

Frequency-Domain based Single-Carrier Waveform Design through Precoder

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Abstract— There are two types of signal waveform: single-carrier (SC) and multi-carrier (MC). A representative of MC signal is orthogonal frequency division multiplexing (OFDM). SC signal can be considered as a linear precoded OFDM signal and has lower peak-to-average power ratio (PAPR) than OFDM signal. There exist many variants between SC and MC signals depending on the linear precoder. The linear precoder can be designed to generate a SC signal having very-low-PAPR based on the minimum variance of instantaneous transmit power (VIP) criterion, or can be extended to generate SC spread spectrum (SS) signal. The frequency-domain based precoder design is presented and a tradeoff between PAPR and bit-error rate (BER) is discussed in this paper.

Keywords— Single-carrier (SC) transmission, spread spectrum (SS), frequency-domain equalization (FDE), peak-to-average power ratio (PAPR)

I. INTRODUCTION

Multi-carrier (MC) signal transmission [1], such as orthogonal frequency division multiplexing (OFDM), is robust against frequency-selective fading [2], but its high peak-to-average power ratio (PAPR) is the main drawback. On the other hand, single-carrier (SC) transmission [3] is attractive for uplink communication because of its lower PAPR. The use of frequency-domain equalization (FDE) can take advantage of the channel frequency selectivity to improve the bit-error rate (BER) performance of SC transmission [4].

OFDM signal can be generated by using inverse discrete Fourier transform (IDFT) [5]. The time-domain based generation of SC signal has been used for long time. Recently, however, SC signal generation in frequency-domain has gained much attention; SC signal can be generated by inserting DFT as a linear precoder before IDFT of OFDM transmitter [6-8]. Because of precoder, the subcarriers (we use this terminology for the ease of explanation) of SC are correlated while those of OFDM are independent. This can be considered as the reason of low-PAPR property of SC.

SC and OFDM signals are considered as two extreme cases. Many low-PAPR variations of SC signal waveform exist between SC and OFDM, where the precoder takes an important role to generate low-PAPR SC signal waveforms. The linear precoder can be a combination of DFT and transmit filter implemented in the frequency domain. Among various transmit filters, square-root Nyquist filter with roll-off factor of higher than 0.5 provides a sufficiently low-PAPR SC signal [9, 10]. Moreover, low-PAPR SC signals can also be

generated [11] based on the minimum variance of instantaneous transmit power (min-VIP) criterion [6]. The use of transmit filter can take an advantage of bandwidth expansion. Joint MMSE-FDE and spectrum combining [12] can be employed at the receiver to obtain additional frequency diversity gain. However, since the bandwidth expansion ratio is limited to two, the achievable frequency diversity gain is limited. It is shown in this paper that a tradeoff among PAPR, spectrum efficiency (SE), and BER depends on the precoder design.

Spread spectrum (SS) technique provides increased frequency diversity gain, but at the cost of SE. In this paper, it is shown that the linear precoder which performs frequency-domain spreading and mapping can be designed. This is called SC with frequency-domain SS (SC-FDSS) [13]. Joint MMSE-FDE and spectrum combining [12] can be employed as the de-spreading technique. There is also a tradeoff among PAPR, SE, and BER similar to the non-spreading SC case. To improve SE, SS can be combined with multi-access, i.e., code-division multi-access (CDMA) [14]. The combination of SC-FDSS and CDMA is expected to provide good BER-PAPR tradeoff among two conventional types of CDMA: direct-sequence CDMA (DS-SS) [e.g., 15] and OFDM-SS [16]. In addition, rake combining can be replaced by FDE to improve the BER performance of DS-SS [17].

The rest of this paper is organized as follows. In Section II, the role of linear precoder is discussed. Section III introduces the precoder design based on min-VIP criterion for OFDM and its BER-PAPR tradeoff. Section IV introduces the transmitter design of SC-FDSS and its performance. Section V concludes the paper.

II. RELATIONSHIP BETWEEN SC AND OFDM

The SC signal is equivalent to precoded OFDM signal [8]. The transmitter structure to generate SC and OFDM signals is illustrated in Fig. 1.

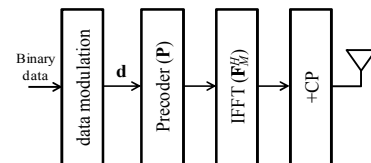


Fig. 1. Transmitter structure to generate SC and MC signals.

When $\mathbf{P}=\mathbf{I}$ (identity matrix of size M -by- M), OFDM signal $\mathbf{s}_{\text{OFDM}}=[s_{\text{OFDM}}(0), s_{\text{OFDM}}(1), \dots, s_{\text{OFDM}}(M-1)]^T$ is generated as

$$\mathbf{s}_{\text{OFDM}} = \mathbf{F}_M^H \mathbf{d}, \quad (1)$$

where $\mathbf{d}=[d(0),d(1),\dots,d(M-1)]^T$ represents the transmit symbol block and \mathbf{F}_M^H denotes the IDFT matrix given by

$$\mathbf{F}_M^H = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{\frac{j2\pi(1)(1)}{M}} & \dots & e^{\frac{j2\pi(1)(M-1)}{M}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{\frac{j2\pi(M-1)(1)}{M}} & \dots & e^{\frac{j2\pi(M-1)(M-1)}{M}} \end{bmatrix}. \quad (2)$$

When $\mathbf{P}=\mathbf{F}_M$ (DFT matrix of size M -by- M), the SC signal $\mathbf{s}_{\text{SC}}=[s_{\text{SC}}(0),s_{\text{SC}}(1),\dots,s_{\text{SC}}(M-1)]^T=\mathbf{d}$ is generated as [6-8]

$$\mathbf{s}_{\text{SC}} = \mathbf{F}_M^H \mathbf{P} \mathbf{d} = \mathbf{d}. \quad (3)$$

Any low PAPR SC signal variant between SC and OFDM can be generated by introducing the frequency-domain filter (or mapping) \mathbf{W} into \mathbf{P} as

$$\mathbf{P} = \mathbf{W} \mathbf{F}_M. \quad (4)$$

The design of precoder \mathbf{P} for reducing the PAPR of SC signal is found in [6, 7]. Note that precoded OFDM considered in [6, 7] is a family of SC signal. In [7], the precoder of size larger than M (i.e., transmit signal bandwidth is wider than the original signal bandwidth) is studied. The use of square-root Nyquist filter [9-10, 18] is one good example for SC [9], in which \mathbf{W} is a diagonal matrix containing the filter transfer function.

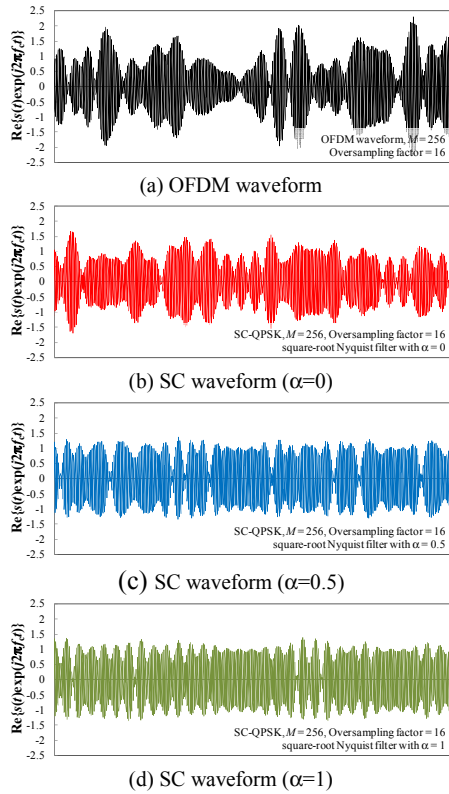


Fig. 2. Comparison of SC and OFDM waveforms when $M=256$.

Figure 2 compares the square-root Nyquist filtered SC signal waveforms for different values of filter roll-off factor α

to OFDM signal waveform with $M=256$ symbols. It is seen that the use of larger α exhibits better PAPR suppression at the cost of the bandwidth expansion by a factor of $1+\alpha$. Joint MMSE-FDE and spectrum combining [12] can be used to improve the BER performance by taking advantage of bandwidth expansion. However, since the maximum bandwidth expansion is limited to two, the BER performance improvement is limited. This brings us to introduce the SS transmission to further improve the BER performance.

In the following sections, we will design the transmit filter \mathbf{W} based on the min-VIP criterion [6, 9]. Then, we will design the transmit filter \mathbf{W} to generate the SC-FDSS signals [13].

III. LOW-PAPR SC SIGNAL DESIGN

SC signal is generally a low-PAPR signal; however, its PAPR may increase when a particular precoder is applied. We use the min-VIP criterion (which has been introduced in [6] for OFDM signal) to determine the frequency-domain filter \mathbf{W} of (4) in order to reduce the PAPR of SC signals.

A. Signal Representation

We assume a single-user SC-FDE block transmission of M data-modulated symbols over N_c subcarriers (assuming $N_c=2M$ for simplicity). The transmitter and receiver structure is illustrated in Fig. 3.

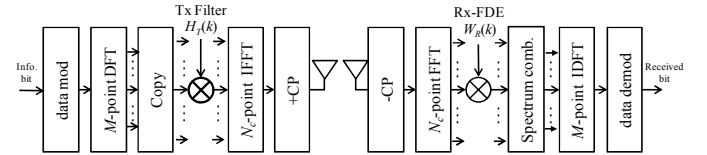


Fig. 3. Transmitter and receiver structure of SC-FDE using low-PAPR transmit filtering.

Decomposing the precoder \mathbf{P} as $\mathbf{P}=\mathbf{H}_T \mathbf{E}_M$, the SC signal $\mathbf{s}=[s(0),s(1),\dots,s(N_c-1)]^T$ can be expressed as

$$\mathbf{s} = \mathbf{F}_{N_c}^H \mathbf{H}_T \mathbf{E}_M \mathbf{d}, \quad (5)$$

where \mathbf{H}_T is an $N_c \times N_c$ diagonal matrix representing transmit filter given by

$$\mathbf{H}_T = \text{diag} \left[0, \dots, H_T \left(\frac{N_c - J}{2} \right), \dots, H_T \left(\frac{N_c + J}{2} - 1 \right), \dots, 0 \right] \quad (6)$$

with $J=(1+\alpha)M$ and \mathbf{E}_M is a row-repeated version of \mathbf{F}_M , which is given by

$$\mathbf{E}_M = \begin{bmatrix} \mathbf{f}_M^T(\frac{M}{2}) \cdots \mathbf{f}_M^T(M-1) \cdots \mathbf{f}_M^T(0) \cdots \mathbf{f}_M^T(M-1) \cdots \mathbf{f}_M^T(0) \cdots \mathbf{f}_M^T(\frac{M}{2}-1) \end{bmatrix}^T, \quad (7)$$

with $\mathbf{f}_M(m)$ being the m -th row of \mathbf{F}_M . The determination of filter coefficient $H_T(k)$ will be described in Sect. III-B. Finally, the cyclic-prefix (CP)-inserted signal block is transmitted.

The transmitted signal goes through an L -path independent symbol-spaced block Rayleigh fading channel [2]. The received signal $\mathbf{r}=[r(0),r(1),\dots,r(N_c-1)]^T$ after CP removal is represented in the time-domain as

$$\mathbf{r} = \sqrt{2E_s/T_s} \mathbf{h} \mathbf{s} + \mathbf{n}, \quad (8)$$

The frequency-domain signal $\hat{\mathbf{D}}$ after joint MMSE-FDE and spectrum combining is finally transformed by M -point IDFT back into time-domain signal $\hat{\mathbf{d}} = [\hat{d}(0), \hat{d}(1), \dots, \hat{d}(M-1)]^T$ as

$$\hat{\mathbf{d}} = \mathbf{F}_M^H \hat{\mathbf{D}}. \quad (22)$$

B. Performance Comparison

A block transmission of SC-FDSS signal with $N_c=256$ subcarriers is considered. The propagation channel is assumed to be a frequency-selective block Rayleigh fading channel having chip-spaced $L=16$ -path uniform power-delay profile. BER-PAPR tradeoff is evaluated and is compared with direct-sequence SS (DS-SS) and MC-SS.

Figure 8 compares the BER-PAPR tradeoffs of DS-SS, MC-SS, and SC-FDSS. Overall, SC-FDSS achieves BER-PAPR tradeoff better than both DS-SS and MC-SS. It is seen from Fig. 8 that SC-FDSS achieves better BER performance than both DS-SS and MC-SS. Performance improvement in SC-FDSS is due to larger frequency-diversity gain through joint MMSE-FDE and spectrum combining. It is interesting to note that PAPR is minimized when $SF=2$ for SC-FDSS. This is because of the interleaving property in time-domain signal of SC-FDSS, which reduces the peak caused by pulse overlapping when $SF < 4$ [10].

However, PAPR of SC-FDSS is larger than DS-SS when $SF=8$, indicating that some PAPR reduction scheme should be employed. A combination of SC-FDSS and min-VIP based precoder design, as described in Sect. III, can be a good approach. This can be left as our future study.

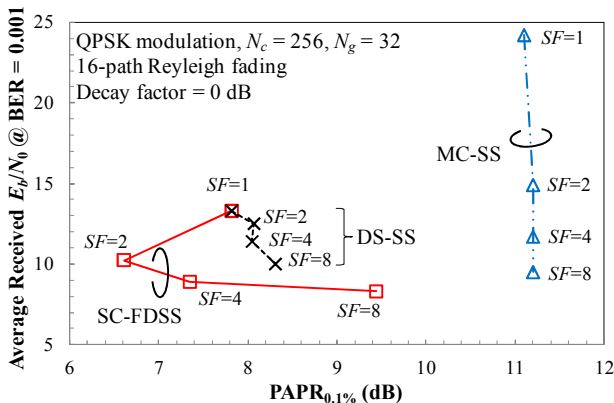


Fig. 8. BER-PAPR tradeoff of SC-FDSS.

V. CONCLUSION

The relationship between SC and OFDM signal waveforms can be described by using the precoder. The PAPR of the SC signal waveform can be controlled by the precoder. There exists the trade-off between PAPR and BER. Precoder design based on min-VIP criterion was presented. Also precoder was extended to include more than two times bandwidth expansion (i.e., spread spectrum). It was confirmed that a good PAPR-BER tradeoff can be achieved for both the non-spreading and spreading cases.

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APPENDIX

Fig. A-1 plots the BER and PAPR performances of filtered SC-FDE using square-root Nyquist filter and the min-VIP based low-PAPR filter for QPSK modulation. It is seen that the min-VIP based low-PAPR filter achieves very similar BER performance to square-root Nyquist filter when $\alpha=0$ and 0.5 while only a slight degradation is seen when $\alpha=1$.

Fig. A-2 plots the complementary cumulative distribution function (CCDF) of PAPR for QPSK modulation. It is clearly seen that the min-VIP based low-PAPR filter achieves lower PAPR than square-root Nyquist filter for all values of α . PAPR reduction is pronounced when $\alpha>0.5$.

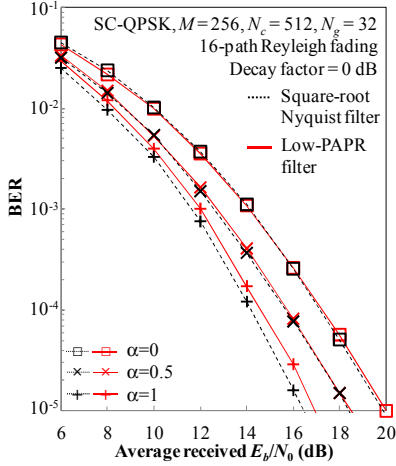


Fig. A-1. BER performance of filtered SC-FDE.

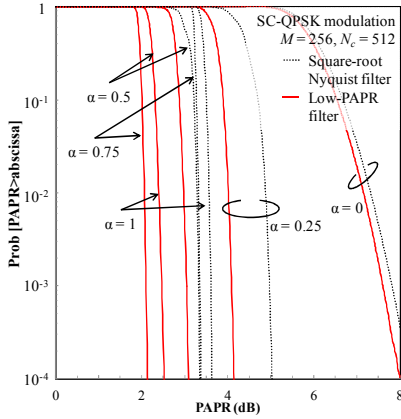
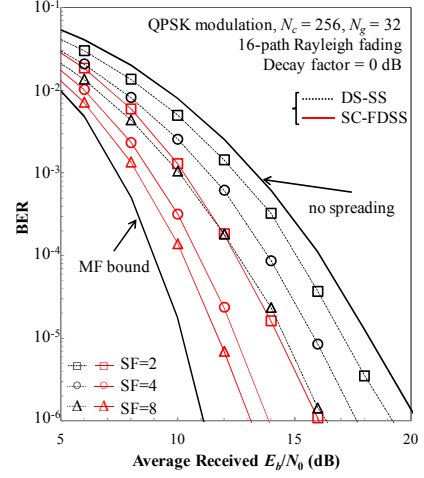


Fig. A-2. PAPR performance of filtered SC-FDE.

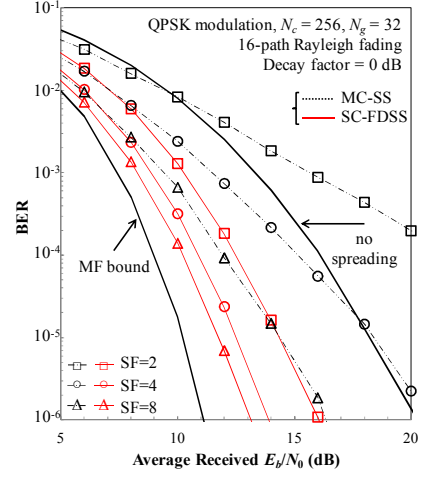
We also provide the BER and PAPR performance comparison of SS signals (i.e., DS-SS, MC-SS, and SC-FDSS). It is seen from Fig. A-3(a) and A-3(b) that SC-FDSS achieve better BER performance than both DS-SS and MC-SS. Performance improvement in SC-FDSS is achieved by joint MMSE-FDE and spectrum combining. In addition, it is also observed that MC-SS provides worse BER performance than SC-FDSS and DS-SS when $SF \leq 4$ due to lower frequency diversity gain.

Fig. A-4 shows the CCDF of PAPR of DS-SS, MC-SS, and SC-FDSS when $SF \leq 8$. PAPR slightly increases when SF increases in DS-SS. However, it is noticed that PAPR is

minimized when $SF=2$ in SC-FDSS. This is because of the interleaving property in time-domain signal of the proposed SC-FDSS, which decreases the peak occurred from pulse overlapping when $SF < 4$ [10]. However, PAPR increases when the interval between each time-domain non-zero signal is excessively wide. In addition, PAPR of SC-FDSS at $SF=8$ is still lower than MC-SS signal at $SF=2$ and 4.



(a) SC-FDSS vs DS-SS



(b) SC-FDSS vs MC-SS

Fig. A-3. BER performances of SS transmission.

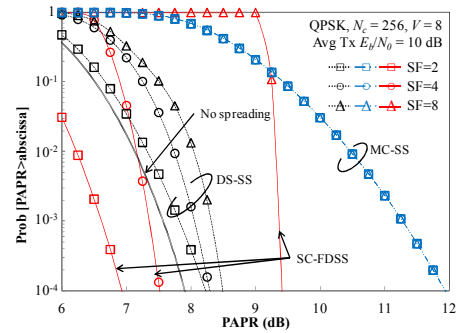


Fig. A-4. PAPR performances of SS signals.