Joint Tomlinson-Harashima Precoding and Transmit Equalization in Time-Domain for Single-Carrier MU-MIMO Block Transmission

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Abstract**—This paper is concerned with cyclic prefix (CP) inserted broadband single-carrier (SC) multi-user multi-input multi-output (SC-MU-MIMO). The use of Tomlinson-Harashima precoding (THP) is effective to remove the inter-user interference (IUI). To remove the inter-symbol interference (ISI) caused by the channel frequency-selectivity, transmit equalization (TE) can be introduced. In the Degen and Rrühl's SC-MU-MIMO scheme (called as FD-SC-TETHP in this paper), THP and TE are performed separately in the frequency-domain. The modulo operation in THP suppresses the power increase caused by the IUI removal. TE can remove the ISI and the amplitude variation in the signals of a block received at each user, though it increases the signal power after equalization. This degrades significantly the received signal-to-noise power ratio (SNR). In this paper, we propose a time-domain SC-TETHP (TD-SC-TETHP), in which the ISI subtraction is performed together with IUI subtraction both in the time-domain. The modulo operation suppresses the power increase caused by both IUI and ISI removal. To prevent the amplitude variation, pre-removal of the received signal amplitude variation is inserted between the modulo operation and the precoding matrix multiplication. Uncoded average bit error rate (BER) performance achievable by our proposed TD-SC-TETHP is evaluated by computer simulation to confirm its superiority to the FD-SC-TETHP. Computational complexity is also discussed.**

Keywords—SC-MU-MIMO; Block Transmission; Tomlinson-Harashima Precoding; Transmit Equalization; Time-Domain;

I. INTRODUCTION

Recently, multi-user (MU) multiple-input multiple-output (MIMO) [1]-[8], [10] has been attracting much attention due to limited available bandwidth in mobile wireless communication. In MU-MIMO, multiple users simultaneously communicate with a base station (BS) having multiple antennas using the same frequency. This paper is concerned with the cyclic prefix (CP) inserted broadband single-carrier (SC) MU-MIMO (SC-MU-MIMO). For uplink broadband SC-MU-MIMO transmissions, a complexity-reduced maximum likelihood (ML) based signal detection technique [1], [2] can be applied. On the other hand, for downlink transmissions, the use of precoding at the transmitter is feasible in order to remove the inter-user interference (IUI) since users cannot generally know other users' channel state information (CSI). Precoding schemes can be classified into linear [3], [4] and nonlinear [5], [6]. It is known that the latter can obtain better transmission quality [7].

To remove the inter-user interference (IUI), the use of Tomlinson-Harashima precoding (THP) [6] is effective. THP is one of the nonlinear precoding schemes. THP carries out a series of IUI subtraction, modulo operation, and precoding matrix multiplication. The precoding matrix multiplication transforms the resultant channel matrix to a lower triangular matrix for IUI subtraction, and consequently the amplitude variation occurs in the signals of a block received at each user. The IUI subtraction causes the signal power increase. The modulo operation suppresses the increase. To remove the inter-symbol interference (ISI) caused by the channel frequency-selectivity, transmit equalization (TE) can be introduced. Degen and Rrühl introduced THP and TE to SC-MU-MIMO scheme (called as FD-SC-TETHP in this paper) [8], in which THP and TE are performed separately in the frequency-domain. The modulo operation suppresses the power increase caused by the IUI removal. TE can remove the ISI and the amplitude variation, though it increases the signal power after equalization. This degrades significantly the received signal-to-noise power ratio (SNR).

In this paper, we propose a time-domain SC-TETHP (TD-SC-TETHP), in which the ISI subtraction is performed together with IUI subtraction both in the time-domain. Although the modulo operation can suppress the power increase caused by both IUI and ISI removal, the signal amplitude variation remains in the signals of a block received at each user. To prevent this amplitude variation, the signal amplitude variations removal is inserted between the modulo operation and the precoding matrix multiplication. This removal increases the signal power, though the power increase is smaller than that caused by TE in FD-SC-TETHP, thereby improving the received SNR. In our proposed TD-SC-TETHP, all operations (IUI/ISI subtraction, modulo operation, pre-removal of the received signal amplitude variations, and precoding matrix multiplication) are performed in the time-domain. In the case of broadband channels having multiple delayed paths, the precoding matrix of conventional THP [6] cannot be directly applied to TD-SC-TETHP. The precoding matrix is modified taking account of the presence of multiple delayed paths. The uncoded average bit error rate (BER) performance achievable by our proposed TD-SC-TETHP is evaluated by computer simulation to confirm its superiority to FD-SC-TETHP. Computational complexity is also discussed.

The remainder of this paper is organized as follows. Section II overviews FD-SC-TETHP proposed by Degen and Rrühl [8] for better understanding of our proposed scheme.

Section III proposes TD-SC-TETHP. In Sections IV and V, computer simulation results and computational complexity comparison are presented, respectively. Finally, Section VI offers some conclusions.

II. FD-SC-TETHP [8]

In this section, we overviews FD-SC-TETHP, which is the conventional THP scheme for SC-MU-MIMO block transmission. Transmitter/receiver structures of FD-SC-TETHP are illustrated in Fig. 1. U users communicate with BS at the same time. The BS uses $N_T(\geq U)$ transmit antennas and each user uses a receive antenna. The time-domain data-modulated signals to all users at time t are written as $U\times 1$ vector $\mathbf{d}(t)=[d_0(t)\dots d_{U-1}(t)]^T$. $[.]^T$ is the transpose operator. At first, the BS performs N_c-point discrete Fourier transform (DFT) to the time-domain data-modulated signals $\{d_u(t); t=0\sim N_c-1\}$, $u=0\sim U-1$, for IUI subtraction in frequency-domain. N_c is the block size. The $k(=0\sim N_c-1)$-th frequency-domain signals to all users after DFT are written as $U\times 1$ vector $\mathbf{D}(k)=[D_0(k)\dots D_{U-1}(k)]^T$.

The BS applies successively the following IUI subtraction and modulo operation to $\mathbf{D}(k)$ in ascending order of user index u. IUI subtraction is given as

$$A_u(k)=D_u(k)-Y_u(k)\,, \tag{1}$$

where $Y_u(k)$ is the u-th user's residual IUI after precoding matrix multiplication at the k-th frequency and described later. Modulo operation suppresses the transmit power increase caused by the IUI subtraction of Eq. (1). Modulo operation is applied to the real and imaginary parts of $A_u(k)$, respectively. However, the frequency-domain signals have the extremely large number of signal points and modulo operation range cannot be decided. The BS performs N_c-point inverse DFT (IDFT) to $\{A_u(k); k=0\sim N_c-1\}$ and converts into the time-domain signals $\{a_u(t); t=0\sim N_c-1\}$ before applying the modulo operation. $x_u(t)$ is the signal after modulo operation to $a_u(t)$ as

$$x_u(t)=\left(a_u(t)\right)\bmod\tau\equiv a_u(t)+\tau z_u(t)\,. \tag{2}$$

τ depends on the modulation level and $\tau=2\sqrt{2}$ in QPSK. The real and imaginary parts of $z_u(t)$ are integral. The BS applies N_c-point DFT to $\{x_u(t); t=0\sim N_c-1\}$ and obtains the u-th user's frequency-domain signals $\{X_u(k); k=0\sim N_c-1\}$. Modulo operation components in time-domain and frequency-domain signals after modulo operation to all users are written as $\mathbf{z}(t)=[z_0(t)\dots z_{U-1}(t)]^T$ and $\mathbf{X}(k)=[X_0(k)\dots X_{U-1}(k)]^T$, respectively.

The BS performs the above successive IUI subtraction and modulo operation, and then multiplies $\mathbf{X}(k)$ by the TE weight matrix $\mathbf{W}(k)=\mathrm{diag}\{W_0(k),\dots,W_{U-1}(k)\}$ of order U at the k-th frequency for ISI removal. The TE weight can also remove the amplitude variation (caused by the precoding matrix multiplication) in the signals of a block received at each user. $\mathbf{W}(k)\mathbf{X}(k)$ after TE is multiplied by the precoding matrix. The $N_T\times 1$ frequency-domain transmit signal vector $\mathbf{S}(k)=[S_0(k)\dots S_{N_T-1}(k)]^T$ after precoding is represented as

$$\mathbf{S}(k)=\sqrt{\frac{UN_c}{\gamma}}\mathbf{F}(k)\mathbf{W}(k)\mathbf{X}(k)\,, \tag{3}$$

where γ is the power normalization coefficient for keeping the transmit power constant given as

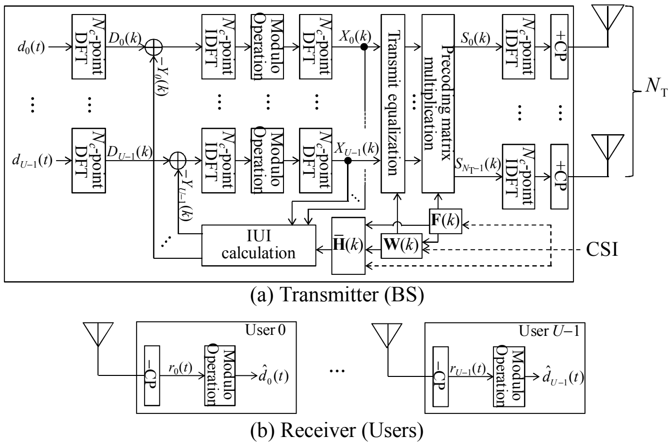

Fig. 1. Transmitter/Receiver structures of FD-SC-TETHP.

$$\gamma=\sum_{k=0}^{N_c-1}\left\|\mathbf{F}(k)\mathbf{W}(k)\mathbf{X}(k)\right\|^2\,. \tag{4}$$

$\|.\|$ is the Euclidean norm of the vector. $\mathbf{F}(k)$ is the $N_T\times U$ precoding matrix which transforms the resultant channel matrix $\mathbf{H}(k)\mathbf{F}(k)$ to a lower triangular matrix for the IUI subtraction, where $\mathbf{H}(k)$ is the $U\times N_T$ channel matrix between all users' receive antennas and the BS transmit antennas at the k-th frequency. The BS applies LQ decomposition [9] to $\mathbf{H}(k)$ in order to obtain $\mathbf{F}(k)$ as

$$\mathbf{H}(k)=\begin{pmatrix}\mathbf{L}(k) & \mathbf{0}\end{pmatrix}\mathbf{Q}(k)=\begin{pmatrix}\mathbf{L}(k) & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{Q}_L(k)\\ \mathbf{Q}_0(k)\end{pmatrix}, \tag{5}$$

$$\mathbf{F}(k)=\mathbf{Q}_L^H(k)\,. \tag{6}$$

$[.]^H$ is the Hermitian transpose operator. $\mathbf{Q}(k)$ and $\mathbf{L}(k)$ are the unitary matrix of order N_T and the lower triangular matrix of order U whose diagonal elements are real numbers, respectively. $\mathbf{Q}_L(k)$ and $\mathbf{Q}_0(k)$ correspond to the $0\sim(U-1)$-th and $U\sim(N_T-1)$-th rows of $\mathbf{Q}(k)$, respectively. The diagonal elements of $\mathbf{L}(k)$ are not constant. Although the amplitude variation occurs in the received signals at each user, the TE can remove this amplitude variation. At last, the BS performs N_c-point IDFT to the frequency-domain transmit signals $\{S_{N_T}(k); k=0\sim N_c-1\}$, $n_T=0\sim N_T-1$. After inserting a CP of N_g symbols into the guard interval (GI), the BS transmits signals from each transmit antenna.

Consequently, $\mathbf{W}(k)$ for removing ISI perfectly can be calculated. The equivalent channel matrix $\overline{\mathbf{H}}(k)$ of order U at the k-th frequency is written from Eq. (3), (5), and (6) as

$$\begin{aligned}\overline{\mathbf{H}}(k)&=\mathbf{H}(k)\mathbf{F}(k)\mathbf{W}(k)\\&=\mathbf{L}(k)diag\{w_0(k),\cdots,w_{U-1}(k)\}.\end{aligned} \tag{7}$$

The diagonal elements of the equivalent channel matrix must be 1 for removing ISI perfectly. Thus, the TE weight matrix $\mathbf{W}(k)$ of order U at the k-th frequency is represented as

$$\mathbf{W}(k)=diag\left\{L_{00}^{-1}(k),\cdots,L_{(U-1)(U-1)}^{-1}(k)\right\}, \tag{8}$$

when the (i,j)-th element of $\mathbf{L}(k)$ is expressed as $L_{ij}(k)$; $i, j=0\sim UN_c-1$. The equivalent channel matrix $\overline{\mathbf{H}}(k)$ of Eq. (7) is rewritten as

$$
\overline{\mathbf{H}}(k)=\begin{pmatrix} 1 & & & \mathbf{0} \\ \frac{L_{10}(k)}{L_{00}(k)} & \ddots & & \\ \vdots & \ddots & \ddots & \\ \frac{L_{(U-1)0}(k)}{L_{00}(k)} & \cdots & \frac{L_{(U-1)(U-2)}(k)}{L_{(U-2)(U-2)}(k)} & 1 \end{pmatrix}. \tag{9}
$$

If $Y_u(k)$ can remove IUI perfectly, the t-th $U\times1$ received signal vector $\mathbf{r}(t)=[r_0(t)\ldots r_{U-1}(t)]^T$ after removing the CP can be represented as

$$
\begin{aligned}
\mathbf{r}(t)&=\sqrt{\frac{2E_s}{T_s}\frac{UN_c}{\gamma}}\big(\mathbf{d}(t)+\tau\mathbf{z}(t)\big)+\mathbf{n}(t) \\
&\equiv C\big(\mathbf{d}(t)+\tau\mathbf{z}(t)\big)+\mathbf{n}(t).
\end{aligned} \tag{10}
$$

where E_s is the average transmit symbol energy and T_s is the symbol duration. $\mathbf{n}(t)=[n_0(t)\ldots n_{U-1}(t)]^T$ is the $U\times1$ noise vector whose elements are the complex Gaussian variables having zero mean and variance $2\sigma^2=2N_0/T_s$ with N_0 being the single-sided power spectrum density of additive white Gaussian noise (AWGN). Each receiver does not require CSI. The received signal is divided by the signal coefficient C of Eq. (10) which is informed from the transmitter. Then, modulo operation is applied and the signals are demodulated.

Consequently, $Y_u(k)$ for perfectly removing IUI can be calculated. The frequency-domain received signals at the k-th frequency are written as $\mathbf{R}(k)=[R_0(k)\ldots R_{U-1}(k)]^T$ after N_c-point DFT is applied to the time-domain received signals $\{r_u(t);\ t=0\sim N_c-1\}$, $u=0\sim U-1$. $\mathbf{R}(k)$ is expressed using $\mathbf{X}(k)$ as $\mathbf{R}(k)=C\overline{\mathbf{H}}(k)\mathbf{X}(k)$ (without noise), i.e., the u-th element $R_u(k)$ of $\mathbf{R}(k)$ are written as

$$
R_u(k)=\begin{cases} CX_u(k) & \text{....................................} for\ u=0 \\ C\left(X_u(k)+\sum_{j=0}^{u-1}\frac{L_{uj}(k)}{L_{jj}(k)}X_j(k)\right) & \text{....} for\ u>0 \end{cases}. \tag{11}
$$

From Eq. (11), $Y_u(k)$ for removing IUI perfectly is given as

$$
Y_u(k)=\begin{cases} 0 & \text{..} for\ u=0 \\ \sum_{j=0}^{u-1}\frac{L_{uj}(k)}{L_{jj}(k)}X_j(k) & \text{........................} for\ u>0 \end{cases}. \tag{12}
$$

III. TD-SC-TETHP

In this section, we propose TD-SC-TETHP for SC-MU-MIMO downlink block transmission. Transmitter structure of TD-SC-TETHP is illustrated in Fig. 2. Receiver structures are the same as those of FD-SC-TETHP. As explained in the previous section, TE in FD-SC-TETHP removes the ISI and the amplitude variation in the signals of a block received at each user. On the other hand, in TD-SC-TETHP, the ISI subtraction is performed together with IUI subtraction both in the time-domain. The modulo operation suppresses the power increase caused by both IUI and ISI removal. The remaining amplitude variation in the signals of a received block at each user is removed between the modulo operation and the precoding matrix multiplication. All operations at the transmitter are performed in the time-domain. The precoding matrix is obtained using the channel matrix where multiple delayed paths are taken into account. The $N_c\times1$ vector composed of the N_c data-modulated signals to the $u(=0\sim U-1)$-th user is written as $\mathbf{d}_u=[d_u(0)\ldots d_u(N_c-1)]^T$. The UN_c data-modulated signal blocks vector to all users is expressed as $\tilde{\mathbf{d}}=[\mathbf{d}_0^T\ldots\mathbf{d}_{U-1}^T]^T$. $\tilde{d}_i$ is the $i(=0\sim UN_c-1)$-th element of $\tilde{\mathbf{d}}$.

The BS applies successively the following IUI/ISI subtraction and modulo operation to $\tilde{\mathbf{d}}$ in ascending order of the index i. IUI/ISI subtraction is given as

$$
\tilde{a}_i=\tilde{d}_i-\tilde{y}_i, \tag{13}
$$

where $\tilde{y}_i$ is the i-th residual IUI and ISI after precoding matrix multiplication and described later. Modulo operation to the real and imaginary parts of $\tilde{a}_i$ suppresses the transmit power increase caused by the IUI/ISI subtraction of Eq. (13). $\tilde{x}_i$ is the signal after modulo operation to $\tilde{a}_i$ as

$$
\tilde{x}_i=(\tilde{a}_i)\bmod\tau\equiv\tilde{a}_i+\tau\tilde{z}_i. \tag{14}
$$

The real and imaginary parts of $\tilde{z}_i$ are integral. Modulo operation components and all users' signal blocks after modulo operation are written as $\tilde{\mathbf{z}}=[\tilde{z}_0\ldots\tilde{z}_{UN_c-1}]^T$ and $\tilde{\mathbf{x}}=[\tilde{x}_0\ldots\tilde{x}_{UN_c-1}]^T$, respectively.

The BS performs the above successive IUI/ISI subtraction and modulo operation, and then multiplies $\tilde{\mathbf{x}}$ by the matrix for the pre-removal of the received signal amplitude variation $\tilde{\mathbf{w}}$ of order UN_c. $\tilde{\mathbf{w}}$ is a diagonal matrix given as $\tilde{\mathbf{w}}=diag\{\tilde{w}_0,\ldots,\tilde{w}_{UN_c-1}\}$. $\tilde{\mathbf{w}}$ is multiplied so that the diagonal elements of the equivalent matrix become 1, and described later. $\tilde{\mathbf{w}}\tilde{\mathbf{x}}$ after the pre-removal is multiplied by the precoding matrix as

$$
\tilde{\mathbf{s}}=\sqrt{\frac{UN_c}{\gamma}}\tilde{\mathbf{f}}\tilde{\mathbf{w}}\tilde{\mathbf{x}}. \tag{15}
$$

$N_TN_c\times1$ vector $\tilde{\mathbf{s}}=[\tilde{s}_0\ldots\tilde{s}_{N_TN_c-1}]^T$ is the N_T transmit signal blocks from all transmit antennas. γ is the power normalization coefficient for keeping the transmit power constant as

$$
\gamma=\left\|\tilde{\mathbf{f}}\tilde{\mathbf{w}}\tilde{\mathbf{x}}\right\|^2. \tag{16}
$$

$\tilde{\mathbf{f}}$ is the $N_TN_c\times UN_c$ precoding matrix which transforms the resultant channel matrix $\tilde{\mathbf{h}}\tilde{\mathbf{f}}$ to a lower triangular matrix for IUI/ISI subtraction. $\tilde{\mathbf{h}}$ is the $UN_c\times N_TN_c$ channel matrix where delay waves are taken into account as

$$
\tilde{\mathbf{h}}=\begin{pmatrix} \mathbf{h}_{00} & \cdots & \mathbf{h}_{0(N_T-1)} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{(U-1)0} & \cdots & \mathbf{h}_{(U-1)(N_T-1)} \end{pmatrix}, \tag{17}
$$

with $\mathbf{h}_{un_T}$ being the $N_c\times N_c$ channel impulse response matrix between the u-th users' receive antenna and the n_T-th BS transmit antenna represented as

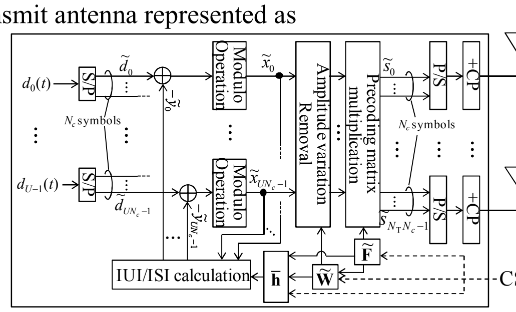

Fig. 2. Transmitter structure of TD-SC-TETHP.

$$\mathbf{h}_{un_T} = \begin{pmatrix} h_{0,un_T} & & & h_{L-1,un_T} & \cdots & h_{1,un_T} \\ h_{1,un_T} & h_{0,un_T} & & & \ddots & \vdots \\ \vdots & h_{1,un_T} & \ddots & \mathbf{0} & & h_{L-1,un_T} \\ h_{L-1,un_T} & \vdots & \ddots & h_{0,un_T} & & \\ & h_{L-1,un_T} & & h_{1,un_T} & \ddots & \\ \mathbf{0} & & \ddots & \vdots & \ddots & h_{0,un_T} \end{pmatrix}, \quad (18)$$

assuming the time delay of $l(=0\sim L-1)$-th path $\tau_l=lT_s$. h_{l,un_T} is the complex-valued path gain of the l-th path and L is the number of delay paths. The BS applies LQ decomposition to $\tilde{\mathbf{h}}$ in order to obtain $\tilde{\mathbf{f}}$ as

$$\tilde{\mathbf{h}} = \begin{pmatrix}\tilde{\mathbf{L}} & \mathbf{0}\end{pmatrix}\tilde{\mathbf{Q}} = \begin{pmatrix}\tilde{\mathbf{L}} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\tilde{\mathbf{Q}}_L \\ \tilde{\mathbf{Q}}_0\end{pmatrix}, \quad (19)$$

$$\tilde{\mathbf{f}} = \tilde{\mathbf{Q}}_L^H. \quad (20)$$

$\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{L}}$ are the unitary matrix of order N_TN_c and the lower triangular matrix of order UN_c whose diagonal elements are real numbers, respectively. $\tilde{\mathbf{Q}}_L$ and $\tilde{\mathbf{Q}}_0$ correspond to the $0\sim(UN_c-1)$-th and $UN_c\sim(N_TN_c-1)$-th rows of $\tilde{\mathbf{Q}}$, respectively. The diagonal elements of $\tilde{\mathbf{L}}$ are not constant, thereby the amplitude variation occurs in the signals of a block received at each user. After inserting a CP of N_g symbols into the GI, the BS transmits signals from each transmit antenna.

Consequently, $\tilde{\mathbf{w}}$ for equalizing all of the diagonal elements of the equivalent channel matrix to 1 can be calculated. The equivalent channel matrix $\bar{\mathbf{h}}$ of order UN_c is written from Eq. (15), (19), and (20) as

$$\begin{aligned}\bar{\mathbf{h}} &= \tilde{\mathbf{h}}\tilde{\mathbf{f}}\tilde{\mathbf{w}} = \tilde{\mathbf{L}}\,diag\{\tilde{w}_0,\cdots,\tilde{w}_{UN_c-1}\} \\ &= \begin{pmatrix}\tilde{L}_{00} & & \mathbf{0} \\ \vdots & \ddots & \\ \tilde{L}_{(UN_c-1)0} & \cdots & L_{(UN_c-1)(UN_c-1)}\end{pmatrix}\begin{pmatrix}\tilde{w}_0 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \tilde{w}_{UN_c-1}\end{pmatrix}.\end{aligned} \quad (21)$$

Thus, the matrix $\tilde{\mathbf{w}}$ of order UN_c is represented as

$$\tilde{\mathbf{w}} = diag\{\tilde{L}_{00}^{-1},\cdots,\tilde{L}_{(UN_c-1)(UN_c-1)}^{-1}\}. \quad (22)$$

The equivalent channel matrix $\bar{\mathbf{h}}$ of Eq. (22) is rewritten as

$$\bar{\mathbf{h}} = \begin{pmatrix} 1 & & & \mathbf{0} \\ \frac{\tilde{L}_{10}}{\tilde{L}_{00}} & \ddots & & \\ \vdots & \ddots & \ddots & \\ \frac{\tilde{L}_{(UN_c-1)0}}{\tilde{L}_{00}} & \cdots & \frac{\tilde{L}_{(UN_c-1)(UN_c-2)}}{\tilde{L}_{(UN_c-2)(UN_c-2)}} & 1 \end{pmatrix}. \quad (23)$$

$UN_c\times1$ vector composed of the all users' received signal blocks is expressed as $\tilde{\mathbf{r}}=[\mathbf{r}_0^T\ldots\mathbf{r}_{U-1}^T]^T$, where $\mathbf{r}_u=[r_u(0)\ldots r_u(N_c-1)]^T$ is the $N_c\times1$ receive block vector of the u-th user. If $\tilde{y}_i$ can remove IUI/ISI perfectly, $\tilde{\mathbf{r}}$ after removing the CP is represented as

$$\begin{aligned}\tilde{\mathbf{r}} &= \sqrt{\frac{2E_s}{T_s}\frac{UN_c}{\gamma}}\left(\tilde{\mathbf{d}}+\tau\tilde{\mathbf{z}}\right)+\tilde{\mathbf{n}} \\ &\equiv C\left(\tilde{\mathbf{d}}+\tau\tilde{\mathbf{z}}\right)+\tilde{\mathbf{n}}.\end{aligned} \quad (24)$$

$\tilde{\mathbf{n}}=[\mathbf{n}_0^T\ldots\mathbf{n}_{U-1}^T]^T$ is the $UN_c\times1$ noise vector, where $\mathbf{n}_u=[n_u(0)\ldots n_u(N_c-1)]^T$ is the $N_c\times1$ noise vector of the u-th user whose elements are the complex Gaussian variables having zero mean and variance $2\sigma^2=2N_0/T_s$. Each receiver does not require CSI. The received signal is divided by the signal coefficient C of Eq. (24) which is informed from the transmitter. Then, modulo operation is applied and the signals are demodulated.

Consequently, $\tilde{y}_i$ for perfectly removing IUI/ISI can be calculated. $\tilde{\mathbf{r}}$ is expressed using $\tilde{\mathbf{x}}$ as $\tilde{\mathbf{r}}=C\bar{\mathbf{h}}\tilde{\mathbf{x}}$ (without noise), i.e., the $i(=0\sim UN_c-1)$-th element $\tilde{r}_i$ of $\tilde{\mathbf{r}}$ are written as

$$\begin{aligned}\tilde{r}_i &= C\left(\tilde{d}_i+\tau\tilde{z}_i\right) \\ &= \begin{cases} C\tilde{x}_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots for\ i=0 \\ C\left(\tilde{x}_i+\sum_{j=0}^{i-1}\frac{\tilde{L}_{ij}}{\tilde{L}_{jj}}\tilde{x}_j\right) \quad \ldots\ldots\ldots\ldots for\ i>0 \end{cases}.\end{aligned} \quad (25)$$

From Eq. (25), $\tilde{y}_i$ for removing IUI/ISI perfectly is given as

$$\tilde{y}_i = \begin{cases} 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots for\ i=0 \\ \sum_{j=0}^{i-1}\frac{\tilde{L}_{ij}}{\tilde{L}_{jj}}\tilde{x}_j \quad \ldots\ldots\ldots\ldots\ldots\ldots for\ i>0 \end{cases}. \quad (26)$$

IV. COMPUTER SIMULATION RESULTS

The computer simulation condition is summarized in Table I. BS transmits signals to U=4 users using N_T=4 transmit antennas. We assume BS can ideally obtain the CSI between all users' receive antennas and the BS transmit antennas. Each user uses a receive antenna for receiving signals. We assume a frequency-selective block Rayleigh fading channel having L=8-path with uniform power delay profile.

Fig. 3 plots the cumulative distribution function (CDF) of the power normalization coefficient $\sqrt{UN_c/\gamma}$ in proposed TD-SC-TETHP. For comparison, Fig. 3 also plots that in FD-SC-TETHP and those when both schemes perform user ordering. In the user ordering, the BS applies the signal processing U! times about the all order combinations of users and transmits the signals when $\sqrt{UN_c/\gamma}$ is maximum. It is seen from Fig. 3 that TD-SC-TETHP has higher power normalization coefficient than FD-SC-TETHP. This is because TE significantly increases the power after the modulo operation in FD-SC-TETHP. On the other hand, in TD-SC-TETHP, modulo operation can suppress the power increase caused by ISI removal and the pre-removal of the amplitude variation does not significantly increase the signal power. It is also seen from Fig. 3 that the user ordering can increase $\sqrt{UN_c/\gamma}$ in both of TD and FD-SC-TETHP.

Fig. 4 plots the uncoded BER performance of TD-SC-TETHP as a function of the average transmit bit energy-to-noise power spectral density ratio (E_b/N_0). For comparison, Fig. 4 also plots those of FD-SC-TETHP and both schemes with the user ordering. It is seen from Fig. 4 that TD-SC-TETHP achieves much better BER performance than FD-SC-TETHP since the power normalization coefficient improves as shown in Fig. 3. It is also seen from Fig.4 that the user ordering can reduce the required E_b/N_0 to achieve BER=10^{-5} by about 16dB in FD-SC-TETHP and about 15dB in TD-SC-TETHP. Thus, TD-SC-TETHP can obtain as much improvement due to the user ordering as FD-SC-TETHP. It is presumed that some kinds of ordering methods in MU-MIMO downlink using THP (e.g., [10]) bring in as much improvement effect to proposed TD-SC-TETHP as FD-SC-TETHP.

V. Computational Complexity Comparison

Table II shows the number of complex multiplications in TD-SC-TETHP at the transmitter. For comparison, the number of complex multiplications in FD-SC-TETHP is also shown in Table II. Although the numbers of complex multiplications for the precoding matrix calculation and the equivalent channel matrix calculation increase with $O(N_c)$ in FD-SC-TETHP, those in TD-SC-TETHP increase with $O(N_c^3)$. The computational complexities of the precoding matrix calculation and the equivalent channel matrix calculation are dominant in all computational complexities since the numbers of complex multiplications for the others increases with $O(N_c)$ or $O(N_c^2)$. Thus, the number of complex multiplications in TD-SC-TETHP is larger than that in FD-SC-TETHP. Note that the user ordering in this paper requires the $U!$ times number of complex multiplications in Table II except for a part of DFT ($=UN_c^2$) and IDFT ($=N_TN_c^2$) in FD-SC-TETHP.

When we set computer simulation condition as Table I without user ordering, for example, the numbers of complex multiplications at transmitter in TD-SC-TETHP and FD-SC-TETHP are 16875392 and 71040, respectively. TD-SC-TETHP require approximately 240 times more complex multiplications at transmitter than that in FD-SC-TETHP.

VI. Conclusion

In this paper, we proposed TD-SC-TETHP, in which the ISI subtraction is performed together with IUI subtraction both in the time-domain, for SC-MU-MIMO downlink block transmission. We showed, by computer simulation, that TD-SC-TETHP achieves much better uncoded BER performance than FD-SC-TETHP at the cost of higher computational complexity. Theoretical analysis, the impact of channel estimation error, comparison of coded BER performance between minimum mean square error (MMSE) based TD and FD-SC-TETHP, and that between uplink and downlink transmissions are left as our important future topics.

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TABLE I. Computer Simulation Condition.

Transmitter & Receiver	Data modulation	QPSK
	Block size	N_c=64
	CP length	N_g=8
	No. of transmit antennas	N_T=4
	No. of users	U=4
	Channel estimation	Ideal
Channel model	Fading	Frequency-selective block Rayleigh
	Power delay profile	8-path uniform
	Time delay of l-th path	$\tau_l=l$ symbols

TABLE II. Number of Complex Multiplications at Transmitter.

	TD-SC-TETHP	FD-SC-TETHP
Precoding matrix calculation	$U^2N_TN_c^3$	$U^2N_TN_c$
IUI calculation		$(U-1)UN_c/2$
IUI/ISI calculation	$(UN_c-1)UN_c/2$	
Precoding matrix multiplication	$UN_TN_c^2$	UN_TN_c
DFT		$2UN_c^2$
IDFT		$(N_T+U)N_c^2$

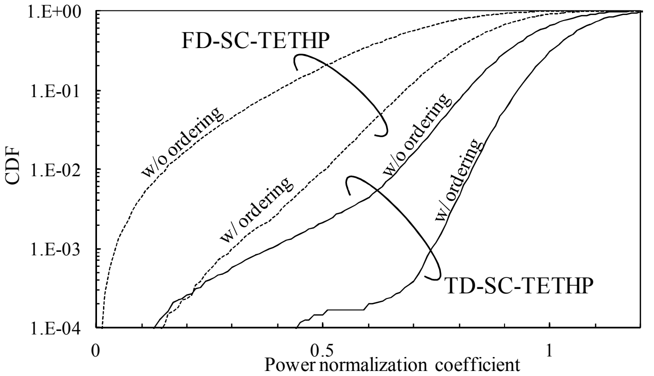

Fig. 3. CDF of power normalization coefficient.

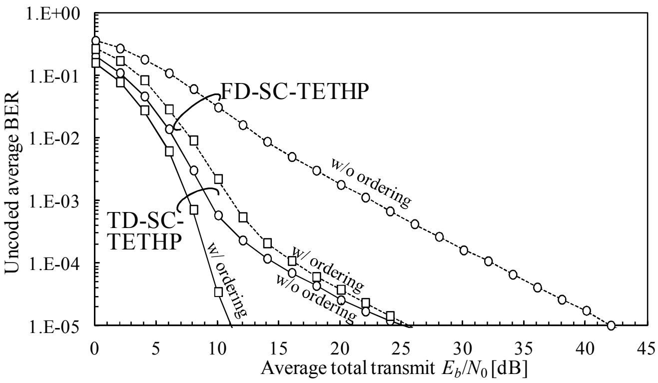

Fig. 4. Average BER performance.