Joint Tx/Rx MMSE Filtering for Single-Carrier MU-MIMO Uplink

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Abstract—**In broadband single-carrier (SC) multi-user multiple-input multiple-output (MU-MIMO) transmissions, inter-symbol interference (ISI) caused by frequency-selectivity of the channel, inter-antenna interference (IAI), and inter-user interference (IUI) degrade the transmission quality. This paper proposes a minimum mean square error based joint transmit and receive linear spatial/frequency filtering (joint Tx/Rx MMSE filtering) for SC-MU-MIMO uplink transmissions assuming that each user equipment (UE) only knows the channel state information (CSI) between itself and the base station (BS) (i.e., CSI sharing among UEs is not considered). By Jointly applying each UE's Tx filtering and BS's Rx filtering, single-user (SU) MIMO channel between each UE and the BS is transformed into multiple eigenmodes, and MMSE based Tx power allocation and Rx frequency-domain equalization (FDE) are applied to each eigenmode. As a consequence, IAI and ISI are suppressed significantly. At the same time, IUI is suppressed by BS's Rx filtering. Furthermore, adaptive rank/modulation control (ARMC) is introduced to utilize the received signal-to-interference plus noise power ratio (SINR) gap among eigenmodes. Computer simulation results confirm the effectiveness of proposed joint Tx/Rx MMSE filtering in SC-MU-MIMO uplink transmissions.**

Keywords—Spatial/frequency filtering; single-carrier; MU-MIMO; uplink;

I. INTRODUCTION

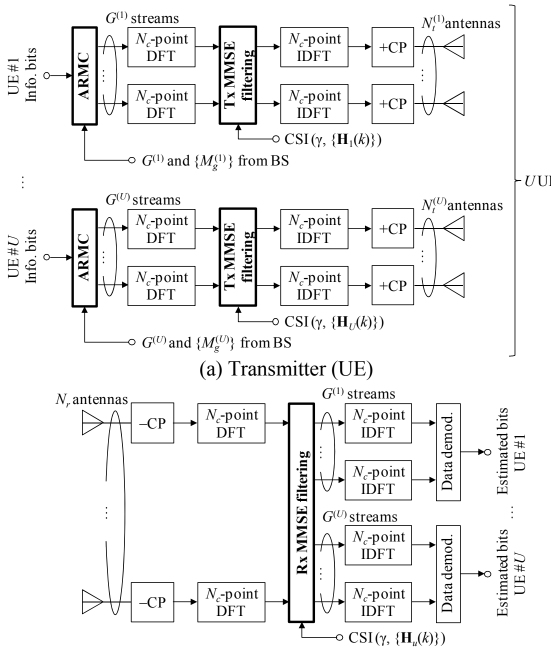
Multi-user multiple-input multiple-output (MU-MIMO) transmission [1-6], where a base station (BS) communicates with multiple user equipments (UEs) using the same time/frequency, is a powerful technique to increase the spectrum efficiency of mobile communications. Broadband single-carrier (SC)-MU-MIMO transmission obtains frequency diversity gain while the transmission quality degrades due to inter-symbol interference (ISI) caused by frequency-selectivity of the channel, inter-antenna interference (IAI), and inter-user interference (IUI) [3-5]. Minimum mean square error based linear receive spatial/frequency filtering (Rx MMSE filtering) was studied for SC-MU-MIMO uplink transmission [3], but the performance improvement is limited due to the residual ISI, IAI, and IUI after the filtering. Iterative interference cancellation [4] or maximum likelihood based detection [5] were studied for further improvement but their computational complexities are extremely high.

Recently, we proposed joint transmit and receive MMSE based linear spatial/frequency filtering (joint Tx/Rx MMSE filtering) for SC single-user (SU)-MIMO transmission [7]. In joint Tx/Rx MMSE filtering, transmitter and receiver share the channel state information (CSI) and jointly apply linear Tx/Rx filtering. Joint Tx/Rx MMSE filtering transforms the SU-MIMO channel between transmitter and receiver into multiple orthogonal channels (i.e. eigenmodes), and at the same time, applies MMSE based Tx power allocation and Rx frequency-domain equalization (FDE) to each eigenmode. As a consequence, IAI is avoided and ISI is significantly suppressed while keeping the computational complexity at the receiver low. By extending joint Tx/Rx MMSE filtering to SC-MU-MIMO uplink transmission, it is expected IAI and ISI are suppressed significantly with low complexity. However, in general, it is difficult to share the CSI among UEs in MU-MIMO transmissions, and hence it is difficult to directly apply joint Tx/Rx MMSE filtering for SC-SU-MIMO to SC-MU-MIMO uplink transmission.

In this paper, we propose a new joint Tx/Rx MMSE filtering for SC-MU-MIMO uplink transmission assuming that each UE only knows the CSI between itself and the BS (i.e., CSI sharing among UEs is not considered). By jointly applying each UE's Tx filtering and BS's Rx filtering, SU-MIMO channel between each UE and the BS is transformed into multiple eigenmodes, and MMSE based Tx power allocation and Rx FDE are applied to each eigenmode. As a consequence, IAI and ISI are suppressed significantly. At the same time, IUI is suppressed by BS's Rx filtering. Note that, unlike SU-MIMO case, it is difficult to decompose the MU-MIMO channel matrix between all UEs and BS to the form of (unitary matrix) × (diagonal matrix) × (unitary matrix) since it requires all CSI at each UE. Therefore, it is difficult to perfectly orthogonalize the MU-MIMO channel by multiplying the Hermitian transpose of the left unitary matrix (i.e., to perfectly remove IAI and IUI). Although residual IAI and IUI remain, there exists large received signal-to-interference plus noise power ratio (SINR) gap among eigenmodes. Therefore, the eigenmodes which have low received SINR limit the overall transmission performance improvement if the same data modulation is applied to all eigenmodes. To avoid this problem and utilize the received SINR gap, adaptive rank/modulation control (ARMC) [7] which adaptively controls the number of eigenmodes to be used for each UE (i.e. rank) and the modulation level on each eigenmode is introduced.

The remainder of this paper is organized as follows. Sect. II presents the system model and signal representation for SC-MU-MIMO uplink transmission with joint Tx/Rx MMSE filtering. Sect. III formulates the optimization problem which minimizes the MSE between the input to each UE's Tx filtering and the output from BS's Rx filtering, and derives the optimal Tx/Rx filter matrices in closed-form. Sect. IV shows the computer simulation results, and Sect. V gives the concluding remarks.

Notations: $E[.]$, diag[.], tr[.], $[.]^T$, and $[.]^H$ denote the ensemble average operation, diagonal matrix, the trace operation, the transpose operation, and the Hermitian transpose operation, respectively. erfc(.) and $(x)^+$ denote the complementary error function and max(0, x), respectively. $\mathbf{I}_N$ is the $N\times N$ identity matrix and $\mathbf{0}_{N\times M}$ is the $N\times M$ zero-matrix.



(a) Transmitter (UE)

(b) Receiver (BS)

Figure 1. System model.

II. SC-MU-MIMO Uplink Transmission with Joint Tx/Rx MMSE Filtering

Fig. 1 shows the system model of SC-MU-MIMO uplink transmission with joint Tx/Rx MMSE filtering. U UEs communicate with a BS which has N_r Rx antennas. UE#u (=1~U) transmits $G^{(u)}$ data streams using $N_t^{(u)} \geq G_u$ Tx antennas with $N_r \geq N_t = \sum_{u=1}^{U} N_t^{(u)} \geq G = \sum_{u=1}^{U} G^{(u)}$. The detail of ARMC which determines {$G^{(u)}$} and the number {$M_g^{(u)}$} of bits in each data symbol of the g(=1~$G^{(u)}$)-th data stream (i.e. modulation level) is described in Sect. II-B.

A. Tx/Rx signals

At the UE#u, information bit sequence is data-modulated to $G^{(u)}$ data symbol sequences by ARMC. Each symbol sequence is divided to N_c-symbol blocks, where N_c is the size of discrete Fourier transform (DFT) and inverse DFT (IDFT), and each symbol block is transformed into a frequency-domain symbol block by N_c-point DFT. The Tx symbol vector $\mathbf{S}_u(k)\in\mathbb{C}^{N_t^{(u)}\times 1}$ at the k(=1~N_c)-th frequency is obtained by applying the Tx MMSE filtering to the frequency-domain data symbol vector $\mathbf{D}_u(k)\in\mathbb{C}^{G^{(u)}\times 1}$ at the k-th frequency, which is expressed as

$$\mathbf{S}_u(k)=[S_1^{(u)}(k)\cdots S_{N_t^{(u)}}^{(u)}(k)]^T=\sqrt{\frac{2E_s}{T_s}}\mathbf{W}_{t,u}(k)\mathbf{D}_u(k), \quad (1)$$

where E_s, T_s, and $\mathbf{W}_{t,u}(k)\in\mathbb{C}^{N_t^{(u)}\times G^{(u)}}$ are the average Tx symbol energy, symbol duration, and Tx filter matrix, respectively. N_c-point IDFT is applied to each Tx symbol block { $S_n^{(u)}(k)$; k=1~N_c}, $n=1\sim N_t^{(u)}$, to transform back into time-domain Tx blocks. Finally, the last N_g symbols of each Tx block are copied as a cyclic prefix (CP) and inserted into the guard interval (GI) at the beginning of each Tx block and then transmitted from $N_t^{(u)}$ antennas.

At the BS, each CP is removed from the signal blocks received by N_r antennas and then, each block is transformed into the frequency-domain Rx signal block by N_c-point DFT. The frequency-domain received signal vector $\mathbf{R}(k)\in\mathbb{C}^{N_r\times 1}$ at the k-th frequency after N_c-point DFT is expressed as

$$\begin{aligned}\mathbf{R}(k)&=\sum_{u=1}^{U}\mathbf{H}_u(k)\mathbf{S}_u(k)+\mathbf{Z}(k)\\&=\sqrt{\frac{2E_s}{T_s}}\left[\mathbf{H}_1(k)\mathbf{W}_{t,1}(k)\cdots\mathbf{H}_U(k)\mathbf{W}_{t,U}(k)\right]\begin{bmatrix}\mathbf{D}_1(k)\\ \vdots\\ \mathbf{D}_U(k)\end{bmatrix}+\mathbf{Z}(k)\\&\equiv\sqrt{\frac{2E_s}{T_s}}\left[\mathbf{H}_1(k)\mathbf{W}_{t,1}(k)\cdots\mathbf{H}_U(k)\mathbf{W}_{t,U}(k)\right]\mathbf{D}(k)+\mathbf{Z}(k),\end{aligned} \quad (2)$$

where $\mathbf{H}_u(k)\in\mathbb{C}^{N_r\times N_t^{(u)}}$ is the channel matrix between UE#u and BS and $\mathbf{Z}(k)\in\mathbb{C}^{N_r\times 1}$ is the noise vector whose elements are zero-mean complex-valued random variables having variance $2N_0/T_s$ with N_0 being the one-sided power spectrum density of additive white Gaussian noise (AWGN). The frequency-domain soft-output symbol vector $\hat{\mathbf{D}}(k)\in\mathbb{C}^{G\times 1}$ is obtained by applying the Rx MMSE filtering on $\mathbf{R}(k)$ as

$$\begin{aligned}\hat{\mathbf{D}}(k)&=[\hat{D}_1(k)\cdots\hat{D}_G(k)]^T=[\hat{\mathbf{D}}_1^T(k)\cdots\hat{\mathbf{D}}_U^T(k)]^T\\&=\mathbf{W}_r(k)\mathbf{R}(k)\equiv\begin{bmatrix}\mathbf{W}_{r,1}(k)\\ \vdots\\ \mathbf{W}_{r,U}(k)\end{bmatrix}\mathbf{R}(k),\end{aligned} \quad (3)$$

where $\mathbf{W}_r(k)\in\mathbb{C}^{G\times N_r}$ is the Rx filter matrix and $\hat{\mathbf{D}}_u(k)\in\mathbb{C}^{G^{(u)}\times 1}$ is the soft-output symbol vector corresponding to $\mathbf{D}_u(k)$. $\mathbf{W}_{r,u}(k)$ is the submatrix of $\mathbf{W}_r(k)$ corresponding to UE#u's signal. N_c-point IDFT is applied to each frequency-domain soft-output symbol block { $\hat{D}_g(k)$; k=1~N_c}, g=1~G and then, the time-domain soft-output symbol blocks are obtained.

B. ARMC

In this paper, the number $G^{(u)}$ of data streams (i.e. rank) and the modulation level (i.e. $M_g^{(u)}$) on each data stream of UE#u are determined based on minimum bit error ratio (BER) criterion.

UE#u's received SINR $\Gamma_n^{(u)}$ of the $n(=1\sim N_t^{(u)})$ -th data stream is expressed as (the derivation is omitted)

$$\Gamma_n^{(u)}=\frac{\left|\tilde{H}_n^{(u)}\right|^2}{\mu_{\mathrm{ISI},n}^{(u)}+\mu_{\mathrm{IAI},n}^{(u)}+\mu_{\mathrm{IUI},n}^{(u)}+\mu_{\mathrm{noise},n}^{(u)}}, \quad (4)$$

$$\begin{cases}\tilde{H}_n^{(u)}=\dfrac{1}{N_c}\displaystyle\sum_{k=1}^{N_c}\hat{H}_{n,n}^{(u)}(k)\\ \hat{H}_{n,n''}^{(u)}(k)=\displaystyle\sum_{m=1}^{N_r}W_{r,n,m}^{(u)}(k)\sum_{n'=1}^{N_t^{(u)}}H_{m,n'}^{(u)}(k)W_{t,n',n''}^{(u)}(k)\end{cases}, \quad (5)$$

where $W_{r,n,m}^{(u)}(k)$, $H_{m,n}^{(u)}(k)$, and $W_{t,n,n'}^{(u)}(k)$ are the (n,m)-th, (m,n)-th, and (n,n')-th elements of $\mathbf{W}_{r,u}(k)$, $\mathbf{H}_u(k)$, and $\mathbf{W}_{t,u}(k)$, respectively. $\mu_{\mathrm{ISI},n}^{(u)}$, $\mu_{\mathrm{IAI},n}^{(u)}$, $\mu_{\mathrm{IUI},n}^{(u)}$, and $\mu_{\mathrm{noise},n}^{(u)}$ are the variances of normalized residual ISI, IAI, IUI, and noise, respectively, expressed as

$$\begin{cases} \mu_{\mathrm{ISI},n}^{(u)} = \dfrac{1}{N_c}\displaystyle\sum_{k=1}^{N_c}\left|\hat{H}_{n,n}^{(u)}(k)\right|^2 - \left|\widetilde{H}_n^{(u)}\right|^2 \\ \mu_{\mathrm{IAI},n}^{(u)} = \dfrac{1}{N_c}\displaystyle\sum_{\substack{n''=1 \\ n''\neq n}}^{N_t^{(u)}}\sum_{k=1}^{N_c}\left|\hat{H}_{n,n''}^{(u)}(k)\right|^2 \\ \mu_{\mathrm{IUI},n}^{(u)} = \dfrac{1}{N_c}\displaystyle\sum_{\substack{u'=1 \\ u'\neq u}}^{U}\sum_{n''=1}^{N_t^{(u')}}\sum_{k=1}^{N_c}\left|\sum_{m=1}^{N_r}W_{r,n,m}^{(u)}(k)\sum_{n'=1}^{N_t^{(u')}}H_{m,n'}^{(u')}(k)W_{t,n',n''}^{(u')}(k)\right|^2 \\ \mu_{\mathrm{noise},n}^{(u)} = \dfrac{\gamma^{-1}}{N_c}\displaystyle\sum_{m=1}^{N_r}\sum_{k=1}^{N_c}\left|W_{r,n,m}^{(u)}(k)\right|^2 \end{cases}, \quad (6)$$

where $\gamma = E_s/N_0$.

When Gray code mapping is used and if the residual ISI + IAI + IUI + noise can be approximated as a complex-valued random variable, the conditional BER $p_{b,n}^{(u)}$ of UE#u's n-th data stream for a given set of modulation level and received SINR $\Gamma_n^{(u)}$ is given as [8]

$$p_{b,n}^{(u)} = a_n^{(u)}\mathrm{erfc}\left(\sqrt{\frac{\Gamma_n^{(u)}}{b_n^{(u)}}}\right), \quad (7)$$

where $a_n^{(u)}$ and $b_n^{(u)}$ are shown in Table I for various modulation levels. When $M_g^{(u)}$ bits are allocated to the symbol of the g(=1~$G^{(u)}$)-th eigenmode of UE#u, the conditional BER P_b averaged over UEs is given as

$$P_b = \frac{1}{U}\sum_{u=1}^{U}\frac{\sum_{g=1}^{G^{(u)}}M_g^{(u)}p_{b,g}^{(u)}}{\sum_{g=1}^{G^{(u)}}M_g^{(u)}} = \frac{1}{U}\sum_{u=1}^{U}\frac{1}{\eta^{(u)}}\sum_{g=1}^{G^{(u)}}a_g^{(u)}\mathrm{erfc}\left(\sqrt{\frac{\Gamma_g^{(u)}}{b_g^{(u)}}}\right), \quad (8)$$

where $\eta^{(u)} = \sum_{g=1}^{G^{(u)}} M_g^{(u)}$ is the data rate (bps/Hz) of UE#u.

The rank $G^{(u)}$ and modulation levels (i.e. $\{M_g^{(u)}\}$) for $G^{(u)}$ data streams are jointly determined as follows. At first, $\Gamma_n^{(u)}$ is calculated from Eq. (4) for all possible combinations of $\{G^{(u)}\}$. Second, for the given data rate $\{\eta^{(u)}\}$, the average conditional BER is calculated from Eq. (8) for all possible combinations of $\{G^{(u)}\}$ and $\{M_g^{(u)}\}$, and then, the optimal combination which minimizes the average conditional BER is found. The optimal combination is informed to each UE and then ARMC is carried out.

TABLE I. $a_n^{(u)}$ AND $b_n^{(u)}$.

Data modulation	$a_n^{(u)}$	$b_n^{(u)}$
BPSK	1/2	1
QPSK	1/2	2
8PSK	1/3	$1/\sin^2(\pi/8)$
16QAM	3/8	10
64QAM	7/24	42
256QAM	15/64	170

III. DERIVATION OF OPTIMAL TX/RX FILTERS

In this section, we derive the optimal Tx and Rx filters which minimize the total MSE of the blocks between the Tx data symbol vector $\mathbf{D}(k)$ and the soft-output symbol vector $\hat{\mathbf{D}}(k)$. For the preparation, some developments of formula are introduced.

$\mathbf{H}_u(k)$ is rewritten by singular value decomposition [9] as

$$\begin{aligned} \mathbf{H}_u(k) &= \mathbf{U}_u(k)\begin{bmatrix}\mathbf{\Lambda}_u^{1/2}(k) \\ \mathbf{0}_{(N_r-N_t^{(u)})\times N_t^{(u)}}\end{bmatrix}\mathbf{V}_u^H(k) \\ &= \begin{bmatrix}\mathbf{U}_{\mathrm{signal},u}(k) & \mathbf{U}_{\mathrm{null},u}(k)\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_u^{1/2}(k) \\ \mathbf{0}_{(N_r-N_t^{(u)})\times N_t^{(u)}}\end{bmatrix}\mathbf{V}_u^H(k) \quad (9) \\ &= \mathbf{U}_{\mathrm{signal},u}(k)\mathbf{\Lambda}_u^{1/2}(k)\mathbf{V}_u^H(k), \end{aligned}$$

where $\mathbf{U}_u(k) \in \mathbb{C}^{N_r\times N_r}$ and $\mathbf{V}_u(k) \in \mathbb{C}^{N_t^{(u)}\times N_t^{(u)}}$ are the unitary matrices whose columns consist of left and right singular vectors of $\mathbf{H}_u(k)$, respectively. $\mathbf{U}_{\mathrm{signal},u}(k) \in \mathbb{C}^{N_r\times N_t^{(u)}}$ and $\mathbf{U}_{\mathrm{null},u}(k) \in \mathbb{C}^{N_r\times(N_r-N_t^{(u)})}$ are the matrices which express the column space and left null-space of $\mathbf{H}_u(k)$, respectively. Here, we assume the rank of $\mathbf{H}_u(k)$ is $N_t^{(u)}$ (i.e. full rank). $\mathbf{\Lambda}_u(k) \in \mathbb{R}^{N_t^{(u)}\times N_t^{(u)}}$ is the diagonal matrix whose $n(=1\sim N_t^{(u)})$-th diagonal component $\Lambda_n^{(u)}(k)$ has the square value of the n-th singular value of $\mathbf{H}_u(k)$. By substituting Eq. (9) and the Tx filter matrix $\mathbf{W}_{t,u}(k) = \mathbf{V}_u(k)\mathbf{P}_u^{1/2}(k)$ which obtains eigenmode transmission [7], Eq. (2) is rewritten as

$$\begin{aligned} \mathbf{R}(k) &= \sqrt{\frac{2E_s}{T_s}}\begin{bmatrix}\mathbf{U}_{\mathrm{signal},1}(k)\cdots\mathbf{U}_{\mathrm{signal},U}(k)\end{bmatrix} \\ &\quad\times\begin{bmatrix}\mathbf{\Lambda}_1^{1/2}(k)\mathbf{P}_1^{1/2}(k) & & \\ & \ddots & \\ & & \mathbf{\Lambda}_U^{1/2}(k)\mathbf{P}_U^{1/2}(k)\end{bmatrix}\mathbf{D}(k) + \mathbf{Z}(k) \quad (10) \\ &\equiv \sqrt{\frac{2E_s}{T_s}}\mathbf{U}_{\mathrm{signal}}(k)\mathbf{Q}(k)\mathbf{D}(k) + \mathbf{Z}(k), \end{aligned}$$

where $\mathbf{P}_u(k) \in \mathbb{R}^{N_t^{(u)}\times G^{(u)}}$ is the matrix whose g-th diagonal element $P_g^{(u)}(k)$ gives the power allocation to the g-th eigenmode and any others are zero. In SC-SU-MIMO transmission [7], the matrix corresponding to $\mathbf{U}_{\mathrm{signal}}(k)$ in Eq. (10) is a unitary matrix. Therefore, the SU-MIMO channel is perfectly diagonalized (i.e. IAI is perfectly removed) by multiplying the Hermitian transpose of the unitary matrix at the receiver. However, the MU-MIMO channel cannot be diagonalized (i.e. IAI and IUI cannot be perfectly removed) since $\mathbf{U}_{\mathrm{signal}}(k)$ is not unitary matrix. In [6], a scheme to remove IAI and IUI perfectly by multiplying the pseudo-inverse matrix of $\mathbf{U}_{\mathrm{signal}}(k)$ (i.e. $\left(\mathbf{U}_{\mathrm{signal}}^H(k)\mathbf{U}_{\mathrm{signal}}(k)\right)^{-1}\mathbf{U}_{\mathrm{signal}}^H(k)$) at BS was proposed for narrowband MU-MIMO transmission. However, ISI enhancement occurs since the scheme is a zero-forcing (ZF) based Rx filtering.

In this paper, therefore, the Tx/Rx filtering which minimizes the total MSE between $\mathbf{D}(k)$ and $\hat{\mathbf{D}}(k)$ is applied.

The minimization of total MSE among UEs under per UE Tx power constraint is formulated as

(P0) $$\min_{\{\mathbf{W}_{t,u}(k),\mathbf{W}_r(k)\}} \varepsilon \tag{11a}$$

$$\text{s.t.} \quad \sum_{k=1}^{N_c} \mathrm{tr}\left(\mathbf{W}_{t,u}(k)\mathbf{W}_{t,u}^H(k)\right) \le N_t^{(u)} N_c, \quad \forall u \tag{11b}$$

where ε is the total MSE between $\mathbf{D}(k)$ and $\hat{\mathbf{D}}(k)$ defined as

$$\varepsilon \equiv E\left[\sum_{k=1}^{N_c} \mathrm{tr}\left\{\left(\mathbf{D}(k) - \hat{\mathbf{D}}(k)\Big/\sqrt{\frac{2E_s}{T_s}}\right)\left(\mathbf{D}(k) - \hat{\mathbf{D}}(k)\Big/\sqrt{\frac{2E_s}{T_s}}\right)^H\right\}\right]. \tag{12}$$

By substituting Eqs.(3) and (10), Eq. (12) is rewritten as

$$\begin{aligned}\varepsilon = &\sum_{k=1}^{N_c} \mathrm{tr}\left\{\begin{matrix}\left(\mathbf{I}_G - \mathbf{W}_r(k)\mathbf{U}_{\text{signal}}(k)\mathbf{Q}(k)\right) \\ \times\left(\mathbf{I}_G - \mathbf{W}_r(k)\mathbf{U}_{\text{signal}}(k)\mathbf{Q}(k)\right)^H\end{matrix}\right\} \\ &+ \gamma^{-1}\sum_{k=1}^{N_c} \mathrm{tr}\left(\mathbf{W}_r(k)\mathbf{W}_r^H(k)\right),\end{aligned} \tag{13}$$

where $E[\mathbf{D}(k)\mathbf{D}^H(k)]=\mathbf{I}_G$ and $E[\mathbf{Z}(k)\mathbf{Z}^H(k)]=(2N_0/T_s)\mathbf{I}_{N_r}$.

At first, the optimal Rx filter matrix $\mathbf{W}_r^*(k)$ is derived assuming Tx filter matrices $\{\mathbf{W}_{t,u}(k)\}$ are given (i.e., $\mathbf{Q}(k)$ is given). Eq. (13) is minimized when $\partial\varepsilon/\partial\mathbf{W}_r(k)=0$ since it is convex on $\mathbf{W}_r(k)$ (the proof is omitted). Therefore, $\mathbf{W}_r^*(k)$ which satisfies the above condition is given by

$$\begin{aligned}\mathbf{W}_r^*(k) = &\left(\mathbf{Q}^T(k)\mathbf{U}_{\text{signal}}^H(k)\mathbf{U}_{\text{signal}}(k)\mathbf{Q}(k) + \gamma^{-1}\mathbf{I}_G\right)^{-1} \\ &\times \mathbf{Q}^T(k)\mathbf{U}_{\text{signal}}^H(k).\end{aligned} \tag{14}$$

Then, by substituting Eq. (14) into Eq. (13) and using the matrix inversion lemma [9], the optimization problem (P0) is rewritten as

(P1) $$\min_{\{\mathbf{P}_u(k)\}} \varepsilon = \sum_{k=1}^{N_c} \mathrm{tr}\left\{\gamma^{-1}\begin{pmatrix}\mathbf{U}_{\text{signal}}^H(k)\mathbf{U}_{\text{signal}}(k) \\ \times\mathbf{Q}(k)\mathbf{Q}^T(k) + \gamma^{-1}\mathbf{I}_{N_t}\end{pmatrix}^{-1}\right\} \tag{15a}$$

$$\text{s.t.} \quad \sum_{k=1}^{N_c} \mathrm{tr}\left(\mathbf{P}_u(k)\mathbf{P}_u^T(k)\right) \le N_t^{(u)} N_c, \quad \forall u \tag{15b}$$

In Eq. (15a), the non-diagonal elements in $\mathbf{U}_{\text{signal}}^H(k)\mathbf{U}_{\text{signal}}(k)$ express IAI and IUI terms, and hence, it is difficult to derive $\{\mathbf{P}_u(k)\}$ which minimize the trace in closed-form.

Although it is possible to obtain the optimal $\{\mathbf{P}_u(k)\}$ using some iterative algorithms such as gradient method, it is impractical since all UEs are required to share the overall CSI with each other.

In this paper, therefore, $\{\mathbf{P}_u(k)\}$ which minimize the virtual MSE assuming IAI and IUI are perfectly removed at BS is derived. Each diagonal element of $\mathbf{U}_{\text{signal}}^H(k)\mathbf{U}_{\text{signal}}(k)$ is 1 due to the property of singular vector, and hence, the minimization of the virtual MSE of UE#u assuming IAI and IUI are perfectly removed at BS is formulated as

(P2) $$\min_{\{\mathbf{P}_u(k)\}} \hat{\varepsilon}^{(u)} = \sum_{k=1}^{N_c}\sum_{g=1}^{G^{(u)}} \frac{\gamma^{-1}}{\Lambda_g^{(u)}(k)P_g^{(u)}(k) + \gamma^{-1}}, \quad \forall u \tag{16}$$

s.t. (15b).

The g-th diagonal element in the optimal $\{\mathbf{P}_u^*(k)\}$ which satisfies KKT condition [10] is derived as

$$P_g^{(u)*}(k) = \left(\frac{1}{\sqrt{\mu^{(u)}}}\frac{1}{\sqrt{\gamma\Lambda_g^{(u)}(k)}} - \frac{1}{\gamma\Lambda_g^{(u)}(k)}\right)^+ \tag{17}$$

where $\mu^{(u)}$ is chosen to satisfy the constraint Eq. (15b).

IV. Computer Simulation

In computer simulation, BS has N_r=4 Rx antennas and U=2 UEs have $N_t^{(u)}=2$ Tx antennas each. Therefore, each UE transmits $G^{(u)}$=1 or 2 data streams simultaneously. The modulation level of each data stream is selected from BPSK, QPSK, 8PSK, 16QAM, 64QAM, and 256QAM as shown in Table I, and then informed from BS to each UE with $G^{(u)}$. The size N_c=128 of DFT/IDFT and the GI length N_c=16. The channel is assumed to be a 16-path frequency-selective block Rayleigh fading having uniform power delay profile. Uncorrelated fading among paths/antennas/UEs is assumed. Each UE knows the perfect CSI between itself and BS, and BS knows the perfect CSI between itself and all UEs.

Fig. 2 shows the average BER performance of SC-MU-MIMO uplink transmission with proposed joint Tx/Rx MMSE filtering ("Proposed") when $\eta^{(u)}$=4 or 8 (bps/Hz) for all UEs. The average BER performance with joint Tx/Rx filtering based on [6] ("ZF-based": the detail is described in Appendix) and conventional Rx MMSE filtering ("Conv.") are also shown for comparison. Note that $G^{(u)}$ and modulation levels of each UE in ZF-based are controlled by ARMC as Proposed, but in Conv., QPSK (when $\eta^{(u)}$=4 (bps/Hz) for all UEs) or 16QAM (when $\eta^{(u)}$=8 (bps/Hz) for all UEs) are applied to $G^{(u)}$= $N_t^{(u)}$ =2 data streams.

It is seen from Fig. 2 that Proposed has better average BER performance than ZF-based and Conv. In Conv., the residual ISI, IAI, and IUI after the filtering limit the performance improvement. On the other hand, in Proposed, by jointly applying each UE's Tx filtering and BS's Rx filtering, SU-MIMO channel between each UE and BS is transformed into multiple eigenmodes, and MMSE based Tx power allocation and Rx FDE are applied to each eigenmode. Therefore, IAI and ISI are suppressed significantly. At the same time, IUI is suppressed by BS's Tx filtering. Furthermore, ARMC utilizes the received SINR gap among eigenmodes (the detail is discussed later). As a consequence, the BER performance is improved significantly. Although ZF-based also transforms the SU-MIMO channel between each UE and BS into multiple eigenmodes and applies MMSE based Tx power allocation and Rx FDE to each eigenmode, ISI enhancement occurs (corresponding to the second term of the denominator in Eq. (A.3)) since it applies the Rx filtering which perfectly removes IAI and IUI. Therefore, the performance cannot be improved even using ARMC.

Fig. 3 shows the distribution of selected rank and modulation levels of each UE. It is seen from Fig. 3 that all Tx bits are allocated to the first eigenmode with high probability. This is because more BER can be reduced by allocating all Tx power to the first eigenmode than allocating Tx power and bits to the second eigenmode which has lower received SINR as well since the first eigenmode has $N_t^{(u)} \times N_r$ -th spatial diversity gain.

V. CONCLUSION

In this paper, we proposed joint Tx/Rx MMSE filtering for SC-MU-MIMO uplink transmission assuming that each UE only knows the CSI between itself and BS (i.e., CSI sharing among UEs is not considered). Computer simulation results showed that proposed joint Tx/Rx MMSE filtering achieves better average BER performance than the joint Tx/Rx filtering which applies ZF-based Rx filtering and conventional Rx MMSE filtering.

REFERENCES

[1] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*, Cambridge University Press, 2007.

[2] D. Gesbert, M. Kountouris, R. W. Heath Jr., C.-B. Chae, and T. Sälzer, "Shifting the MIMO paradigm," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 36-46, Oct. 2007.

[3] T. Chiba, Kazuaki Takeda, Kazuki Takeda, and F. Adachi, "Uplink capacity of a cellular system using multi-user single-carrier MIMO multiplexing combined with frequency-domain equalization and transmit power control," *Wirel. Pers. Commun.*, vol. 58, no. 3, pp. 455-468, Jun. 2011.

[4] S. Okuyama, T. Yamamoto, Kazuki Takeda, and F. Adachi, "Iterative MMSE detection with interference cancellation for up-link HARQ using frequency-domain filtered SC-FDMA MIMO multiplexing," *IEICE Trans. Commun.*, vol. E94-B, no. 12, pp. 3559-3568, Dec. 2011.

[5] T. Yamamoto, K. Adachi, S. Sun, and F. Adachi, "Recursive QR packet combining for uplink single-carrier multi-user MIMO HARQ using near ML detection," *Wirel. Commun. Mob. Comput.*, vol. 12, no. 18, pp. 1652-1663, Dec. 2012.

[6] W. Liu, L. L. Yang, and L. Hanzo, "SVD-assisted multiuser transmitter and multiuser detector design for MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 1016-1021, Feb. 2009.

[7] S. Kumagai, T. Obara, T. Yamamoto, and F. Adachi, "Joint Tx/Rx MMSE filtering for single-carrier MIMO transmission," *IEICE Trans. Commun.*, vol. E97-B, no. 9, pp. 1967-1976, Sep. 2014.

[8] J. G. Proakis and M. Salehi, *Digital Communications*, 5th ed., McGraw-Hill, 2008.

[9] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge, 1985.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, 2006.

APPENDIX: JOINT TX/RX FILTERING BASED ON [6]

In [6], joint Tx/Rx MMSE filtering which perfectly removes IAI and IUI when perfect CSI is available was proposed for narrowband MU-MIMO uplink. Here we extend this method to broadband SC-MU-MIMO uplink transmission.

When Tx filter matrix of UE#u is given by $\mathbf{W}_{t,u}(k) = \mathbf{V}_u(k)\mathbf{P}_u^{1/2}(k)$ as proposed method, Rx filter matrix $\mathbf{W}_r(k) \in \mathbb{C}^{G \times N_r}$ at the BS is expressed using the matrix $\mathbf{W}_{\mathrm{IAI/IUI}}(k) \in \mathbb{C}^{G \times N_r}$ which removes IAI and IUI perfectly and the diagonal matrix $\mathbf{W}_{\mathrm{FDE}}(k) \in \mathbb{R}^{G \times G}$ which applies Rx FDE to each eigenmode as

$$\mathbf{W}_r(k) = \mathbf{W}_{\mathrm{FDE}}(k)\mathbf{W}_{\mathrm{IAI/IUI}}(k), \quad \text{(A.1)}$$

$$\begin{cases} \mathbf{W}_{\mathrm{IAI/IUI}}(k) = \left(\mathbf{U}_{\mathrm{signal}}^H(k)\mathbf{U}_{\mathrm{signal}}(k)\right)^{-1}\mathbf{U}_{\mathrm{signal}}(k) \\ \mathbf{W}_{\mathrm{FDE}}(k) = \mathrm{diag}\left[\dfrac{\sqrt{\Lambda_1^{(1)}(k)P_1^{(1)}(k)}}{\Lambda_1^{(1)}(k)P_1^{(1)}(k) + \gamma^{-1}\left\|\mathbf{W}_{\mathrm{IAI/IUI},1}(k)\right\|^2} \cdots \dfrac{\sqrt{\Lambda_{G^{(U)}}^{(U)}(k)P_{G^{(U)}}^{(U)}(k)}}{\Lambda_{G^{(U)}}^{(U)}(k)P_{G^{(U)}}^{(U)}(k) + \gamma^{-1}\left\|\mathbf{W}_{\mathrm{IAI/IUI},G}(k)\right\|^2}\right] \end{cases}, \quad \text{(A.2)}$$

where $\|\mathbf{W}_{\mathrm{IAI/IUI},g}\|^2$ is the square value of the Euclidean norm of the g-th row vector of $\mathbf{W}_{\mathrm{IAI/IUI}}(k)$. By substituting Eq. (A.1) into Eq. (13), the total MSE $\varepsilon^{(u)}$ of UE#u is expressed as

$$\varepsilon^{(u)} = \sum_{k=1}^{N_c}\sum_{g=1}^{G^{(u)}} \frac{\gamma^{-1}}{\Lambda_g^{(u)}(k)P_g^{(u)}(k) + \gamma^{-1}\left\|\mathbf{W}_{\mathrm{IAI/IUI},\sum_{u'=1}^{u-1}G^{(u')}+g}\right\|^2}, \forall u. \quad \text{(A.3)}$$

It is impractical to calculate $\|\mathbf{W}_{\mathrm{IAI/IUI},g}\|^2$ in Eq. (A.3) at each UE since it requires all UEs' CSI. In this paper, therefore, $\{\mathbf{P}_u(k)\}$ which minimize Eq. (A.3) assuming $\|\mathbf{W}_{\mathrm{IAI/IUI},g}\|^2=1$, $\forall g$, are used. In this case, the optimization problem to derive the optimal $\{\mathbf{P}_u(k)\}$ is the same as (P2), and hence, the g-th diagonal element of $\mathbf{P}_u(k)$ is given by Eq. (17) as the proposed method.

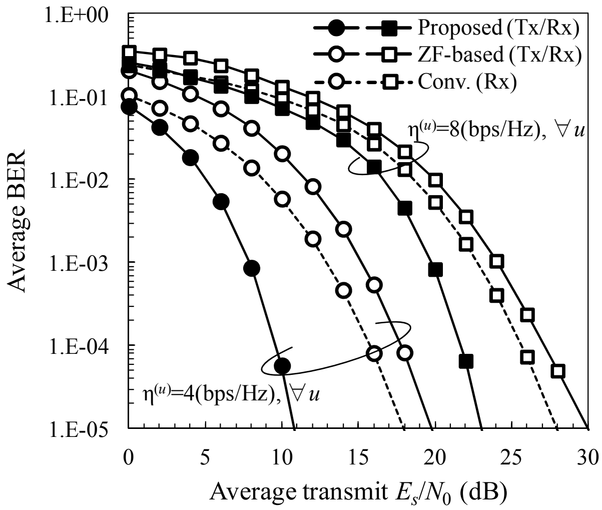
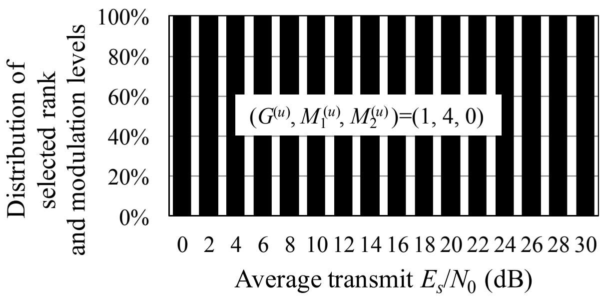
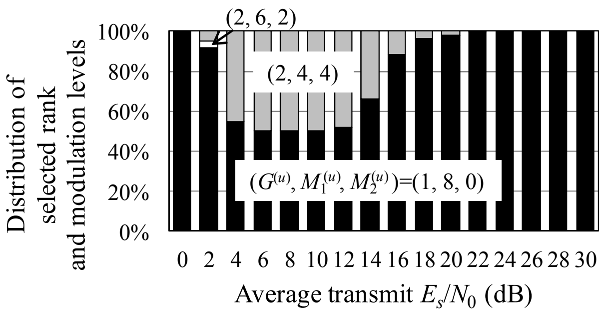


Figure 2. Average BER performance.



(a) $\eta^{(u)}$=4 (bps/Hz) for all UEs



(b) $\eta^{(u)}$=8 (bps/Hz) for all UEs

Figure 3. Distribution of selected rank and modulation levels.