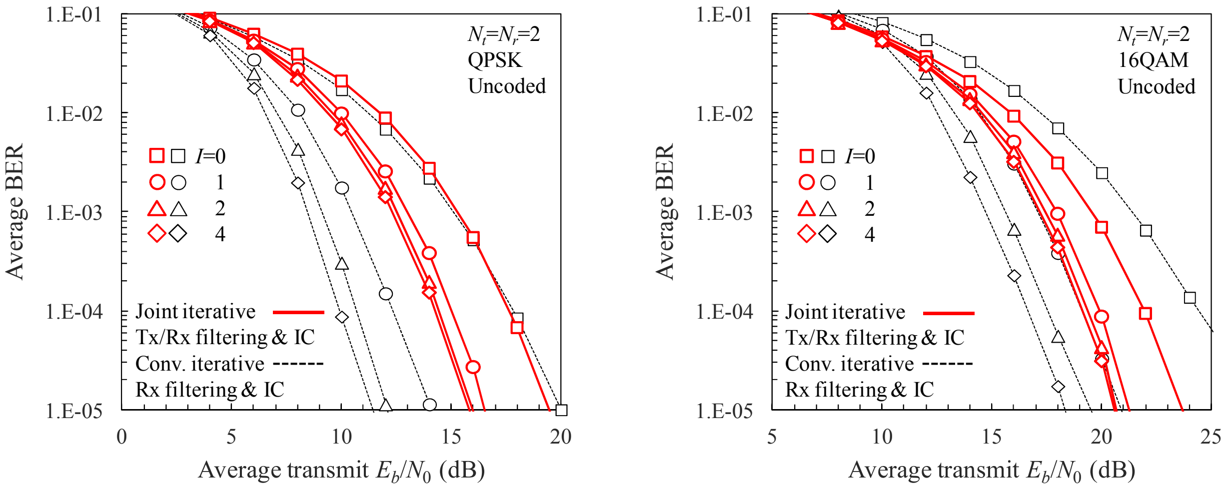
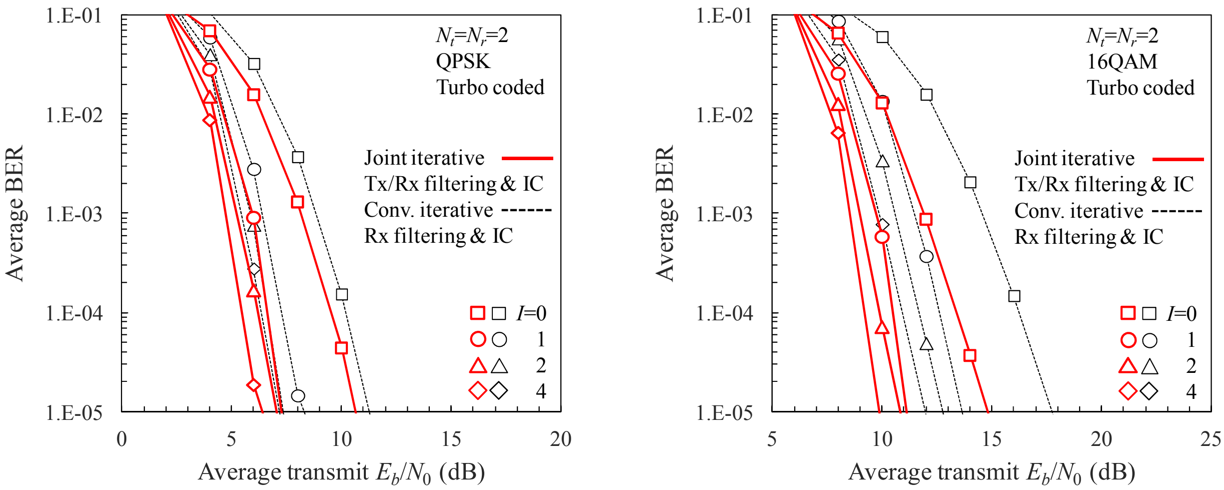


(a) Uncoded



(b) Turbo coded

Figure 2. Average BER performance (left figure: QPSK, right figure: 16QAM).