Tapped delay line model for band-limited multipath channel in DS-CDMA mobile radio

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It is explained that a correlated tapped delay line model needs to be assumed for transmission performance estimation using Rake combining, even though the propagation channel itself can be characterised as a wide sense stationary uncorrelated scattering channel. Fading correlation between different tap gains can be computed by taking into account the transmit and receive chip pulse shaping filters. A simple example, showing how correlation affects performance estimation in Rake combining, is presented, assuming a two-path Rayleigh fading propagation channel.

Problem: In mobile wireless communications, owing to reflection by obstacles such as buildings, there are many propagation paths between a transmitter and a receiver. If the time delays of multipaths cannot be neglected, the propagation channel is called a frequency selective channel. Assuming a wide sense stationary uncorrelated scattering (WSSUS) channel, the equivalent lowpass impulse response of the propagation channel at time t, owing to an excitation at time $t - \tau$, can be expressed as [1]

$$h_c(t,\tau) = \sum_{m=0}^{\infty} \xi_{c,m}(t)\delta(\tau - \tau_{c,m}) \qquad (1)$$

where $\xi_{c,m}(t)$ and $\tau_{c,m}$ are, respectively, the complex-valued random path gain and time delay of the mth path, $\xi_{c,m}(t)$ satisfies $\sum_{m=0}^{\infty} E[|\xi_{c,m}(t)|^2] = 1$ with $E[\cdot]$ denoting ensemble averaging, and $\delta(x)$ is the delta function. For the WSSUS channel assumption, $|\xi_{c,m}(t)|$ are statistically independent random processes.

When observing the propagation channel over a limited bandwidth, the band-limited channel is seen as a different propagation channel from the real one. For signal transmission covering the bandwidth of B Hz, the equivalent channel can be completely represented by a tapped delay line model [2]:

$$h(t,\tau) = \sum_{l=0}^{\infty} \xi_l(t)\delta\left(\tau - \frac{l}{B}\right) \qquad (2)$$

This model is widely used for performance estimation of a direct sequence code division multiple access (DS-CDMA) system with Rake combining and a chip rate of $B = 1/T_c$. However, as we will show in the following, the random path gain $\xi_l(t)$ in eqn. 2 is a linear combination of random path gains $\{\xi_{c,m}(t)\}$ of eqn. 1, and therefore it may not necessarily be independent. In addition, assessing the DS-CDMA transmission performance with Rake combining requires knowledge of the statistical properties of $\{\xi_{c,m}(t)\}$ which we address in this Letter.

Tap gain representation: Let the overall impulse response of the transmit and receive chip pulse shaping filters of the DS-CDMA transmission system be denoted as $h_f(t)$. The channel seen at the receive chip filter output, i.e. eqn. 2, can be represented as $h(t, \tau) = h_c(t, \tau) \otimes h_f(t)$, where $\otimes$ denotes time convolution operation. Substituting eqn. 1 into this gives

$$h(t,\tau) = \sum_{m=0}^{\infty} \xi_{c,m}(t) h_f(t - \tau_{c,m}) \qquad (3)$$

In the DS-CDMA receiver, the receive chip filter output is sampled at a rate of $1/T_c$ and fed to spreading code sequence matched filters [3] to resolve the multipath signal components. The timing of spreading code sequence replica of the lth matched filter or Rake finger is shifted by lT_c, $l = 0, 1, ...,$ from the reference timing. Hence, applying the tapped delay line model, the lth tap gain in eqn. 2 is given by

$$\xi_l(t) = \sum_{m=0}^{\infty} \xi_{c,m}(t) h_f(lT_c - \tau_{c,m}) \qquad (4)$$

Tap gain correlation: We assume that $\{\xi_{c,m}(t)\}$ are zero-mean random processes. The fading correlation coefficient between the lth and nth taps is given by $\rho_{l,n} = E[\xi_l(t)\xi_n^*(t)]/\sqrt{\{E[|\xi_l(t)|^2]E[\xi_n(t)|^2]\}}$. Recalling that $\{\xi_{c,m}(t)\}$ are statistically independent random processes, we have

$$E\left[\xi_l(t)\xi_n^*(t)\right] = \int_{-\infty}^{\infty} \Omega(\tau) h_f(lT_c - \tau) h_f^*(nT_c - \tau) d\tau \quad (5)$$

where $\Omega(\tau) = \sum_{m=0}^{\infty} E[|\xi_{c,m}(t)|^2]\delta(\tau - \tau_{c,m})$ is the power delay profile of the propagation channel (seen when the measurement bandwidth is infinite). The tap gain correlation $\rho_{l,n}$ can be computed using

$$\rho_{l,n} = \int_{-\infty}^{\infty} \Omega(\tau) h_f(lT_c - \tau) h_f^*(nT_c - \tau) d\tau \Big/ \sqrt{\Gamma_l \Gamma_n} \quad (6)$$

where $\Gamma_l = \int_{-\infty}^{\infty} \Omega(\tau)|h_f(lT_c - \tau)|^2 d\tau$ is the variance of the lth tap gain. if the overall transmit and receive chip filter impulse response is given, $\rho_{l,n}$ of the tapped delay line model can be computed using eqn. 6 for the arbitrary power delay profile $\Omega(\tau)$ of the propagation channel.

Table 1: Tap gain variance and correlation for double spike power delay profile and rectangular-spectrum chip shaping

		$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$
$\tau_0/T_c = 0.5$	Γ_l	0.703	0.203	0.0225	8.11×10^{-3}	4.14×10^{-3}
	$\rho_{l,0}$	1	+0.537	–0.537	+0.537	–0.537
$\tau_0/T_c = 1.0$	Γ_l	0.5	0.5	0	0	0
	$\rho_{l,0}$	1	0	0	0	0
$\tau_0/T_c = 1.25$	Γ_l	0.516	0.405	0.0450	8.27×10^{-3}	3.35×10^{-3}
	$\rho_{l,0}$	1	–0.177	–0.177	+0.177	–0.177

Numerical example: For simplicity, we assume a two-path independent Rayleigh faded propagation channel, i.e. $\Omega(\tau) = 0.5\delta(\tau) + 0.5\delta(\tau - \tau_0)$ and an ideal rectangular-spectrum chip pulse shaping with bandwidth of $B = 1/T_c$, i.e. $h_f(t) = \sin(\pi Bt)/(\pi Bt)$. The equivalent channel seen at the receive chip filter output has a continuous impulse response, which can be represented using the tapped delay line model of eqn. 2 for a band-limited DS-CDMA system. We assume that the receiver sampling timing is locked to the first path. The values of Γ_l and $\rho_{l,0}$ are computed for $l = 0$ to 4 when $\tau_0/T_c = 0.5$, 1, and 1.25, and are listed in Table 1. The first two taps are dominant contributors in a tapped delay line model of eqn. 2. Therefore, we use the first two taps for two-finger Rake combining. How the correlation between the first two taps affects the Rake combining performance is discussed below. For maximal ratio combining (MRC), the combiner output signal-to-noise ratio (SNR) is the sum of SNRs at two finger outputs. From the MRC combining theory [4], we know that a Rake combiner with correlated tap gains having different variances is equivalent to one with independently faded tap gains but with equal variance of $\sqrt{[\{1 - |\rho_{1,0}|^2\}\Gamma_1\Gamma_0]}$, where $\Gamma_0 = 0.5 + 0.5|h_f(-\tau_0)|^2$ and $\Gamma_1 = 0.5|h_f(T_c - \tau_0)|^2$. Therefore, with correlated tap gains, the MRC combining performance is inferior to MRC combining with independent tap gains by Δ (dB) $= 5\log_{10}(1 - |\rho_{1,0}|^2)$. From Table 1, we obtain Δ = 0.739, 0, and 0.0691 dB when $\tau_0/T_c = 0.5$, 1, and 1.25, respectively.

Summary: We have explained that a correlated tapped delay line model needs to be assumed for transmission performance estimation using Rake combining, even when the propagation channel itself can be characterised as a WSSUS channel. Correlation between tap gains can be computed by taking into account the transmit and receive chip pulse shaping filters. A simple example was presented assuming that the propagation channel has a double spike power delay profile and that an overall transmit and receive chip pulse shaping has a rectangular-spectrum shape. For this propagation channel, a two-finger Rake combining with correlated tap gains was found to have a poorer performance by as much as 0.739 dB, when compared to Rake combining with independent tap gains with $\tau_0/T_c = 0.5$. This clearly suggests that the tap gain correlations must be taken into account for estimating the transmission performance with Rake combining.

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