Beamspace Channel Estimation for 3D Lens-Based Millimeter-Wave Massive MIMO Systems

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***Abstract*—Lens-based mmWave massive MIMO can significantly reduce the number of required RF chains without obvious performance loss, where the accurate information of beamspace channel is required. However, existing beamspace channel estimation schemes are based on the 2D beamspace channel model. In this paper, we consider the more general 3D beamspace channel model, and propose an adaptive support detection (ASD)-based channel estimation scheme. The basic idea is to decompose the 3D beamspace channel estimation problem into several sub-problems, and each one only deals with a sparse channel component. For each channel component, we first adaptively detect its support with high accuracy by exploiting the horizontal and vertical sparsity of 3D beamspace channel. Then, we remove the influence of this channel component to detect the support of the next channel component. After the support detections of all channel components, we can estimate the nonzero elements of the beamspace channel with low pilot overhead. Simulation results verify that the proposed scheme enjoys satisfying accuracy, even with low SNR.**

I. Introduction

Millimeter-wave (mmWave) massive multiple-input multiple-output (MIMO) has been regarded as a promising technique for 5G wireless communications [1], since it can considerably increase the data rate thanks to its higher spectrum efficiency and wider bandwidth. However, in traditional MIMO systems, each antenna usually requires one dedicated radio-frequency (RF) chain. Due to the large number of antennas [1] and the high energy consumption of RF chain [2], mmWave massive MIMO systems will involve unaffordable hardware complexity and energy consumption. To solve this problem, lens-based mmWave massive MIMO has been recently proposed [3]. By employing the lens antenna array, the signals from different directions (beams) can be concentrated on different antennas, which means that the traditional spatial channel is transformed into beamspace [4]. As the number of effective prorogation paths in mmWave communications is limited [2], the beamspace channel is sparse [3]. Therefore, it is possible to select few dominant beams to significantly reduce the MIMO dimension as well as the number of required RF chains [5].

However, lens-based mmWave massive MIMO systems require to obtain the information of high-dimension beamspace channel by using a limited number of RF chains, which is challenging in practice [3]. To this end, two categories of schemes have been proposed very recently [6]–[8]. The first category is to reduce the effective dimension of the beamspace channel to simplify the estimation problem [6], [7]. Specifically, the selected beams are first determined by beam training to reduce the beamspace channel dimension. Then, the dimension-reduced beamspace channel can be estimated by the classical least squares (LS) algorithm with low pilot overhead. The second category is to exploit the sparsity of beamspace channel to further reduce the pilot overhead [8]. The key idea is to estimate the support (i.e., the index set of nonzero elements in a sparse vector) of the sparse beamspace channel with high accuracy by using the structural channel properties. However, all existing schemes are designed based on the 2D beamspace channel model, where only the horizontal degrees of freedom (DoFs) can be exploited. For the more general 3D beamspace channel model that can exploit both horizontal and vertical DoFs to achieve better performance [5], the beamspace channel estimation problem has not been addressed in the literature to the best of our knowledge.

In this paper, we consider the 3D beamspace channel model, and propose an adaptive support detection (ASD)-based channel estimation scheme for 3D lens-based millimeter-wave massive MIMO systems[1]. Specially, we first decompose the total 3D beamspace channel estimation problem into several sub-problems, and each sub-problem only considers one sparse channel component. Then, for each sparse channel component, we propose an ASD algorithm to detect its support, which consists of two stages. In the first stage, we utilize the special structural sparsity of 3D beamspace channel to obtain an initial support. In the second stage, according to the different power diffusions of 3D beamspace channel in the horizontal and vertical directions, we adaptively adjust the initial support to improve the support accuracy. After the support detections of all channel components, the nonzero elements of the 3D beamspace channel can be estimated with low pilot overhead. Simulation results show that our scheme enjoys satisfying accuracy, even with low signal-to-noise ratio (SNR). This makes our scheme attractive for mmWave communications, where low SNR is the typical case before beamforming [2].

Notation: Lower-case and upper-case boldface letters $\mathbf{a}$ and $\mathbf{A}$ denote a vector and a matrix, respectively; $\mathbf{A}^H$, $\mathbf{A}^{-1}$, and

[1]The simulation codes are provided to reproduce the results in this paper at: http://oa.ee.tsinghua.edu.cn/dailinglong/.

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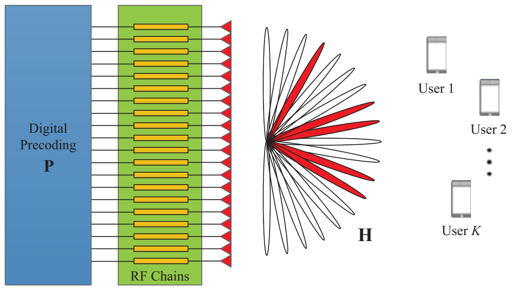
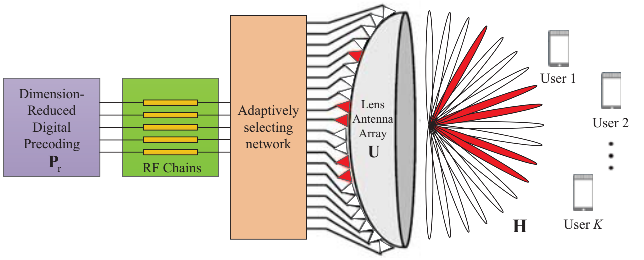


(a)



(b)

Fig. 1. Comparison of system architectures: (a) traditional 3D spatial MIMO; (b) 3D lens-based MIMO.

$\mathrm{tr}(\mathbf{A})$ denote the conjugate transpose, inversion, and trace of $\mathbf{A}$, respectively; $\|\mathbf{A}\|_F$ denotes the Frobenius norm of $\mathbf{A}$; $|a|$ denotes the amplitude of scalar a; $\mathrm{Card}(\mathcal{A})$ denotes the cardinality of set $\mathcal{A}$; $\otimes$ denotes the kronecker product; $\lceil\cdot\rceil$ is the ceil operator; Finally, $\mathbf{I}_K$ is the $K\times K$ identity matrix.

II. System Model

We consider a typical time division duplexing (TDD) mmWave massive MIMO system, where the base station (BS) employs N antennas and N_{RF} RF chains to simultaneously serve K single-antenna users [3]–[5].

A. Traditional 3D spatial MIMO

As shown in Fig. 1 (a), for the traditional 3D spatial MIMO, the $K\times 1$ received signal vector $\mathbf{y}^{\mathrm{DL}}$ for all K users in the downlink can be presented by

$$\mathbf{y}^{\mathrm{DL}} = \mathbf{H}^T\mathbf{P}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}^T \in \mathbb{C}^{K\times N}$ is the downlink channel matrix, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K]$ is the uplink channel matrix according to the channel reciprocity [9], $\mathbf{h}_k$ of size $N\times 1$ is the channel vector between the BS and the kth user as will be discussed later, $\mathbf{s}$ of size $K\times 1$ is the signal vector for all K users satisfying $\mathbb{E}\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_K$, $\mathbf{P}$ of size $N\times K$ is the precoding matrix satisfying the total transmit power constraint ρ as $\mathrm{tr}\left(\mathbf{P}\mathbf{P}^H\right) \le \rho$, $\mathbf{n} \sim \mathcal{CN}\left(0, \sigma^2_{\mathrm{DL}}\mathbf{I}_K\right)$ is the $K\times 1$ additive white Gaussian noise vector, where σ^2_{DL} is the downlink noise power. Obviously, from Fig. 1 (a) we have $N_{\mathrm{RF}} = N$, which is usually large for mmWave massive MIMO systems.

Next, we will introduce the channel vector $\mathbf{h}_k$ of the kth user. Unlike the traditional 2D channel model [6]–[8], in this paper we adopt the widely used 3D Saleh-Valenzuela channel model for mmWave communications as [3]–[5]

$$\mathbf{h}_k = \sqrt{\frac{N}{L+1}}\sum_{l=0}^{L}\beta_k^{(l)}\mathbf{a}\left(\varphi_k^{(l)}, \theta_k^{(l)}\right) = \sqrt{\frac{N}{L+1}}\sum_{l=0}^{L}\mathbf{c}_l, \tag{2}$$

where $\mathbf{c}_0 = \beta_k^{(0)}\mathbf{a}\left(\varphi_k^{(0)}, \theta_k^{(0)}\right)$ is the line-of-sight (LoS) component of $\mathbf{h}_k$ with $\beta_k^{(0)}$ presenting the complex gain and $\varphi_k^{(0)}$ ($\theta_k^{(0)}$) denoting the spatial azimuth (elevation), $\mathbf{c}_l = \beta_k^{(l)}\mathbf{a}\left(\varphi_k^{(l)}, \theta_k^{(l)}\right)$ for $1\le l\le L$ is the lth non-line-of-sight (NLoS) component of $\mathbf{h}_k$, and L is the total number of NLoS components which can be usually obtained by channel measurement [10], $\mathbf{a}(\varphi, \theta)$ is the 2D array steering vector. For the typical uniform planar array (UPA) with N_1 elements in horizon and N_2 elements in vertical ($N = N_1N_2$), we have

$$\mathbf{a}(\varphi, \theta) = \mathbf{a}_{\mathrm{az}}(\varphi) \otimes \mathbf{a}_{\mathrm{el}}(\theta), \tag{3}$$

where $\mathbf{a}_{\mathrm{az}}(\varphi) = \frac{1}{\sqrt{N_1}}\left[e^{-j2\pi\varphi i}\right]$ for $i\in\mathcal{I}(N_1)$, $\mathbf{a}_{\mathrm{el}}(\theta) = \frac{1}{\sqrt{N_2}}\left[e^{-j2\pi\theta j}\right]$ for $j\in\mathcal{I}(N_2)$, and we define $\mathcal{I}(n) = \{p-(n-1)/2,\ p=0,1,\cdots,n-1\}$. The spatial azimuth (elevation) is defined as $\varphi \triangleq \frac{d_1}{\lambda}\sin\tilde{\varphi}$ ($\theta \triangleq \frac{d_2}{\lambda}\sin\tilde{\theta}$) [5], where $\tilde{\varphi}$ ($\tilde{\theta}$) is the physical azimuth (elevation), λ is the wavelength of carrier, and d_1 (d_2) is the horizontal (vertical) antenna spacing. At mmWave frequencies, we usually have $d_1 = d_2 = \lambda/2$ [1].

B. 3D lens-based MIMO

The 3D spatial channel (2) can be transformed into the beamspace by employing a lens antenna array [3] as shown in Fig. 1 (b). Mathematically, this lens antenna array plays the role of an $N\times N$ spatial discrete fourier transform matrix $\mathbf{U}$, which contains the array steering vectors of N orthogonal directions (beams) covering the entire 3D angular space as [3]

$$\mathbf{U} = \left[\mathbf{a}\left(i/N_1, j/N_2\right)\right]^H_{i\in\mathcal{I}(N_1), j\in\mathcal{I}(N_2)}, \tag{4}$$

where i/N_1 for $i\in\mathcal{I}(N_1)$ and j/N_2 for $j\in\mathcal{I}(N_2)$ are the spatial azimuths and elevations predefined by lens antenna array, respectively. Then, the system model of 3D lens-based MIMO can be represented by

$$\tilde{\mathbf{y}}^{\mathrm{DL}} = \mathbf{H}^T\mathbf{U}^T\mathbf{P}\mathbf{s} + \mathbf{n} = \tilde{\mathbf{H}}^T\mathbf{P}\mathbf{s} + \mathbf{n}, \tag{5}$$

where $\tilde{\mathbf{y}}^{\mathrm{DL}}$ is the received downlink signal vector in the beamspace, $\tilde{\mathbf{H}}^T = (\mathbf{U}\mathbf{H})^T$ is the downlink beamspace channel matrix whose N columns correspond to N orthogonal beams predefined by lens antenna array. Note that $\tilde{\mathbf{H}}^T$ has a sparse structure [3]–[5] due to the limited number of effective progogation paths in mmWave communications. Therefore, it is possible to select few dominant beams by using the adaptive selecting network[2] as shown in Fig. 1 (b). Then, the dimension of lens-based MIMO system can be reduced as $\tilde{\mathbf{y}}^{\mathrm{DL}} \approx \tilde{\mathbf{H}}_{\mathrm{r}}^T\mathbf{P}_{\mathrm{r}}\mathbf{s} + \mathbf{n}$, where $\tilde{\mathbf{H}}_{\mathrm{r}} = \tilde{\mathbf{H}}(b,:)_{b\in\mathcal{B}}$, $\mathcal{B}$ denotes the index set of selected beams, and $\mathbf{P}_{\mathrm{r}}$ is the dimension-reduced digital precoding matrix. As the dimension of $\mathbf{P}_{\mathrm{r}}$ is much smaller than that of $\mathbf{P}$ in (1), the number of required RF

[2]The adaptive selecting network proposed in [8] is realized by the low-cost 1-bit phase shifter network. During the data transmission, it can realize beam selection by turning off some phase shifters. During the channel estimation, it can be adaptively used as a combiner to obtain the efficient measurements of the beamspace channel as will be discussed later.

chains can be significantly reduced as shown in Fig. 1 (b)[3]. However, to achieve the near-optimal performance, the BS needs to acquire the 3D high-dimension beamspace channel with limited number of RF chains, which is challenging.

III. 3D Beamspace Channel Estimation

In this section, we first formulate the 3D beamspace channel estimation problem as a typical sparse recovery problem. After that, we prove the special structural sparsity of 3D beamspace channel. Finally, based on this property, an ASD-based channel estimation scheme is proposed.

A. Problem formulation

In TDD systems, the channel estimation is executed in the uplink, where each user needs to transmit the known pilot sequences to the BS over Q instants, during which the beamspace channel remains unchanged [9]. In this paper, we assume that the Q instants are divided into M blocks and each block consists of K instants ($Q = MK$). We define $\mathbf{\Psi}_m$ of size $K \times K$ as the pilot matrix for the mth block, which contains K mutually orthogonal pilot sequences transmitted by K users over K instants [9]. Obviously, we have $\mathbf{\Psi}_m \mathbf{\Psi}_m^H = \mathbf{I}_K$. Then, according to the channel reciprocity [9] in TDD systems, the received uplink signal matrix $\tilde{\mathbf{Y}}_m^{\rm UL}$ of size $N \times K$ at the BS in the mth block can be presented as

$$\tilde{\mathbf{Y}}_m^{\rm UL} = \mathbf{U}\mathbf{H}\mathbf{\Psi}_m + \mathbf{N}_m = \tilde{\mathbf{H}}\mathbf{\Psi}_m + \mathbf{N}_m, \quad m = 1, 2, \cdots, M, \tag{6}$$

where $\mathbf{N}_m$ is the $N \times K$ noise matrix in the mth block with the uplink noise power $\sigma_{\rm UL}^2$. After that, by using the adaptive selecting network as shown in Fig. 1 (b), the BS can employ an analog combiner $\mathbf{W}_m$ of size $K \times N$ to combine $\tilde{\mathbf{Y}}_m^{\rm UL}$ (6), and obtain $\mathbf{R}_m$ of size $K \times K$ in the baseband sampled by $N_{\rm RF} = K$ RF chains as

$$\mathbf{R}_m = \mathbf{W}_m \tilde{\mathbf{Y}}_m^{\rm UL} = \mathbf{W}_m \tilde{\mathbf{H}} \mathbf{\Psi}_m + \mathbf{W}_m \mathbf{N}_m. \tag{7}$$

Finally, by multiplying the known pilot matrix $\mathbf{\Psi}_m^H$ on the right side of (7), the $K \times K$ measurement matrix $\mathbf{Z}_m$ of the beamspace channel $\tilde{\mathbf{H}}$ can be obtained by

$$\mathbf{Z}_m = \mathbf{R}_m \mathbf{\Psi}_m^H = \mathbf{W}_m \tilde{\mathbf{H}} + \mathbf{N}_m^{\rm eff}, \tag{8}$$

where $\mathbf{N}_m^{\rm eff} = \mathbf{W}_m \mathbf{N}_m \mathbf{\Psi}_m^H$ is the effective noise matrix.

We consider the beamspace channel estimation problem of the kth user without loss of generality. After the pilot transmission, an $Q \times 1$ measurement vector $\bar{\mathbf{z}}_k$ is obtained as

$$\bar{\mathbf{z}}_k = \begin{bmatrix} \mathbf{z}_{1,k} \\ \vdots \\ \mathbf{z}_{M,k} \end{bmatrix} = \begin{bmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_M \end{bmatrix} \tilde{\mathbf{h}}_k + \begin{bmatrix} \mathbf{n}_{1,k}^{\rm eff} \\ \vdots \\ \mathbf{n}_{M,k}^{\rm eff} \end{bmatrix} \triangleq \bar{\mathbf{W}}\tilde{\mathbf{h}}_k + \bar{\mathbf{n}}_k, \tag{9}$$

where $\tilde{\mathbf{h}}_k$ is the kth column of $\tilde{\mathbf{H}}$ defined as the beamspace channel of the kth user, $\mathbf{z}_{m,k}$ and $\mathbf{n}_{m,k}^{\rm eff}$ are the kth column of $\mathbf{Z}_m$ and $\mathbf{N}_m^{\rm eff}$ in (8), respectively. $\bar{\mathbf{z}}_k$, $\bar{\mathbf{W}}$, and $\bar{\mathbf{n}}_k$ are of size $Q \times 1$, $Q \times N$, and $Q \times 1$, respectively. Since $\tilde{\mathbf{h}}_k$ is a sparse vector, (9) can be regarded as a typical sparse signal recovery problem [11], which can be solved by classical CS algorithms, such as orthogonal matching pursuit (OMP) [12]. However, when the uplink SNR is low, which is the typical case in mmWave communications due to the lack of beamforming gain and the low transmit power of users [2], $\tilde{\mathbf{h}}_k$ will be overwhelmed by noise. As a result, the performance of classical CS algorithms is usually poor. To this end, we propose an ASD-based channel estimation scheme in this paper, which can achieve better performance than classical CS algorithms.

[3] In this paper, we assume $N_{\rm RF} = K$ to guarantee the spatial multiplexing gains of K users without loss of generality.

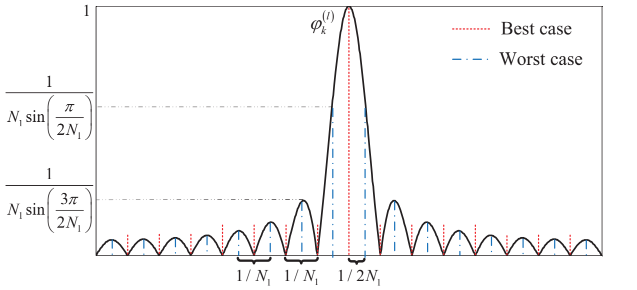


Fig. 2. The amplitude distribution of $\Upsilon\left(N_1, i\Delta\varphi - \varphi_k^{(l)}\right)$ for $i \in \mathcal{I}(N_1)$.

B. Special structural sparsity of 3D beamspace channel

In this section, we prove the special structural sparsity of 3D beamspace channel, which is the basis of our scheme. To do this, we first define $\tilde{\mathbf{c}}_l = \mathbf{U}\mathbf{c}_l$ as the l th channel component of $\tilde{\mathbf{h}}_k$ in the beamspace. Then, we reshape the $N \times 1$ vector $\tilde{\mathbf{c}}_l$ as an $N_1 \times N_2$ matrix $\tilde{\mathbf{C}}_l$, where $\tilde{\mathbf{C}}_l(i,j) = \tilde{\mathbf{c}}_l(N_2(i-1)+j)$. Based on $\tilde{\mathbf{C}}_l$, we have the following **Lemma 1**.

Lemma 1. *Define $\mathbf{S}$ as the sub-matrix of $\tilde{\mathbf{C}}_l$, which extracts the V_1 strongest rows and V_2 strongest columns from $\tilde{\mathbf{C}}_l$. Assume V_1 and V_2 are two even integers without loss of generality. Then, the ratio between the power of $\mathbf{S}$ and the power of $\tilde{\mathbf{C}}_l$ can be lower-bounded by*

$$\frac{\|\mathbf{S}\|_F^2}{\left\|\tilde{\mathbf{C}}_l\right\|_F^2} \geq \frac{4}{N^2} \sum_{i=1}^{V_1/2} \frac{1}{\sin^2\left(\frac{(2i-1)\pi}{2N_1}\right)} \sum_{j=1}^{V_2/2} \frac{1}{\sin^2\left(\frac{(2j-1)\pi}{2N_2}\right)}. \tag{10}$$

Proof: According to the definitions of $\tilde{\mathbf{c}}_l$ and $\tilde{\mathbf{C}}_l$, we have

$$\begin{aligned} \tilde{\mathbf{C}}_l(i,j)/\beta_k^{(l)} &= \mathbf{a}^H(i\Delta\varphi, j\Delta\theta)\,\mathbf{a}\left(\varphi_k^{(l)}, \theta_k^{(l)}\right) \\ &= \left[\mathbf{a}_{\rm az}^H(i\Delta\varphi) \otimes \mathbf{a}_{\rm el}^H(j\Delta\theta)\right]\left[\mathbf{a}_{\rm az}\left(\varphi_k^{(l)}\right) \otimes \mathbf{a}_{\rm el}\left(\theta_k^{(l)}\right)\right] \\ &= \left[\mathbf{a}_{\rm az}^H(i\Delta\varphi)\,\mathbf{a}_{\rm az}\left(\varphi_k^{(l)}\right)\right] \otimes \left[\mathbf{a}_{\rm el}^H(j\Delta\theta)\,\mathbf{a}_{\rm el}\left(\theta_k^{(l)}\right)\right] \\ &= \Upsilon\left(N_1, i\Delta\varphi - \varphi_k^{(l)}\right)\Upsilon\left(N_2, j\Delta\theta - \theta_k^{(l)}\right), \end{aligned} \tag{11}$$

where $\Upsilon(n,x) = \frac{\sin n\pi x}{n \sin \pi x}$. Fig. 2 shows the amplitude distribution of $\Upsilon\left(N_1, i\Delta\varphi - \varphi_k^{(l)}\right)$ for $i \in \mathcal{I}(N_1)$, where the set of red dash lines (or blue dot dash lines) presents the set of spatial azimuths pre-defined by lens antenna array. From Fig. 2, we can observe that when $\varphi_k^{(l)}$ exactly equals one pre-defined spatial azimuth, all the power of $\tilde{\mathbf{C}}_l$ is concentrated on one row, which is the best case. In contrast, the worst power diffusion will happen when the distance between $\varphi_k^{(l)}$ and one pre-

Input:
Measurement vector: $\bar{\mathbf{z}}_k$ in (9);
Analog combiner: $\bar{\mathbf{W}}$ in (9);
Total number of channel components: $L+1$;
Extracted numbers of rows and columns: $V_1 = V_2$.
Initialization: $\tilde{\mathbf{c}}_l^{\mathrm{e}} = \mathbf{0}_{N\times 1}$ for $0 \le l \le L$, $\bar{\mathbf{z}}_k^{(0)} = \bar{\mathbf{z}}_k$.
for $0 \le l \le L$
1. Detect the index p^* of the strongest element of $\tilde{\mathbf{c}}_l$;
2. Adaptively detect the support $\mathrm{supp}\left(\tilde{\mathbf{c}}_l\right)$ of $\tilde{\mathbf{c}}_l$ based on p^*, V_1, and V_2; Estimate $\tilde{\mathbf{c}}_l$ as $\tilde{\mathbf{c}}_l^{\mathrm{e}}$ based on $\mathrm{supp}\left(\tilde{\mathbf{c}}_l\right)$;
3. Remove the influence of $\tilde{\mathbf{c}}_l$;
4. $l = l+1$;
end for
5. Obtain the total support $\mathcal{S}_{\mathrm{T}}$ of $\tilde{\mathbf{h}}_k$;
6. Estimate $\tilde{\mathbf{h}}_k$ as $\tilde{\mathbf{h}}_k^{\mathrm{e}}$ based on $\mathcal{S}_{\mathrm{T}}$;
Output: Estimated beamspace channel for user k: $\tilde{\mathbf{h}}_k^{\mathrm{e}}$.

Algorithm 1: Proposed SD-based channel estimation.

defined spatial azimuth equals $1/2N_1$. In this case, the power of V_1 strongest rows of $\tilde{\mathbf{C}}_l/\beta_k^{(l)}$ is $\frac{2}{N_1^2}\sum_{i=1}^{V_1/2}\sin^{-2}\left(\frac{(2i-1)\pi}{2N_1}\right)$.

Similarly, when the distance between $\theta_k^{(l)}$ and one predefined spatial elevation equals $1/2N_2$, the power of V_2 strongest columns of $\tilde{\mathbf{C}}_l/\beta_k^{(l)}$ is $\frac{2}{N_2^2}\sum_{j=1}^{V_2/2}\sin^{-2}\left(\frac{(2j-1)\pi}{2N_2}\right)$.

Based on the analysis above and the fact $\left\|\tilde{\mathbf{C}}_l\right\|_F^2 = \left(\beta_k^{(l)}\right)^2$, we can derive the conclusion in (10). ■

From **Lemma 1**, we can derive two important conclusions. The first one is that $\tilde{\mathbf{C}}_l$ can be considered as a block sparse matrix, since the most power of $\tilde{\mathbf{C}}_l$ is concentrated on a sub-matrix $\mathbf{S}$ with much smaller size. For example, when $N_1 = N_2 = 32$, $V_1 = V_2 = 8$, the lower-bound (10) is about 91%. This means that we can retain only a small number (e.g., $V_1V_2 = 64$) of dominant elements of $\tilde{\mathbf{C}}_l$ and regard other elements as zero without obvious performance loss. The second one is that the indices of V_1 strongest rows and V_2 strongest columns of $\tilde{\mathbf{C}}_l$ can be uniquely determined by the position of the strongest element of $\tilde{\mathbf{C}}_l$, denoted as $\tilde{\mathbf{C}}_l\left(i^*, j^*\right)$. Specifically, according to Fig. 2, the index sets of the extracted rows and columns of $\tilde{\mathbf{C}}_l$ can be presented by $\mathcal{R} = \left\{i^* - \frac{V_1}{2}, \cdots, i^* + \frac{V_1-2}{2}\right\}$ and $\mathcal{C} = \left\{j^* - \frac{V_2}{2}, \cdots, j^* + \frac{V_2-2}{2}\right\}$, respectively. After the indices of extracted rows and columns have been determined, the support of the sparse vector $\tilde{\mathbf{c}}_l$ can be directly obtained according to the relationship $\tilde{\mathbf{C}}_l\left(i,j\right) = \tilde{\mathbf{c}}_l\left(N_2\left(i-1\right)+j\right)$.

C. Proposed ASD-based channel estimation scheme

Based on **Lemma 1**, in this section, we propose an ASD-based channel estimation scheme. The pseudo-code of our scheme is summarized in **Algorithm 1**, which can be explained in details as below.

Firstly, we decompose the 3D beamspace channel estimation problem into a series of sub-problems, each of which only considers one sparse channel component $\tilde{\mathbf{c}}_l$. We estimate each channel component $\tilde{\mathbf{c}}_l$ for $0 \le l \le L$ in an iterative procedure. During the lth iteration, we first detect the index p^* of the strongest element of $\tilde{\mathbf{c}}_l$ in step 1 as $p^* = \underset{1\le n\le N}{\arg\max}\left|\bar{\mathbf{w}}_n^H\bar{\mathbf{z}}_k^{(l)}\right|$, where $\bar{\mathbf{w}}_n$ is the nth column of $\bar{\mathbf{W}}$. Then, the indices of the strongest row and column of the equivalently $\tilde{\mathbf{C}}_l$ can be computed as $i^* = \lceil p^*/N_2 \rceil$ and $j^* = p^* - N_2\left(i^*-1\right)$.

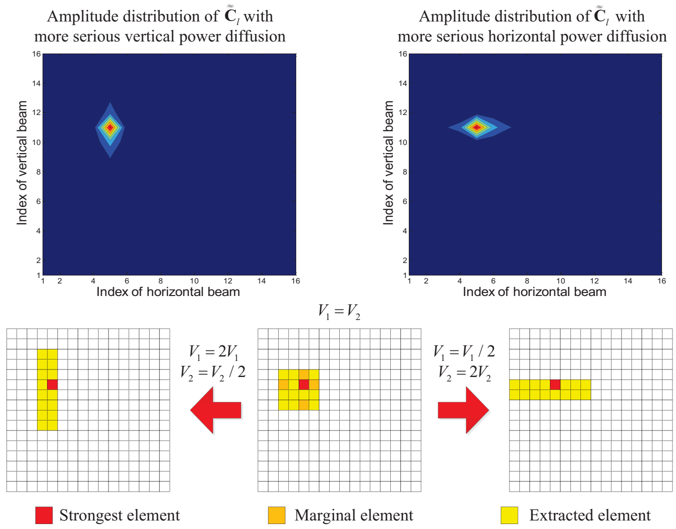


Fig. 3. Illustration of the adaptive support detection.

Next, in step 2, we adaptively detect the support of $\tilde{\mathbf{c}}_l$, which is equivalent to detect the index sets $\mathcal{R}$ and $\mathcal{C}$ of the extracted rows and columns of $\tilde{\mathbf{C}}_l$. This procedure consists of two stages as illustrated in Fig. 3. In the first stage, since we have no priori information for the power diffusions of 3D beamspace channel in the horizontal and vertical directions, we set $V_1 = V_2$. Then, based on i^* and j^*, we can obtain $\mathcal{R}$ and $\mathcal{C}$ as discussed in Section III-B. After that, the support $\mathrm{supp}\left(\tilde{\mathbf{c}}_l\right)$ of $\tilde{\mathbf{c}}_l$ can be directly obtained, and the nonzero elements of $\tilde{\mathbf{c}}_l$ can be estimated by the classical LS algorithm as

$$\tilde{\mathbf{c}}_l^{\mathrm{e}}\left(\mathrm{supp}\left(\tilde{\mathbf{c}}_l\right)\right) = \left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\bar{\mathbf{z}}_k^{(l)},\ \mathbf{A} = \bar{\mathbf{W}}(:,b)_{b\in\mathrm{supp}(\tilde{\mathbf{c}}_l)}. \tag{12}$$

In the second stage, we form $\tilde{\mathbf{C}}_l^{\mathrm{e}}$ based on $\tilde{\mathbf{c}}_l^{\mathrm{e}}$. Then, as shown in Fig. 3, we define four marginal nonzero elements of $\tilde{\mathbf{C}}_l^{\mathrm{e}}$ as $M_1 = \left|\tilde{\mathbf{C}}_l^{\mathrm{e}}\left(i^*, j^*-V_2/2\right)\right|$, $M_2 = \left|\tilde{\mathbf{C}}_l^{\mathrm{e}}\left(i^*, j^*+V_2/2-1\right)\right|$, $M_3 = \left|\tilde{\mathbf{C}}_l^{\mathrm{e}}\left(i^*-V_1/2, j^*\right)\right|$, $M_4 = \left|\tilde{\mathbf{C}}_l^{\mathrm{e}}\left(i^*+V_1/2-1, j^*\right)\right|$. If $\min\left(M_1, M_2\right) < \min\left(M_3, M_4\right)$, we can conclude that the power diffusion of the 3D beamspace channel in the vertical direction is more serious than that of the horizontal direction. In this case, we set $V_1 = 2V_1$, $V_2 = V_2/2$, and re-estimate $\tilde{\mathbf{c}}_l^{\mathrm{e}}$ using the method described above. This procedure will be repeated until $\min\left(M_1, M_2\right) > \min\left(M_3, M_4\right)$. Conversely, if $\min\left(M_1, M_2\right) > \min\left(M_3, M_4\right)$, the power diffusion in the horizontal direction is more serious. Accordingly, we will set $V_1 = V_1/2$, $V_2 = 2V_2$, and re-estimate $\tilde{\mathbf{c}}_l^{\mathrm{e}}$ until $\min\left(M_1, M_2\right) < \min\left(M_3, M_4\right)$.

After $\tilde{\mathbf{c}}_l^{\mathrm{e}}$ has been estimated, in step 3, the influence of this channel component is removed by $\bar{\mathbf{z}}_k^{(l+1)} = \bar{\mathbf{z}}_k^{(l)} - \bar{\mathbf{W}}^H\tilde{\mathbf{c}}_l^{\mathrm{e}}$. Such procedure will be repeated ($l = l+1$ in step 4) until the last channel component is considered.

Finally, after the support detections of all channel components, we can obtain the total support $\mathcal{S}_{\mathrm{T}}$ of $\tilde{\mathbf{h}}_k$ as

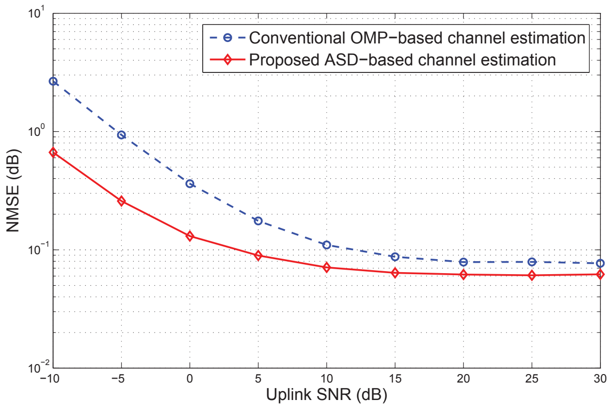


Fig. 4. NMSE performance comparison.

$\mathcal{S}_{\rm T} = \bigcup_{0\leq l\leq L} \text{supp}\left(\tilde{\mathbf{c}}_l\right)$ in step 5. Then, by using LS algorithm, the nonzero elements of $\tilde{\mathbf{h}}_k$ can be estimated in step 6 as $\tilde{\mathbf{h}}_k^{\rm e}\left(\mathcal{S}_{\rm T}\right) = \left(\mathbf{A}_{\rm T}^H\mathbf{A}_{\rm T}\right)^{-1}\mathbf{A}_{\rm T}^H\bar{\mathbf{z}}_k$, where $\mathbf{A}_{\rm T} = \bar{\mathbf{W}}(:,b)_{b\in\mathcal{S}_{\rm T}}$.

The key difference between **Algorithm 1** and classical CS algorithms [12] lies in the support detection. For classical CS algorithms, the positions of all nonzero elements are estimated by an iterative way, which will be more and more inaccurate as the magnitude of nonzero element decreases. In **Algorithm 1**, we only estimate the position of the strongest element. Then, based on the structural sparsity of 3D beamspace channel, the accurate support can be directly obtained with higher probability. Additionally, we can observe from **Algorithm 1** that the most complicated part of our scheme is running the LS algorithm in step 2 and step 6, and the number of running times is determined by L, V_1, and V_2. Since L, V_1, and V_2 are usually small as discussed above, the computational complexity of ASD-based channel estimation scheme is considerably low.

IV. Simulation Results

In this section, we consider a typical mmWave massive MIMO system, where the BS equips $N_{\rm RF} = 16$ RF chains and a lens antenna array with $N_1 = 32$ elements in horizon and $N_2 = 32$ elements in vertical (totally $N = 1024$ antennas) to simultaneously serve $K = 16$ users. For the kth user, the spatial 3D channel is generated as follows [3]–[5]: 1) one LoS component and $L = 2$ NLoS components; 2) $\beta_k^{(0)} \sim \mathcal{CN}(0,1)$, and $\beta_k^{(l)} \sim \mathcal{CN}\left(0,10^{-2}\right)$ for $l = 1,2$; 3) $\varphi_k^{(l)}$ and $\theta_k^{(l)}$ for $l = 0,1,2$ follow the i.i.d. uniform distribution within $[-0.5, 0.5]$. $\bar{\mathbf{W}}$ in (9) is designed as the Bernoulli random matrix which can be realized by the adaptive selecting network with 1-bit phase shifters, i.e., each element of $\bar{\mathbf{W}}$ is randomly selected from $\frac{1}{\sqrt{Q}}\{-1,+1\}$ with equal probability.

Fig. 4 shows the normalized mean square error (NMSE) performance comparison between the proposed ASD-based channel estimation scheme and the conventional OMP-based channel estimation scheme (i.e., using OMP [12] to solve (9)), where the number of instants Q for pilot transmission is $Q = 256$. For ASD-based channel estimation scheme, we set $V_1 = V_2 = 8$, while for OMP-based channel estimation scheme, we assume that the sparsity level of the 3D beamspace channel equals $V_1V_2\,(L{+}1) = 192$. We can observe from Fig. 4 that ASD-based channel estimation scheme can achieve obvious improvement in accuracy compared with OMP-based channel estimation scheme, especially when the uplink SNR is low (e.g., less than 15 dB). As the uplink SNR in channel estimation is usually low for mmWave communications due to the lack of beamforming gain and the low transmit power of users [2], we can claim that our scheme is more attractive for lens-based mmWave massive MIMO systems. Moreover, as the number of instants $Q = 256$ for pilot transmission is much smaller than the dimension of the 3D beamspace channel $N = 1024$, we can further conclude that our scheme enjoys low pilot overhead.

V. Conclusions

In this paper, we propose an ASD-based channel estimation scheme for 3D lens-based mmWave massive MIMO systems with low pilot overhead. Its key idea is to decompose the total 3D beamspace channel estimation problem into a series of sub-problems, each of which only considers one sparse channel component. For each sparse channel component, we adaptively detect its support with high accuracy based on the different power diffusions of 3D beamspace channel in the horizontal and vertical directions. Analysis shows that the complexity of our scheme is similar to that of the LS algorithm. Simulation results verify that our scheme enjoys higher accuracy than the conventional OMP-based channel estimation scheme, especially in the low SNR region.

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