Vector Perturbation for Single-Carrier MU-MIMO Downlink

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***Abstract*— In this paper, we propose two vector perturbation (VP) schemes for single-carrier (SC) multi-user (MU) multiple-input multiple-output (MIMO) downlink block transmission. The first VP scheme (called SC-VP-1) adds a perturbation vector to each user's signal block in time-domain and then, multiplies a precoding matrix to perturbation vector-added signal blocks in time-domain. The precoding matrix used in this paper takes into account delay waves unlike the conventional VP. In order to reduce the computational complexity of perturbation vector search, a combination of QR decomposition and M algorithm is applied. On the other hand, in the second VP scheme (called SC-VP-2), each user's perturbation vector-added time-domain signal block is transformed into the frequency-domain signal block before multiplying a precoding matrix. In SC-VP-2, unlike conventional VP and SC-VP-1, an equivalent time-domain expression for the frequency-domain precoding matrix multiplication is used for QR decomposition and M algorithm based perturbation vector search. The uncoded bit error rate (BER) performance achievable by SC-VP-1 and SC-VP-2 is evaluated by computer simulation. It is shown that SC-VP-1 and SC-VP-2 provide the same BER performance. It is also shown that SC-VP achieves better BER performance than both SC-MU-MIMO using channel inversion (CI) and orthogonal frequency division multiplexing (OFDM) MU-MIMO using VP (OFDM-VP). Computational complexity is compared among SC-VP-1, SC-VP-2, and OFDM-VP.**

Keywords—SC-MU-MIMO; Vector Perturbation; QR decomposition; M algorithm

I. INTRODUCTION

In multi-user (MU) multiple-input multiple-output (MIMO) [1]-[7], multiple users simultaneously communicate with a base station (BS) having multiple antennas using the same frequency. Recently, MU-MIMO has been attracting much attention since available bandwidth is limited. The broadband single-carrier (SC) block transmission can take advantage of the frequency-selectivity of the channel and obtain the frequency-diversity gain to improve the transmission performance. The combination of MU-MIMO and SC block transmission makes high-speed data transmissions possible with limited available bandwidth.

For uplink broadband SC-MU-MIMO transmissions, a complexity-reduced maximum likelihood (ML) based signal detection technique [1], [2] can be applied. On the other hand, for downlink transmissions, users cannot generally know other users' channel state information (CSI) and hence the use of precoding is feasible [3]-[7]. Channel inversion (CI) is a linear precoding scheme [5]. When the channel is in an ill condition (e.g., high fading correlation among users), the received signal-to-noise power ratio (SNR) achievable with CI degrades, thereby degrading the transmission performance. Vector perturbation (VP) [6], which is a nonlinear precoding scheme, can avoid SNR degradation in an ill condition by finding and adding the optimal perturbation vector. VP is widely used in orthogonal frequency-division multiplexing (OFDM) transmission but can be apply to SC transmission as well. In SC-MU-MIMO using VP, the number of perturbation vector candidates is larger than OFDM-MU-MIMO using VP (OFDM-VP) since a perturbation vector is added taking into account the frequency-selectivity of the channel. Thus, in terms of the received SNR improvement due to applying VP, SC transmission may be better than OFDM transmission. However, to the best of authors' knowledge, VP applied to SC-MU-MIMO has not yet been studied.

In this paper, we propose two VP schemes for SC-MU-MIMO downlink block transmission. The first VP scheme (SC-VP-1) adds a perturbation vector to each user's signal block in time-domain and then, multiplies a precoding matrix to perturbation vector-added signal blocks in time-domain. In case of the channel having delay waves, the precoding matrix of conventional VP cannot be applied to SC-VP-1 directly. Therefore, the precoding matrix used in this paper takes into account delay waves. Note that the computational complexity of perturbation vector search is extremely high in SC-VP-1 since the perturbation vector is searched taking into account all users' signal blocks. In order to reduce the computational complexity of perturbation vector search, a combination of QR decomposition and M algorithm [7] is applied. The QR decomposition and M algorithm can significantly reduce the computational complexity of perturbation vector search while achieving a transmission performance close to that using optimal perturbation vectors. Therefore, the combination of QR decomposition and M algorithm makes it possible that applying VP to SC-MU-MIMO downlink block transmission.

However, calculating the precoding matrix of SC-VP-1 requires the inverse matrix calculation of order UN_c (where U is the number of users communicating with BS at the same time and N_c is the block size). This inverse matrix calculation is high computational complexity of $O(U^3N_c^3)$. In the second VP scheme (SC-VP-2), each user's perturbation vector-added time-domain signal block is transformed into the frequency-domain signal block before the precoding matrix multiplication. The computational complexity of inverse matrix calculation can be reduced to $O(U^3N_c)$ by the above transformation. SC-VP-2 still requires the same computational complexity of perturbation vector search as SC-VP-1. As noted above, each user's perturbation vector-added time-domain signal block is transformed into the frequency-domain in SC-VP-2. A time-domain perturbation component spreads into all frequency components. Noting that since multiple precoding matrices are used in SC-VP-2, the computational complexity cannot be reduced enough by applying the QR decomposition and M algorithm directly. Thus, unlike conventional VP and SC-VP-1, an equivalent time-domain expression for frequency-domain precoding matrix multiplication (in following, a time-domain equivalent precoding matrix) is used for QR decomposition and M algorithm based perturbation vector search. The uncoded bit error rate (BER) performance achievable by SC-VP-1 and SC-VP-2 is evaluated by computer simulation. It is shown that SC-VP-1 and SC-VP-2 provide the same BER performance. It is also shown that SC-VP achieves better BER performance than both SC-CI and OFDM-VP. Computational complexity of SC-VP-1 and SC-VP-2 is compared to OFDM-VP.

The remainder of this paper is organized as follows. Section II overviews narrowband MU-MIMO downlink using VP. Section III proposes SC-VP-1 and SC-VP-2. In Section IV, computer simulation results are presented. Finally, Section V offers some conclusions.

II. DOWNLINK NARROWBAND MU-MIMO USING VP [6]

In this section, we overviews conventional narrowband MU-MIMO downlink using VP. In OFDM-VP, perturbation vector addition and precoding matrix multiplication are applied to each subcarrier. U users communicate with BS at the same time. The BS uses N_T transmit antennas and each user uses a receive antenna. $[.]^T$ is the transpose operator. The transmit signals to all users are written as $N_T\times 1$ vector $\mathbf{s}=[s_0\cdots s_{N_T-1}]^T$. In VP, the $N_T\times U$ precoding matrix $\mathbf{f}$ is multiplied to the $U\times 1$ signal vector $\mathbf{x}=[x_0\cdots x_{U-1}]^T$ after perturbation vector addition for suppressing the inter-user interference (IUI) as

$$\mathbf{s}=\sqrt{U/\gamma}\,\mathbf{f}\mathbf{x}\,, \tag{1}$$

where $\mathbf{f}$ is given as

$$\mathbf{f}=\mathbf{h}^H\left(\mathbf{h}\mathbf{h}^H\right)^{-1}, \tag{2}$$

where $\mathbf{h}$ is the $U\times N_T$ channel matrix between all users' receive antennas and the BS transmit antennas. $\mathbf{f}$ is the same as CI. $\sqrt{U/\gamma}$ is the power normalization coefficient for keeping the transmit power constant, where $\gamma=\|\mathbf{fx}\|^2$. $\|.\|$ is the Euclidean norm of the vector. In VP, a perturbation vector is added to the data-modulated signal vector before multiplying the precoding matrix. SNR degradation in an ill condition can be avoided by finding and adding the optimal perturbation vector. $\mathbf{x}$ is represented as

$$\mathbf{x}=\mathbf{d}+\tau\mathbf{l}\,, \tag{3}$$

where $\mathbf{d}=[d_0\cdots d_{U-1}]^T$ is $U\times 1$ vector composed of the data-modulated signals to all users. τ depends on the modulation level and $\tau=2\sqrt{2}$ in QPSK. The $U\times 1$ perturbation vector $\mathbf{l}=[l_0\cdots l_{U-1}]^T$ is determined to minimize the Euclidean norm of $\mathbf{s}$ as

$$\mathbf{l}=\arg\min_{\mathbf{l}'}\left(\left\|\mathbf{fx}\right\|^2\right)=\arg\min_{\mathbf{l}'}\left(\left\|\mathbf{f}\left(\mathbf{d}+\tau\mathbf{l}'\right)\right\|^2\right). \tag{4}$$

$U\times 1$ received signal vector is represented as

$$\begin{aligned}\mathbf{r}&=\sqrt{2E_s/T_s}\,\mathbf{h}\mathbf{s}+\mathbf{n}\\&=\sqrt{(2UE_s/T_s)/\gamma}\,\mathbf{h}\mathbf{f}\left(\mathbf{d}+\tau\mathbf{l}\right)+\mathbf{n}\,,\\&=\sqrt{(2UE_s/T_s)/\gamma}\left(\mathbf{d}+\tau\mathbf{l}\right)+\mathbf{n}\end{aligned} \tag{5}$$

where E_s is the average transmit symbol energy and T_s is the symbol duration. $\mathbf{n}=[n_0\cdots n_{U-1}]^T$ is the $U\times 1$ noise vector whose elements are the complex Gaussian variables having zero mean and variance $2\sigma^2=2N_0/T_s$ with N_0 being the single-sided power spectrum density of additive white Gaussian noise (AWGN). Each receiver does not require CSI. The received signal is divided by the signal coefficient of Eq. (5) (the first coefficient of the right side) which is informed from the transmitter. Then, modulo operation is applied for removing the perturbation vector. At last, the signals are demodulated.

III. TWO NEW VP SCHEMES

A. SC-VP-1

In this subsection, we propose a VP scheme for SC-MU-MIMO downlink block transmission. In SC-VP-1, the precoding matrix obtained using the channel matrix (delay waves are taken into account) is used. Transmitter/receiver structures of SC-VP-1 is illustrated in Fig. 1. The transmit signal block which is composed of N_c transmit signals from the n_T-th transmit antenna is written as $N_c\times 1$ vector $\mathbf{s}_{n_T}=[s_{n_T}(0)\cdots s_{n_T}(N_c-1)]^T$. The N_T transmit signal blocks from all transmit antennas are given as $N_TN_c\times 1$ vector $\tilde{\mathbf{s}}=[\mathbf{s}_0^T\cdots\mathbf{s}_{N_T-1}^T]^T$. The precoding matrix multiplication and the $N_TN_c\times UN_c$ precoding matrix can be given as

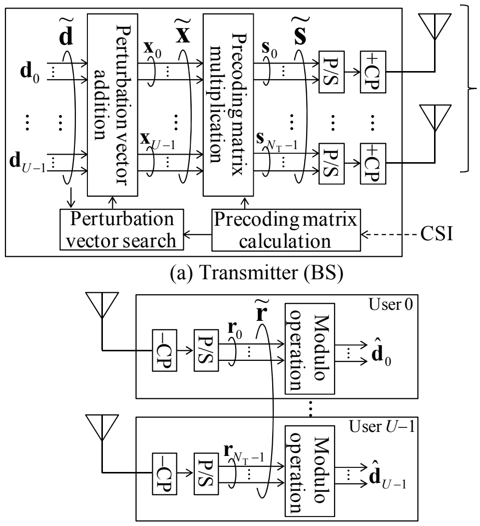
$$\tilde{\mathbf{s}}=\sqrt{UN_c/\gamma}\,\tilde{\mathbf{f}}\tilde{\mathbf{x}}\,, \tag{6}$$

$$\tilde{\mathbf{f}}=\tilde{\mathbf{h}}^H\left(\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H\right)^{-1}, \tag{7}$$

respectively. $\sqrt{UN_c/\gamma}$ is the power normalization coefficient for keeping the transmit power constant, where $\gamma=\|\tilde{\mathbf{f}}\tilde{\mathbf{x}}\|^2$. The $UN_c\times N_TN_c$ channel matrix $\tilde{\mathbf{h}}$ is the channel matrix expression which is different from that of conventional VP and is given as

$$\tilde{\mathbf{h}}=\begin{pmatrix}\mathbf{h}_{00} & \cdots & \mathbf{h}_{0(N_T-1)}\\ \vdots & \ddots & \vdots\\ \mathbf{h}_{(U-1)0} & \cdots & \mathbf{h}_{(U-1)(N_T-1)}\end{pmatrix}, \tag{8}$$

where $\mathbf{h}_{un_T}$ is the $N_c\times N_c$ channel impulse response matrix between the u-th users' receive antenna and the n_T-th BS transmit antenna. $\mathbf{h}_{un_T}$ is represented as



(a) Transmitter (BS)

(b) Receiver (Users)

Fig. 1. Transmitter/receiver structures of SC-VP-1.

$$\mathbf{h}_{un_T}=\begin{pmatrix} h_{0,un_T} & & & h_{L-1,un_T} & \cdots & h_{1,un_T} \\ h_{1,un_T} & h_{0,un_T} & & & \ddots & \vdots \\ \vdots & h_{1,un_T} & \ddots & \mathbf{0} & & h_{L-1,un_T} \\ h_{L-1,un_T} & \vdots & \ddots & h_{0,un_T} & & \\ & h_{L-1,un_T} & & h_{1,un_T} & \ddots & \\ \mathbf{0} & & \ddots & \vdots & \ddots & h_{0,un_T} \end{pmatrix}, \quad (9)$$

where h_{l,un_T} is the complex-valued path gain of the $l(=0\sim L-1)$-th path and L is the number of delay paths. $\widetilde{\mathbf{x}}=[\mathbf{x}_0^T\cdots\mathbf{x}_{U-1}^T]^T$ is the $UN_c\times1$ signal blocks vector after perturbation vector addition and represented as

$$\widetilde{\mathbf{x}}=\widetilde{\mathbf{d}}+\tau\widetilde{\mathbf{l}}, \quad (10)$$

where $\widetilde{\mathbf{d}}$ is the UN_c data-modulated signal blocks vector to all users given as $\widetilde{\mathbf{d}}=[\mathbf{d}_0^T\cdots\mathbf{d}_{U-1}^T]^T$ with $\mathbf{d}_u=[d_u(0)\cdots d_u(N_c-1)]^T$ being the $N_c\times1$ vector composed of the N_c data-modulated signals to the $u(=0\sim U-1)$-th user. $\widetilde{\mathbf{l}}$ is the $UN_c\times1$ perturbation vector and can be given as $\widetilde{\mathbf{l}}=[\mathbf{l}_0^T\cdots\mathbf{l}_{U-1}^T]^T$ with $\mathbf{l}_u=[l_u(0)\cdots l_u(N_c-1)]^T$ being the u-th $N_c\times1$ perturbation vector. $\widetilde{\mathbf{l}}$ is determined to minimize the Euclidean norm of $\widetilde{\mathbf{s}}$ as

$$\widetilde{\mathbf{l}}=\arg\min_{\widetilde{\mathbf{l}}'}\left(\left\|\widetilde{\mathbf{f}}\widetilde{\mathbf{x}}\right\|^2\right)=\arg\min_{\widetilde{\mathbf{l}}'}\left(\left\|\widetilde{\mathbf{f}}\left(\widetilde{\mathbf{d}}+\tau\widetilde{\mathbf{l}}'\right)\right\|^2\right). \quad (11)$$

After inserting a cyclic prefix (CP) of N_g symbols into the guard interval (GI), the BS transmits signal blocks from each transmit antenna. $\mathbf{r}_u$ is the u-th $N_c\times1$ received signal block vector after removing the GI. The $UN_c\times1$ received signal blocks vector $\widetilde{\mathbf{r}}=[\mathbf{r}_0^T\cdots\mathbf{r}_{U-1}^T]^T$ is represented as

$$\begin{aligned}\widetilde{\mathbf{r}}&=\sqrt{2E_s/T_s}\,\widetilde{\mathbf{h}}\widetilde{\mathbf{s}}+\widetilde{\mathbf{n}}\\&=\sqrt{(2UN_cE_s/T_s)/\gamma}\,\widetilde{\mathbf{h}}\widetilde{\mathbf{f}}\left(\widetilde{\mathbf{d}}+\tau\widetilde{\mathbf{l}}\right)+\widetilde{\mathbf{n}},\\&=\sqrt{(2UN_cE_s/T_s)/\gamma}\left(\widetilde{\mathbf{d}}+\tau\widetilde{\mathbf{l}}\right)+\widetilde{\mathbf{n}}\end{aligned} \quad (12)$$

where $\widetilde{\mathbf{n}}=[\mathbf{n}_0^T\cdots\mathbf{n}_{U-1}^T]^T$ is the $UN_c\times1$ noise vector. $\mathbf{n}_u=[n_u(0)\cdots n_u(N_c-1)]^T$ is the $N_c\times1$ noise vector whose elements are the complex Gaussian variables having zero mean and variance $2\sigma^2=2N_0/T_s$. Each receiver does not require CSI. The received signal is divided by the signal coefficient of Eq. (12) (the first coefficient of the right side) which is informed from the transmitter. Modulo operation is applied for removing the perturbation vector. At last, the signals are demodulated.

The computational complexity of perturbation vector search expressed as Eq. (11) is extremely high since the transmit power is normalized taking into account all users' signal blocks. In order to reduce the computational complexity of perturbation vector search, a combination of QR decomposition and M algorithm [7] is applied. The QR decomposition is performed to the precoding matrix represented as Eq. (7) as

$$\widetilde{\mathbf{f}}=\widetilde{\mathbf{Q}}\begin{pmatrix}\widetilde{\mathbf{R}}\\\mathbf{0}\end{pmatrix}, \quad (13)$$

where $\widetilde{\mathbf{Q}}$ is the unitary matrix of order N_TN_c and $\widetilde{\mathbf{R}}$ is the upper triangular matrix of order UN_c. The $j(=0\sim UN_c-1)$-th element of $\widetilde{\mathbf{d}}$ and $\widetilde{\mathbf{l}}$ are written as $\widetilde{d}_j$ and $\widetilde{l}_j$, respectively. When the (i,j)-th element of $\widetilde{\mathbf{R}}$ is given as $\widetilde{R}_{ij}$; $i, j=0\sim UN_c-1$, Eq. (11) is rewritten as

$$\begin{aligned}\widetilde{\mathbf{l}}&=\arg\min_{\widetilde{\mathbf{l}}'}\left(\left\|\widetilde{\mathbf{R}}\left(\widetilde{\mathbf{d}}+\tau\widetilde{\mathbf{l}}'\right)\right\|^2\right)\\&=\arg\min_{\widetilde{\mathbf{l}}'}\begin{pmatrix}\left|\widetilde{R}_{(UN_c-1)(UN_c-1)}\left(\widetilde{d}_{UN_c-1}+\tau\widetilde{l}_{UN_c-1}\right)\right|^2+\\\left|\sum_{j=UN_c-2}^{UN_c-1}\widetilde{R}_{(UN_c-2)j}\left(\widetilde{d}_j+\tau\widetilde{l}_j\right)\right|^2+\\\cdots+\left|\sum_{j=0}^{UN_c-1}\widetilde{R}_{0j}\left(\widetilde{d}_j+\tau\widetilde{l}_j\right)\right|^2\end{pmatrix},\end{aligned} \quad (14)$$

The first term of the right side of Eq. (14) depends on only $\widetilde{l}_{UN_c-1}$ in the component of $\widetilde{\mathbf{l}}$. Thus, $\widetilde{l}_{UN_c-1}$ is determined to minimize the first coefficient. Then, the second term depends on $\widetilde{l}_{UN_c-1}$ and $\widetilde{l}_{UN_c-2}$. Since $\widetilde{l}_{UN_c-1}$ has already been determined, $\widetilde{l}_{UN_c-2}$ is determined to minimize the second term. $\widetilde{\mathbf{l}}$ can be determined by successively carrying out this operation to the UN_c-th term. However, $\widetilde{l}_i$ may not minimize the terms from $(i+1)$-th to UN_c-th. Therefore, M candidates of the perturbation elements are survived in ascending order of the sum of the terms from $(i+1)$-th to UN_c-th of Eq. (14) when determining $\widetilde{l}_i$ to find a near optimal perturbation vector (M algorithm). When K candidates are searched for each perturbation vector, the number of search candidates using QR decomposition and M algorithm is $MKUN_c$. When we set, for example, M=5, K=9, U=4, and N_c=64, the number $MKUN_c$ of search candidates is equal to 11520 while the number of all search candidates without QR decomposition and M algorithm is $K^{UN_c}=9^{4\times64}\approx2\times10^{244}$. Thus, the number of all search candidates can be reduced significantly compared to K^{UN_c}.

B. SC-VP-2

In this subsection, we propose a VP scheme using the frequency-domain channel gain for SC-MU-MIMO downlink block transmission. Transmitter structures of SC-VP-2 is illustrated in Fig. 2. Receiver structures of SC-VP-2 is the same as that of SC-VP-1. The time-domain data-modulated signals to all users and the perturbation vector at time t are written as $U\times1$ vector $\mathbf{d}(t)=[d_0(t)\cdots d_{U-1}(t)]^T$ and $U\times1$ vector $\mathbf{l}(t)=[l_0(t)\cdots l_{U-1}(t)]^T$, respectively. The $U\times1$ signal vector $\mathbf{x}(t)$ after perturbation vector addition is given as

$$\mathbf{x}(t)=\mathbf{d}(t)+\tau\mathbf{l}(t). \quad (15)$$

The BS performs N_c-point discrete Fourier transform (DFT) to the time-domain data-modulated signals $\{x_u(t);\ t=0\sim N_c-1\}$, $u=0\sim U-1$, for multiplying the precoding matrix after adding the perturbation vector. The $k(=0\sim N_c-1)$-th frequency-domain signals to all users are written as $U\times1$ vector $\mathbf{X}(k)=[X_0(k)\cdots X_{U-1}(k)]^T$. After N_c-point DFT, the precoding matrix is multiplied by $\mathbf{X}(k)$. The $N_T\times1$ signal vector $\mathbf{S}(k)$ after multiplying the precoding matrix is represented as

$$\mathbf{S}(k)=\sqrt{UN_c/\gamma}\,\mathbf{F}(k)\mathbf{X}(k), \quad (16)$$

where the power normalization coefficient and the $N_T\times U$

precoding matrix $\mathbf{F}(k)$ at the k-th frequency is given as

$$\gamma = \sum_{k=0}^{N_c-1} \left\| \mathbf{F}(k)\mathbf{X}(k) \right\|^2 , \tag{17}$$

$$\mathbf{F}(k) = \mathbf{H}^H(k)\left(\mathbf{H}(k)\mathbf{H}^H(k)\right)^{-1} , \tag{18}$$

respectively. $\mathbf{H}(k)$ is the $U \times N_T$ channel matrix between all users' receive antennas and the BS transmit antennas at the k-th frequency. The BS performs N_c-point inverse DFT (IDFT) to $\{S_{n_T}(k);\ k=0\sim N_c-1\}$, $n_T=0\sim N_T-1$. After inserting a CP of N_g symbols into the GI, the BS transmits signals from each transmit antenna. The t-th $U\times1$ received signal vector $\overline{\mathbf{r}}(t)=[r_0(t)\cdots r_{U-1}(t)]^T$ after removing the GI is represented as

$$\overline{\mathbf{r}}(t) = \sqrt{(2UN_cE_s/T_s)/\gamma}\left(\mathbf{d}(t)+\tau\mathbf{l}(t)\right)+\mathbf{n}(t) , \tag{19}$$

where $\mathbf{n}(t)=[n_0(t)\cdots n_{U-1}(t)]^T$ is the $U\times1$ noise vector whose elements are the complex Gaussian variables having zero mean and variance $2\sigma^2=2N_0/T_s$. Each receiver does not require CSI. The received signal is divided by the signal coefficient of Eq. (19) (the first coefficient of the right side) which is informed from the transmitter. Modulo operation is applied for removing the perturbation vector. At last, the signals are demodulated.

SC-VP-2 also requires the same computational complexity as SC-VP-1 for searching all candidates of perturbation vector. Each user's perturbation vector-added time-domain signal block is transformed into the frequency-domain signal block before the precoding matrix multiplication as noted above. A perturbation component spreads into all frequency components. Noting that since multiple precoding matrices are used in SC-VP-2, the computational complexity cannot be reduced enough by applying the QR decomposition and M algorithm directly. Thus, unlike conventional VP and SC-VP-1, a time-domain equivalent precoding matrix is obtained before perturbation vector search. The $N_TN_c\times1$ signal vector $\mathbf{S}=[\mathbf{S}^T(0)\cdots\mathbf{S}^T(N_c-1)]^T$ composed of all user and frequency components after precoding matrix multiplication are written as

$$\mathbf{S} = \sqrt{\frac{UN_c}{\gamma}}\begin{pmatrix}\mathbf{F}(0) & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & \mathbf{F}(N_c-1)\end{pmatrix}\begin{pmatrix}\mathbf{X}(0)\\ \vdots \\ \mathbf{X}(N_c-1)\end{pmatrix} = \sqrt{\frac{UN_c}{\gamma}}\mathbf{F}\mathbf{X} \tag{20}$$

where the signal vector to all users at the k-th frequency $\mathbf{X}(k)$ is arranged in order of frequency in $\mathbf{X}$. Then, $\mathbf{X}$ is rearranged in order of user of the $N_c\times1$ signal vector $\overline{\mathbf{X}}_u=[X_u(0)\cdots X_u(N_c-1)]^T$ to the u-th user at all frequencies. The $N_c\times1$ signal vector at all frequencies after precoding matrix multiplication is written as $\overline{\mathbf{S}}_{n_T}=[S_{n_T}(0)\cdots S_{n_T}(N_c-1)]^T$. The $N_TN_c\times1$ signal vector at all frequencies after precoding matrix multiplication $\overline{\mathbf{S}}=[\overline{\mathbf{S}}_0^T\cdots\overline{\mathbf{S}}_{N_T-1}^T]^T$ is rewritten from $\mathbf{S}$ as

$$\overline{\mathbf{S}} = \sqrt{\frac{UN_c}{\gamma}}\begin{pmatrix}\overline{\mathbf{F}}_{00} & \cdots & \overline{\mathbf{F}}_{0(U-1)}\\ \vdots & \ddots & \vdots\\ \overline{\mathbf{F}}_{(N_T-1)0} & \cdots & \overline{\mathbf{F}}_{(N_T-1)(U-1)}\end{pmatrix}\begin{pmatrix}\overline{\mathbf{X}}_0\\ \vdots\\ \overline{\mathbf{X}}_{U-1}\end{pmatrix} = \sqrt{\frac{UN_c}{\gamma}}\overline{\mathbf{F}}\overline{\mathbf{X}} \tag{21}$$

where $\overline{\mathbf{F}}_{yz}$ is the diagonal matrix of order N_c, whose k-th diagonal component is the (y,z)-th element $F_{yz}(k)$ of $\mathbf{F}(k)$. Using DFT matrix $\boldsymbol{\Delta}$ of order N_c, whose (y,z)-th element is $\exp(-j2\pi yz/N_c)/\sqrt{N_c}$, $\overline{\mathbf{X}}$ can be represented as

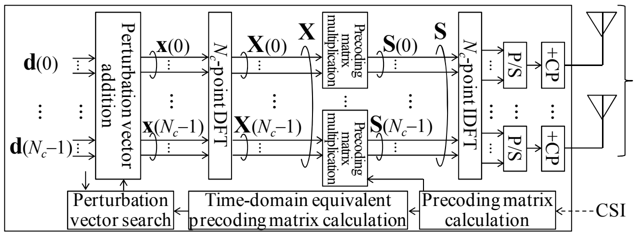


Fig. 2. Transmitter structures of SC-VP-2.

$$\overline{\mathbf{X}} = \begin{pmatrix}\boldsymbol{\Delta} & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & \boldsymbol{\Delta}\end{pmatrix}\begin{pmatrix}\mathbf{x}_0\\ \vdots\\ \mathbf{x}_{U-1}\end{pmatrix} = \overline{\boldsymbol{\Delta}}\tilde{\mathbf{x}} . \tag{22}$$

Thus, Eq. (21) is rewritten as

$$\overline{\mathbf{S}} = \sqrt{UN_c/\gamma}\,\overline{\mathbf{F}}\overline{\boldsymbol{\Delta}}\tilde{\mathbf{x}} \equiv \sqrt{UN_c/\gamma}\,\bar{\mathbf{f}}\tilde{\mathbf{x}} . \tag{23}$$

Eq. (23) is similar to Eq. (6). $\bar{\mathbf{f}}$ can be regarded as the $N_TN_c\times UN_c$ time-domain equivalent precoding matrix. After calculating $\bar{\mathbf{f}}$, $\tilde{\mathbf{f}}$ in Eq. (11) and (13) are replaced by $\bar{\mathbf{f}}$ and then, perturbation vector search using the QR decomposition and M algorithm is applied as the same way.

IV. Computer Simulation Results

The computer simulation condition is summarized in Table I. BS transmits signals to U=4 users using N_T=4 transmit antennas. We assume BS can ideally obtain the CSI between all users' receive antennas and the BS transmit antennas. Each user uses a receive antenna for receiving signals. We assume a frequency-selective block Rayleigh fading channel having L=8-path with uniform power delay profile.

Fig. 3 plots the cumulative distribution function (CDF) of the power normalization coefficient $\sqrt{UN_c/\gamma}$ in SC-VP-1 and SC-VP-2 at the number M=50 of survival paths in each stage. For comparison, Fig. 3 also plots that in SC-CI and the CDF of the power normalization coefficient $\sqrt{U/\gamma}$ in CI for OFDM-MU-MIMO (OFDM-CI) and OFDM-VP (with searching all candidates). It is seen from Fig. 3 that the CDFs of the power normalization coefficient in SC-VP-1 and SC-VP-2 are equivalent and have a better distribution than SC-CI since adding a near optimal perturbation vector increases the power normalization coefficient.

Fig. 4 plots the uncoded BER performance of SC-VP-1 and SC-VP-2 as a function of the average transmit bit energy-to-noise power spectral density ratio (E_b/N_0) at M=50. For comparison, Fig. 4 also plots those of SC-CI, OFDM-CI, and OFDM-VP (with searching all candidates). It is seen from Fig. 4 that the SC-VP-1 and SC-VP-2 provide the same BER performance and achieve better BER performance than SC-CI and OFDM-VP. It is also seen from Fig.4 that SC-VP provides larger improvement than OFDM-VP in terms of the average BER improvement from the CI. In SC-VP, a perturbation vector is added with taking into account the frequency-selectivity of the channel. The variance of the power normalization coefficient in SC-VP is larger than that in OFDM-VP owing to the frequency-diversity. As a result, the power normalization coefficient improvement by the selected perturbation vector in SC-VP is larger than that in OFDM-VP.

Fig. 5 plots the average BER of SC-VP as a function of M at the average transmit E_b/N_0=10dB. Increasing the number of survival paths in each stage improves the BER since a nearer optimal perturbation vector can be found. When the computer simulation condition in Table I is used, the average BER improvement by increasing M is saturated around about M=40.

Table II shows the number of complex multiplications in SC-VP-1 and SC-VP-2 at the transmitter. For comparison, the number of complex multiplications in OFDM-VP is also shown in Table II. a is the minimum integer satisfied with $M \le K^a$. Although the number of complex multiplications in SC-VP-1 depends on N_c^3, that in SC-VP-2 depends on N_c for precoding matrix calculation. SC-VP-2 requires fewer multiplications for precoding matrix calculation than SC-VP-2. In SC-VP-2, a time-domain equivalent precoding matrix is required. However, in terms of the total number of complex multiplications for the time-domain equivalent precoding matrix calculation and the precoding matrix calculation, SC-VP-2 requires fewer multiplications than SC-VP-1. The number of complex multiplications of perturbation vector search and that of QR decomposition are the same between SC-VP-1 and SC-VP-2. That of DFT/IDFT depends on N_c^2. Thus, the number of complex multiplications in SC-VP-2 is smaller than that in SC-VP-1. Since SC-VP-1 and SC-VP-2 provide the same BER performance, SC-VP-2 is considered the better scheme. The number of complex multiplications of precoding matrix calculation and multiplication in SC-VP-2 is the same as that in OFDM-VP. However, the number of complex multiplications of perturbation vector search in SC-VP-2 is larger than that in OFDM-VP. In addition, SC-VP-2 requires the time-domain equivalent precoding matrix calculation and QR decomposition. Thus, the number of complex multiplications in SC-VP-2 is larger than that in OFDM-VP. When we set computer simulation condition as Table I and M=50 for example, the numbers of complex multiplications at transmitter in SC-VP-1, SC-VP-2, and OFDM-VP are 1357025259, 1306678259 and 6752256, respectively. SC-VP-1 and SC-VP-2 require approximately 200 and 190 times more complex multiplications at transmitter than that in OFDM-VP, respectively.

V. CONCLUSION

In this paper, we proposed two VP schemes (SC-VP-1 and SC-VP-2) applying QR decomposition and M algorithm for SC-MU-MIMO downlink. We showed, by computer simulation, that SC-VP-1 and SC-VP-2 provide the same BER performance. We also showed that SC-VP-1 and SC-VP-2 achieve better uncoded BER performance than SC-CI and OFDM-VP. SC-VP-1 and SC-VP-2 and OFDM-VP were compared in terms of the number of complex multiplications. The impact of channel estimation error and performance comparisons of coded case between SC-VP and OFDM-VP and between uplink and downlink transmissions are left as our important future topics.

REFERENCES

[1] M. Itagaki, K. Takeda, and F. Adachi, “Frequency-domain QRM-MLD block signal detection for multi-user single-carrier MIMO uplink,” in Proc. of 2010 International Conference on Network Infrastructure and Digital Content (IC-NIDC2010), Beijing, China, Sept. 2010.

[2] T. Yamamoto, K. Adachi, S. Sun, and F. Adachi, “Recursive QR packet combining for uplink single-carrier multi-user MIMO HARQ using near ML detection,” Wireless Communications and Mobile Computing, Wiley, Dec. 2012, DOI: 10.1002/wcm.2334.

[3] M. Morelli, M.-O. Pan and C.-C. J. Kuo, “Frequency-domain pre-equalization for single-carrier space-division multiple-access downlink transmissions,” in Proc. of IEEE 63rd Vehicular Technology Conference (VTC2006-Spring), Melbourne, Australia, May, 2006.

[4] C. Degen and L. Rrühl, “Linear and successive predistortion in the frequency domain: performance evaluation in SDMA systems,” in Proc. of 2005 IEEE Wireless Communications and Networking Conference (WCNC2005), New Orleans, USA, Mar. 2005.

[5] B. M. Hochwald, B. Peel and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multiantenna multiuser communication-Part I: channel inversion and regularization,” IEEE Trans. Commun., vol. 53, no. 1, pp. 195-202, Jan. 2005.

[6] B. Peel, B. M. Hochwald and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multiantenna multiuser communication-Part II: Perturbation,” IEEE Trans. Commun., vol. 53, no. 3, pp. 195-202, Mar. 2005.

[7] J. Z. Zhang and K. J. Kim, “Near-capacity MIMO multiuser precoding with QRD-M algorithm,” in Proc. of IEEE ACSSC, California, U.S.A., Nov. 2005.

TABLE I. COMPUTER SIMULATION CONDITION.

Transmitter & Receiver	Data modulation	QPSK
	Block size	N_c=64
	CP length	N_g=8
	No. of transmit antennas	N_T=4
	No. of users	U=4
	No. of each perturbation component candidates	K=9
	No. of survival paths in each stage	M=1~100
	Channel estimation	Ideal
Channel model	Fading	Frequency-selective block Rayleigh
	Power delay profile	8-path uniform
	Time delay of l-th path	τ_l=l symbols

TABLE II. NUMBER OF COMPLEX MULTIPLICATIONS AT TRANSMITTER.

	OFDM-VP	SC-VP-1	SC-VP-2
Precoding matrix calculation	$2U^2(U+N_T)N_c$	$2U^2(U+N_T)N_c^3$	$2U^2(U+N_T)N_c$
Time-domain equivalent precoding matrix calculation			$U^2N_TN_c^3$
QR decomposition		$U^2N_TN_c^3$	$U^2N_TN_c^3$
Perturbation vector search	$K^UUN_TN_c$	$a(a+1)K^a/2+MK\sum_{i=1}^{UN_c-a}(a+i)(a+i+1)/2$	$a(a+1)K^a/2+MK\sum_{i=1}^{UN_c-a}(a+i)(a+i+1)/2$
Precoding matrix multiplication	UN_TN_c	$UN_TN_c^2$	UN_TN_c
DFT			UN_c^2
IDFT	$N_TN_c^2$		$N_TN_c^2$

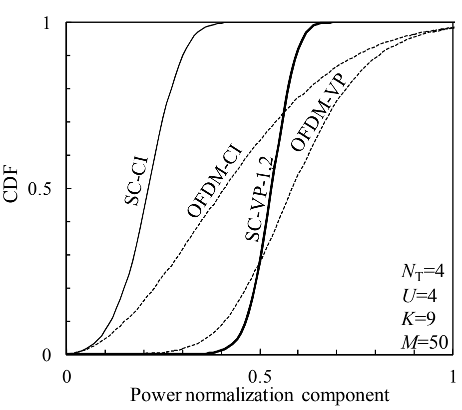


Fig. 3. CDF of power normalization coefficient.

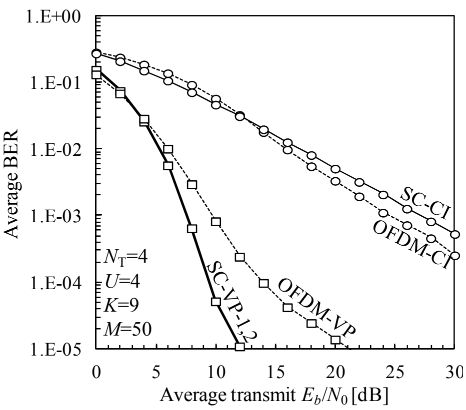


Fig. 4. Average BER performance.

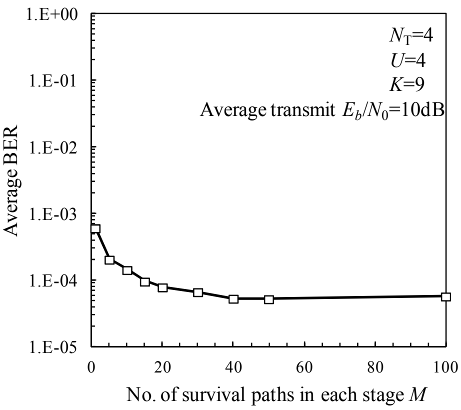


Fig. 5. Average BER as a function of M.