# A Low-Complexity Phase Rotation Estimation using Fourth-Power Constellation for Blind SLM

Amnart Boonkajay<sup>†</sup> and Fumiyuki Adachi<sup>††</sup>

Research Organization of Electrical Communication (ROEC), Tohoku University

2-1-1 Katahira, Aoba-ku, Sendai, Miyagi, 980-8577 Japan

E-mail: <sup>†</sup>amnart@riec.tohoku.ac.jp, <sup>††</sup>adachi@ecei.tohoku.ac.jp

Abstract—Blind selected mapping (blind SLM) is an effective peak-to-average power ratio (PAPR) reduction technique, in which the phase rotation sequence which has been used at the transmitter is blindly estimated. The Euclidean distance between the received symbols after de-mapping and the original QAM constellation is used for the estimation, which requires high computational complexity. In this paper, we introduce a phase rotation sequence estimation based on the minimum Euclidean distance of the fourth-power constellation. The use of fourthpower constellation reduces symbol candidates in minimum Euclidean distance calculation and hence, contributes to complexity reduction. A set of phase rotation sequences constructed by random selection from  $\{0^\circ, 135^\circ\}$  are also introduced to further reduce the complexity and maintain high estimation accuracy. Simulation result confirms that the proposed phase rotation and sequence estimation technique can reduce the complexity without degrading the uncoded bit error rate (BER) performance for the given PAPR reduction.

### Index Terms-PAPR, single carrier, selected mapping

#### I. INTRODUCTION

The development of the fifth-generation (5G) mobile communications systems [1] is intensified recently aiming at initiation of 5G services in around 2020. Even in 5G systems, the low peak-to-average power ratio (PAPR) waveform design remains important, in particular, for battery-powered user equipments (UEs). Single-carrier (SC) signals have lower PAPR compared to orthogonal frequency division multiplexing (OFDM) signals [2]. However, PAPR reduction technique is also necessary for SC transmission since PAPR of SC signals increases when a transmit filtering is employed [3].

Selected mapping (SLM) [4] is an efficient and simple PAPR reduction scheme. SLM selects the waveform having the lowest PAPR among many candidates generated by applying phase rotation to the original transmit signal. The SLM [4], originally proposed for OFDM, requires side information transmission. SLM without side information (called blind SLM) compatible with both SC and OFDM was proposed in [5]. Its applications to space-time block coded transmit diversity (STBC-TD) and multiuser multi-input multi-output (MU-MIMO) were discussed in [6] and [7], respectively.

The blind SLM in [5-7] employs a maximum likelihood (ML) phase rotation sequence estimation based on minimum Euclidean distance between the de-mapped symbols and the original signal constellation. The ML estimation works effectively, but it requires high computational complexity. A 2-step

phase rotation sequence estimation based on Viterbi algorithm [8] was proposed to reduce the computational complexity, however, its complexity reduction capability is obvious only when the number of phase rotation sequences is larger than the number of subcarriers.

To remedy the above complexity problem in the blind SLM, we introduce an ML phase rotation sequence estimation based on minimum Euclidean distance of the fourth-power constellation. The use of the fourth-power constellation can reduce the number of candidates in the Euclidean distance calculation significantly and hence, contributes to computational complexity reduction. Moreover, it is recommended in [9] that the use of the fourth-power constellation together with a set of phase rotations  $\{0^\circ, 135^\circ\}$  can further reduce the complexity while maintaining high estimation accuracy. Note that Ref. [9] uses the fourth-power constellation and the above phase rotation sets for embedding the side information into data transmission, i.e., [9] uses the phase rotations  $\{0^{\circ}, 180^{\circ}\}$  to generate waveform candidates, selects the one with the lowest PAPR as transmit signal, then embeds the side information by applying  $45^{\circ}$  or  $135^{\circ}$  phase rotation to some subcarriers. However, the approach in [9] considers OFDM only and has a disadvantage that false detection is caused when some of the embedded subcarriers suffers from frequency-selective fading. In contrast, our proposed phase rotation sequence estimation considers an average Euclidean distance from all subcarriers.

In this paper, performance evaluation of the blind SLM using the above phase rotation sets and the proposed ML phase rotation sequence estimation using the fourth-power constellation is carried out by computer simulation in aspects of PAPR, BER and computational complexity and assuming SC uplink MIMO transmission (UE to base station (BS)). The simulation results confirm that the proposed ML phase rotation sequence estimation can reduce the complexity without degrading the uncoded BER performance for the given PAPR reduction.

### II. OVERVIEW OF CONVENTIONAL BLIND SLM

Here, we briefly describe the concept of blind SLM in [5-7]. We describe only the signal representation for STBC-TD and MU-MIMO, where the representation for single-antenna transmission (SISO) is obtained by setting the number of BS antennas ( $N_{\rm BS}$ ) and UE antennas ( $N_{\rm UE}$ ), J and Q to be 1.

In single-user STBC-TD, we assume that the transmit filtering is not used (i.e., band-limiting filter only). In MU-



Fig. 1. Transceiver system model with blind SLM (SC uplink).

MIMO, we assume that an eigenmode transmit filtering based on singular-value decomposition (SVD) is employed at the UE and a minimum mean-square error based multiuser filtering is employed at the BS, where the derivation of filter weights were already discussed in [3], [10]. The transceiver system models equipped with blind SLM can be depicted by Fig. 1.

#### A. SLM algorithm

Assuming that a time-domain transmit waveform is  $\{s(n); n = 0 \sim N_c - 1\}$ , PAPR is calculated over a V-times oversampled block and is given by

$$PAPR(\{s(n)\}) = \frac{\max\{|s(n)|^2, n = 0, \frac{1}{V}, \frac{2}{V}, \dots, N_c - 1\}}{\frac{1}{N_c} \sum_{n=0}^{N_c - 1} |s(n)|^2}.$$
 (1)

In STBC-TD transmission, information sequence to be transmitted is data-modulated and is divided into J blocks, obtaining the *j*-th block of  $N_c$ -length data symbol  $\{d_j(n); n =$  $0 \sim N_c - 1, j = 0 \sim J - 1$ .  $\{d_j(n)\}$  is phase-rotated by multiplying with the selected phase rotation sequence  $\{\Phi_{\hat{m}(i)}(n); n = 0 \sim N_c - 1\}$ , yielding the phase-rotated block  $\{d_{i,\hat{m}}(n); n = 0 \sim N_c - 1, j = 0 \sim J - 1\}. \{d_{i,\hat{m}}(n)\}$ is then transformed into frequency-domain components block  $\{D_{i,\hat{m}(i)}(k); k = 0 \sim N_c - 1\}$  by  $N_c$ -point DFT. After that,  $\{D_{i,\hat{m}(i)}(k)\}\$  are passed through transmit signal processing e.g. STBC coding, obtaining the frequency-domain transmit signal at the  $n_{\rm UE}$ -th transmit antenna ( $n_{\rm UE} = 0 \sim N_{\rm UE} - 1$ ) and the q-th timeslot  $(q=0\sim Q-1)$  as  $\{S_{n_{\mathrm{UE}},q,\hat{m}(j)}(k);k=0\sim$  $N_c - 1$  and its corresponding time-domain waveform after applying inverse DFT (IDFT) as  $\{s_{n_{\rm UE},q,\hat{m}(j)}(n); n = 0 \sim$  $N_c-1$ }. If we assume that  $N_{\text{UE}}=2$ , J=Q=2 and  $S_{n_{\text{UE}},q,\hat{m}(j)}(k)$ can be described by the following matrix representations.

$$\begin{bmatrix} S_{0,0,\hat{m}(j)}(k) & S_{0,1,\hat{m}(j)}(k) \\ S_{1,0,\hat{m}(j)}(k) & S_{1,1,\hat{m}(j)}(k) \end{bmatrix} \\ = \sqrt{\frac{2E_s}{T_s}} \begin{bmatrix} D_{0,\hat{m}(j)}(k) & -D_{1,\hat{m}(j)}^*(k) \\ D_{1,\hat{m}(j)}(k) & D_{0,\hat{m}(j)}^*(k) \end{bmatrix} ,$$
(2)

where  $E_s$  and  $T_s$  are symbol energy and symbol duration.

In MU-MIMO transmission, information sequence of the u-th user  $(u = 0 \sim U - 1)$  is data-modulated into Gstreams of  $N_c$ -length block  $\{\mathbf{d}_u(n); n = 0 \sim N_c - 1\}$  with  $\mathbf{d}_u(n) = [d_{u,0}(n), \ldots, d_{u,g}(n), \ldots, d_{u,G-1}(n)]^T$ . The timedomain block  $\{\mathbf{d}_u(n)\}$  is then multiplied by the selected phase rotation sequence to obtain  $\{\mathbf{d}_{u,\hat{m}(u)}(n) = \Phi_{\hat{m}(u)}(n)\mathbf{d}_u(n)\}$ .  $\{\mathbf{d}_{u,\hat{m}(u)}(n)\}$  is then transformed to frequency-domain components block  $\{\mathbf{D}_{u,\hat{m}(u)}(k); k = 0 \sim N_c - 1\}$  by  $N_c$ point DFT. Transmit filtering is applied afterward, obtaining an  $N_{\text{UE}} \times 1$  frequency-domain component vector at the k-th subcarrier of the u-th user as

$$\mathbf{S}_{u,\hat{m}(u)}(k) = \sqrt{\frac{2E_s}{T_s}} \mathbf{W}_{T,u}(k) \mathbf{D}_{u,\hat{m}(u)}(k), \tag{3}$$

where  $\mathbf{W}_{T,u}(k)$  is an eigenmode transmit filtering weight at the k-th subcarrier with dimension of  $N_{\text{UE}} \times G$  [3], [10].  $\mathbf{S}_{u,\hat{m}(u)}(k)$  is then transformed back into time-domain signal by IDFT to obtain the transmit waveforms through  $N_{\text{UE}}$ antennas as  $\{\mathbf{s}_{u,\hat{m}(u)}(n); n = 0 \sim N_c - 1\}$  with  $\mathbf{s}_{u,\hat{m}(u)}(n) = [s_{u,0,\hat{m}(u)}(n), \dots, s_{u,N_{\text{UE}}-\hat{m}(u)}(n)]^T$ .

In the case of SC uplink STBC-TD without transmit filtering (i.e., employing band-limiting filter only), the PAPR of signals before and after STBC coding are exactly the same. This is because the STBC coding employs only complex conjugate operations [6]. Therefore, we can select an individual phase rotation sequence for each of  $\{d_j(n)\}$  prior to applying STBC coding. The selected phase rotation sequence for the *j*-th data block,  $\{\Phi_{\hat{m}(j)}(n)\}$  with the corresponding sequence index  $\hat{m}(j)$ , is determined by

$$\hat{m}(j) = \arg\min_{m=0\sim M-1} \left( \mathsf{PAPR}(\{\Phi_m(n)d_j(n)\}) \right), \tag{4}$$

where  $\{\Phi_m(n); n = 0 \sim N_c - 1, m = 0 \sim M - 1\}$  is the *m*th phase sequence in a codebook and is generated randomly as  $\Phi_m(n) \in \{e^{j0}, e^{j2\pi/3}, e^{j4\pi/3}\}$ , except the first sequence is defined as  $\{\Phi_0(n) = e^{j0}; n = 0 \sim N_c - 1\}$  [5].

Meanwhile, Eq. (4) is not available for MU-MIMO transmission since the signals before and after transmit filtering have different PAPR. In this case, a selection criterion which minimizes the maximum PAPR value (called Mini-max criterion) among all  $N_{\rm UE}$  transmit antennas is used. A common phase rotation  $\{\Phi_{\hat{m}(u)}(n)\}$  with the corresponding sequence index  $\hat{m}(u)$  can be defined as follows.

$$\hat{m}(u) = \arg\min_{m=0 \sim M-1} \left( \max_{n_{\rm UE}=0 \sim N_{\rm UE}-1} \text{PAPR}\left( \{ s_{u,n_{\rm UE},m}(n) \} \right) \right).$$
(5)

The selection criterion in Eq. (5) is sub-optimal and hence, PAPR increases when  $N_{\rm UE}$  increases. However, it can keep the phase rotation estimation simple and no major changes on filtering weights calculation is required [7].

### B. Phase rotation sequence estimation

Phase rotation sequence estimation is employed after the receive signal processing such as STBC decoding or multiuser minimum mean square error (MMSE) based receive filtering. Phase rotation sequence estimation is done by calculating Euclidean distance between the de-mapped signal (i.e. multiplied by the complex conjugate of phase rotation sequence) and original constellation. If the de-mapping is done correctly, the de-mapped signal should be very close to the original

constellation and hence, its Euclidean distance from the nearest QAM symbol is very small. The phase rotation sequence associated with the de-mapped signal having the minimum averaged Euclidean distance is selected.

In STBC-TD, assuming the *j*-th time-domain received block before de-mapping is  $\{\hat{d}_j(n); n = 0 \sim N_c - 1, j = 0 \sim J - 1\}$ , the phase rotation sequence estimation can be expressed as

$$\tilde{m}(j) = \arg\min_{m=0\sim M-1} \left( \sum_{n=0}^{N_c-1} \min_{\mathbb{C}\in\Psi_{\rm mod}} \left| \Phi_m^*(n) \hat{d}_j(n) - \mathbb{C} \right|^2 \right), \quad (6)$$

where  $\Psi_{\text{mod}}$  is the original data-modulated constellation (e.g. QAM mapping). Finally, the received symbols prior to hard decision is obtained by applying the de-mapping as  $\{\tilde{d}_j(n) = \Phi_{\tilde{m}(j)}^*(n)\hat{d}_j(n); n = 0 \sim N_c - 1, j = 0 \sim J - 1\}.$ 

In MU-MIMO transmission, the received vector after employing multiuser filtering of the *u*-th UE is denoted by  $\{\hat{\mathbf{d}}_u(n); n = 0 \sim N_c - 1\}$  with  $\hat{\mathbf{d}}_u(n) = [\hat{d}_{u,0}(n), \dots, \hat{d}_{u,g}(n), \dots, \hat{d}_{u,G-1}(n)]^T$ . Then, the phase rotation sequence estimation is carried out separately for each user and is expressed as

$$\tilde{m}(u) = \arg\min_{m=0\sim M-1} \left( \sum_{n=0}^{N_c - 1} \sum_{g=0}^{G-1} \min_{\mathbb{C} \in \Psi_{\text{mod}}} \left| \Phi_m^*(n) \hat{d}_{u,g}(n) - \mathbb{C} \right|^2 \right)$$
(7)

Finally, the received symbols prior to hard decision is obtained by  $\{\tilde{\mathbf{d}}_u(n) = \Phi^*_{\tilde{m}(u)}(n)\hat{\mathbf{d}}_u(n); n = 0 \sim N_c - 1\}.$ 

### III. PROPOSED ML PHASE ROTATION ESTIMATION

The blind SLM in [5-7] requires high complexity. Although the 2-step phase rotation sequence estimation [8] can reduce the complexity at the receiver, its complexity reduction capability becomes obvious only if M is very large. To improve the blind SLM, we introduce a new set of phase rotation with a phase rotation sequence estimation based on minimum Euclidean distance of the fourth-power constellation. Here, we mention that there is no major changes on the signal representations and the SLM algorithms in [5-8], also in Eqs. (4) and (5). The modifications are needed only at the codebook design and the phase rotation estimation.

We assume the ML sequence estimation and use the fourthpower constellation (i.e.,  $(I + iQ)^4$ ) instead of the original constellation (i.e., (I + jQ)). Figs. 2(a) and 2(b) show the comparison of the original and the fourth-power constellation assuming 16QAM modulation and the phase rotations of  $\Phi_m(n) \in \{e^{j0}, e^{j2\pi/3}, e^{j4\pi/3}\}$  (in other words, 3-value phase rotations of {0°, 120°, 240°}). In addition, Ref. [9] indicates that  $\Phi_m(n) \in \{e^{j0}, e^{j\pi/4}\}$  or  $\{e^{j0}, e^{j3\pi/4}\}$  (in other words, 2value phase rotations of  $\{0^\circ, 45^\circ\}$  or  $\{0^\circ, 135^\circ\}$ ) are attractive since they also enlarge the Euclidean distance between correct and incorrect de-mapped symbols, which may contribute to estimation accuracy and BER improvement. Their constellations can be depicted by Figs. 2(c) and 2(d). It is observed from Fig. 2 that the number of symbol candidates significantly reduces, leading to computational complexity reduction, even the fourth-power operations is also used. Note that we cannot



use  $\Phi_m(n) \in \{e^{j0}, e^{j\pi/4}, e^{j3\pi/4}\}$  since they cause overlaps between correct and incorrect de-mappings. By employing the above concept, we modify the codebook generation and the phase rotation sequence estimation as follows.

#### A. Phase rotation sequence generation

Phase rotation sequence generation is simply modified by randomly generating the predefined codebook as  $\Phi_m(n) \in$  $\{e^{j0}, e^{j3\pi/4}\}, m = 1 \sim M - 1$ , except the first sequence as "all 1" for representing the original waveform. Meanwhile, we checked by simulation and found that the use of  $\{e^{j0}, e^{j\pi/4}\}$ increases the PAPR, hence it is not adopted in this paper.

#### B. Phase rotation sequence estimation

By substituting the fourth-power constellation in the ML estimation equations, Eqs. (6) and (7) can be rewritten as Eq. (8) for STBC-TD and Eq. (9) for MU-MIMO, respectively.

$$\tilde{m}(j) = \arg\min_{m=0\sim M-1} \left( \sum_{n=0}^{N_c-1} \min_{\mathbb{C}\in\Psi_{\rm mod}^4} \left| (\Phi_m^*(n)\hat{d}_j(n))^4 - \mathbb{C} \right|^2 \right), (8)$$

$$\tilde{m}(u) = \arg\min_{m=0\sim M-1} \left( \sum_{n=0}^{N_c-1} \sum_{g=0}^{C-1} \min_{\mathbb{C}\in\Psi_{\rm mod}^4} \left| (\Phi_m^*(n)\hat{d}_{u,g}(n))^4 - \mathbb{C} \right|^2 \right)$$
(9)

where  $\Psi^4_{mod}$  is the fourth-power modulated constellation. Eqs. (8) and (9) applies the fourth-power operation to the received signal, meaning that the effect of noise also increases. However, larger error magnitude when the de-mapping is incorrect can be obtained and hence, the phase rotation sequence estimation can work effectively even in the low-SNR region.

In addition, by observing Fig. 2(d) and referring [9], the difference of real value is relatively larger than that of imaginary value. This is because the phase-rotated fourth-power constellation (i.e.,  $\times$  marks) becomes 180° different from the

$$\tilde{m}(j) = \arg \min_{m=0 \sim M-1} \left( \sum_{n=0}^{N_c-1} \min_{\mathbb{C} \in \Psi_{\text{mod}}^4} \left| \Re\{ (\Phi_m^*(n)\hat{d}_j(n))^4 \} - \Re\{\mathbb{C}\} \right|^2 \right),$$
(10)

$$\tilde{m}(u) = \arg\min_{m=0\sim M-1} \left( \sum_{n=0}^{N_c-1} \sum_{g=0}^{G-1} \min_{\mathbb{C}\in\Psi_{\text{mod}}^4} \left| \Re\{(\Phi_m^*(n)\hat{d}_{u,g}(n))^4\} - \Re\{\mathbb{C}\} \right|^2 \right).$$
(11)

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Modulation	Data modulation	16QAM, 64QAM	
	FFT/IFFT block size	$N_c = 128$	
	Cyclic prefix length	$N_g = 16$	
User equipment	Tx filter	Eigenmode (MU-MIMO)	
	No. of UE antennas	$N_{\rm UE} = 2$	
Blind SLM parameter	Phase sequence type	Random polyphase	
	No. of sequences	$M = 1 \sim 256$	
	Phase sequence	Maximum likalihood	
	estimation method	Waxiniuni-likelinood	
	Oversampling rate	V = 8	
Channel	Fading type	Frequency-selective	
		block Rayleigh	
	Power delay profile	symbol-spaced	
		16-path uniform	
Base station	No. of BS antennas	$N_{\rm BS} = 4$	
	Rx filter	MMSE (STBC-TD)	

TABLE I Simulation parameters

original fourth-power constellation (i.e., • marks). Hence, it is sufficient to use the difference of real values instead of conventional Euclidean distance calculation. As a result, Eqs. (8) and (9) can be simplified as Eq. (10) for STBC-TD and Eq. (11) for MU-MIMO, respectively, where  $\Re{\cdot}$  represent the real value of complex number.

#### **IV. PERFORMANCE EVALUATION**

Simulation parameters are summarized in Table I. SC uplink MIMO transmissions without channel coding are considered for simplicity. Propagation path-loss and shadowing loss are not considered. Phase rotation codebook is generated randomly as  $\Phi_m(n) \in \{e^{j0}, e^{j2\pi/3}, e^{j4\pi/3}\}$  (referred as conventional blind SLM [5]) and  $\Phi_m(n) \in \{e^{j0}, e^{j3\pi/4}\}$  (referred as proposed  $\{0^\circ, 135^\circ\}$ ). Performance evaluation is discussed in terms of PAPR, BER and computational complexity, and then compared with the conventional blind SLM , i.e. blind SLM using the phase sequence estimation based on (I + jQ). We assume U=2 and  $G=N_{\rm UE}=2$  for MU-MIMO transmission, in which UEs employ eigenmode transmit filtering based on singular value decomposition (SVD) and the BS employs multiuser MMSE receive filtering. This MU-MIMO scheme is called MMSE-SVD [3]. Ideal channel estimation is assumed.

## A. PAPR vs. computational complexity

PAPR performance is evaluated by measuring the PAPR value at complementary cumulative distribution function (CCDF) equals  $10^{-3}$ , called PAPR<sub>0.1%</sub>. Complexity is evaluated by counting the number of real-valued addition operations and assuming that the complexity of real-valued multiplication is approximately 3 times of real-valued addition [11], [12]. Complexity of the phase rotation sequence estimation per one data stream is summarized in Table II, where the complexity of 2-step estimation is calculated based on 27 states [8].



Fig. 3.  $PAPR_{0.1\%}$  versus computational complexity.

TABLE II							
OMPUTATIONAL	COMPLEXITY	PER	DATA	STREAM			

		No. of real-valued	No. of real-valued
		multiplications	additions
Conv. blind	16QAM	$M \times (36N_c + 1)$	$M \times (51N_c + 1)$
SLM (ML)	64QAM	$M \times (132N_c + 1)$	$M \times (195N_c + 1)$
Conv. blind	16QAM	$38 \times N_{tb}$	$(51 \times N_{tb}) + MN_c$
SLM (2-step)	64QAM	$134 \times N_{tb}$	$(195 \times N_{tb}) + MN_c$
Proposed blind	16QAM	$M \times (15N_c + 1)$	$M \times (12N_c + 1)$
SLM (ML)	64QAM	$M \times (22N_c + 1)$	$M \times (19N_c + 1)$

**Remark:**  $N_{tb}$  is the number of branches used in Viterbi algorithm for one received block (maximum is  $729 \times N_c$ ).

Figs. 3(a) and 3(b) shows the PAPR<sub>0.1%</sub> versus computational complexity of SC uplink STBC-TD and MU-MIMO using blind SLM. PAPR reduces when M increases, but the total complexity also increases. The 2-step estimation can reduce the complexity while maintaining the same PAPR as that of conventional blind SLM [8], but the complexity reduction capability is obvious only when M > 64. The proposed ML phase rotation sequence estimation can reduce the complexity even when  $M \leq 64$  due to less number of



Fig. 4. BER performance of SC uplink with blind SLM.

symbol candidates in the Euclidean distance calculation.

The use of phase rotations  $\{0^{\circ}, 135^{\circ}\}$  with the ML phase rotation sequence estimation based on Eqs. (10) and (11) can keep PAPR the same as that of conventional blind SLM and with less complexity. Assuming M=64 and 16QAM (64QAM) modulation, the proposed ML estimation can lower the PAPR by 2.9 dB (3.1 dB) for STBC-TD and 3.3dB (3.3 dB) for MU-MIMO transmission, respectively. Its complexity is only 35% (14%) of the conventional ML estimation [5] and 50% (21%) of the conventional 2-step estimation [8]. The complexity reduction capability is more obvious in 64QAM since the impact of complexity reduction obtained from symbol candidate reduction is more obvious than the impact of complexity increasing due to the fourth-power calculation (which is irrespective to modulation level).

#### B. BER

Fig. 4 shows the uncoded BER performance of SC uplink STBC-TD and MU-MIMO with the blind SLM using phase rotation sets  $\{0^{\circ}, 135^{\circ}\}$ , together with the ML phase rotation sequence estimation using the fourth-power constellation, as a function of average received bit energy-per-noise power spectrum density  $(E_b/N_0)$ . The BER performances of transmission using the conventional ML estimation [6-7] and without blind SLM are also plotted for comparison. It is seen that when M=64, the use of blind SLM, both with the conventional and the proposed ML phase rotation sequence estimation, achieves the same BER performance compared to the transmission without SLM when the received  $E_b/N_0$  is sufficiently high (for example  $E_b/N_0 > 0$  dB for STBC-TD and 6 dB for MU-MIMO). As a result, the use of phase rotation set  $\{0^\circ, 135^\circ\}$ together with the fourth-power constellation is more attractive due to its lower complexity, while maintaining the same PAPR and BER performances as the conventional blind SLM.

#### V. CONCLUSION

A blind SLM technique consisting of 2-value phase rotation sets  $\{0^\circ, 135^\circ\}$  and ML phase rotation sequence estimation using the fourth-power constellation was introduced in this paper. The use of fourth-power constellation can reduce the number of symbol candidates in minimum Euclidean distance computation, leading to computational complexity reduction. Simulation results assuming SC uplink transmissions with M = 64 and 16QAM (64QAM) modulation confirmed that the blind SLM with phase rotation sets  $\{0^{\circ}, 135^{\circ}\}$  and proposed ML phase rotation sequence estimation can reduce the PAPR of STBC-TD by 2.9 dB (3.1 dB) and the PAPR of MU-MIMO by 3.3 dB (3.3 dB). Computational complexity of phase rotation estimation is only 35% (14%) of the conventional blind SLM using ML estimation, and 50% (21%) of 2-step estimation. It was also confirmed that there is no significant BER degradation compared to the transmission without blind SLM when  $E_b/N_0 > 6$  dB.

In addition, the use of 2-value phase rotation sets  $\{0^\circ, 135^\circ\}$ and the fourth-power constellation also has potential to reduce the complexity of 2-step estimation, especially when M > 64. We leave it as our future studies since it involves many design parameters such as the maximum number of states.

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