LETTER

Time- and Frequency-Domain Expressions for Rake Combiner Output SNR

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SUMMARY The frequency- and time-domain expressions are derived for the signal-to-noise power ratio (SNR) of an ideal Rake combiner output in a direct sequence spread spectrum (DS-SS) mobile communication system. The derived SNR expressions make it possible to estimate the SNR statistics after Rake combining for an arbitrary spreading chip rate in the frequency-selective multipath channel.

key words: *spread spectrum, Rake combiner, multipath channel, mobile communication*

1. Introduction

A mobile radio propagation channel is characterized by a frequency selective multipath channel consisting of many propagation paths with different time delays. To overcome the adverse effects of frequency selective multipath channel, the direct sequence spread spectrum (DS-SS) technique can be applied. The direct sequence code division multiple access (DS-CDMA), which has been adopted as the wireless access technique for the 3rd generation mobile communication systems known as IMT-2000 [1], is a multiple access based on DS-SS technique. In a DS-SS receiver, the multipath channel can be resolved into many frequency non-selective channels (or propagation paths), which are then coherently combined by a Rake combiner to improve the transmission performance [2]. It was experimentally confirmed that increasing the spreading chip rate can improve the multipath resolution capability and thereby, the transmission performance [3]. In this paper, the frequency- and time-domain expressions are derived for the signal-to-noise power ratio (SNR) after ideal Rake combining. The derived SNR expressions make it possible to estimate the SNR statistics after Rake combining for an arbitrary spreading chip rate in the frequency selective multipath channel. The effect of spreading chip rate is discussed assuming the ITU-R defined Vehicular B propagation model [4].

2. Expressions for Rake Combiner Output SNR

The DS-spread signal may be expressed using the equivalent lowpass representation as

$$s_T(t)=\sqrt{2S}\sum_{n=-\infty}^{\infty}d_k c_n h_T(t-nT_c), \qquad (1)$$

where $h_T(t)$ is the impulse response of the transmit chip pulse shaping filter having the transfer function $H_T(f)$, S is the average signal power, $\{d_k\}$ is the complex-valued data symbol sequence with $|d_k|=1$ and $k=\lfloor n/N \rfloor$ (N is the spreading factor and $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x), $\{c_n\}$ is the complex-valued random spreading chip sequence with $|c_n|=1$, $E[c_n c_m^*]=1(0)$ if $n=m$ (otherwise) with * denoting complex conjugate and $E[\cdot]$ being the expectation operation, and T_c is the chip duration.

The DS-spread signal is transmitted over a multipath channel. Let $h(t,\tau)$ be the impulse response of the multipath channel at time t due to an excitation at time t-τ. An ideal Rake combining receiver is assumed that has a perfect knowledge of $h(t,\tau)$ and coherently combines all multipath components. The ideal Rake combiner structure is illustrated in Fig. 1 for a discrete time delay model [6]:

$$h(t,\tau)=\sum_{l=0}^{\infty}\xi_l(t)\delta(\tau-\tau_l), \qquad (2)$$

with $\xi_l(t)$ and τ_l being the complex path gain and time delay of the lth propagation path and $\delta(\cdot)$ being the delta function. The Rake finger is a cascade comprising a chip-matched filter (MF) and a code-MF [5]. The chip-MF output can be expressed using the integral representation as

$$r(t)=\int_{-\infty}^{\infty}s_R(\tau)h(t,\tau)d\tau+w(t), \qquad (3)$$

where $s_R(t)$ and $w(t)$ are given by

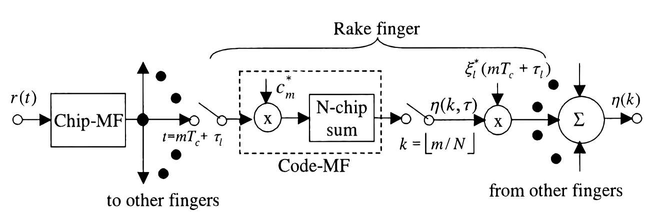

Fig. 1 Rake combiner structure. Discrete time delay model is assumed.

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$$\begin{cases} s_R(t) = \sqrt{2S} \displaystyle\sum_{n=-\infty}^{\infty} d_k c_n h_c(t - nT_c) \\ w(t) = \displaystyle\int_{-\infty}^{\infty} n(\tau) h_R(t-\tau) d\tau \end{cases} . \quad (4)$$

In Eq. (4), $h_c(t) = h_T(t) \otimes h_R(t)$ is the overall transmit and receive chip pulse shaping filter impulse response with $\otimes$ denoting the convolution operation, where $h_R(t)$ is the transfer function of the receive filter and is given by $h_R(t) = (1/T_c)h_T^*(-t)$, and $n(t)$ is the equivalent lowpass additive white Gaussian noise with power spectrum density of N_0. Denoting the transfer function of the receive chip-MF as $H_R(f)$ with $H_R(0) = 1$, the overall transfer function of transmit and receive chip filters is given by $H_c(f) = H_T(f)H_R(f)$, where $H_R(f) = (1/T_c)H_T^*(f)$.

Hereafter, it is assumed that the channel variation in time is slow and thus, the time dependence of $h(t,\tau)$ is omitted for simplicity purpose and the notation $h(\tau)$ is used. The chip-MF output is sampled and fed to code-MF's to resolve the multipath signal components having different time delays. The sampling timing is shifted by τ from the reference timing to retrieve the multipath signal component having time delay τ. The code-MF correlates the sampled sequence $\{r(mT_c+\tau)\}$ with $\{c_m\}$ to obtain $\eta(k,\tau)$:

$$\eta(k,\tau) = \sum_{m=kN}^{(k+1)N-1} r(mT_c+\tau)c_m^*. \quad (5)$$

Then, the output from each Rake finger is multiplied with the complex conjugate of the corresponding channel impulse response at time delay τ and is combined to yield the output of Rake combiner based on the maximal ratio combining (MRC) [2]. The integral representation of the Rake combiner output may be expressed as

$$\eta(k) = \int_{-\infty}^{\infty} \eta(k,\tau)h^*(\tau)d\tau. \quad (6)$$

In the following, the expressions for the SNR of $\eta(k)$ are derived for a large spreading factor, i.e., $N \gg 1$. From Eqs. (3)–(5) and since $E[c_n c_m^*] = 1(0)$ if $n = m$ (otherwise), the signal component $\eta_s(k,\tau)$ in the Rake finger output $\eta(k,\tau)$ is given by

$$\begin{aligned} \eta_s(k,\tau) &= \sqrt{2S}d_k \sum_{m=kN}^{(k+1)N-1} \sum_{n=-\infty}^{\infty} c_n c_m^* \\ &\quad \cdot \left[\int_{-\infty}^{\infty} h(\tau')h_c((m-n)T_c+\tau-\tau')d\tau'\right] \\ &\approx \sqrt{2S}d_k N \int_{-\infty}^{\infty} h(\tau')h_c(\tau-\tau')d\tau'. \quad (7) \end{aligned}$$

To derive Eq. (7), it was assumed that $N \gg 1$ and interpath interference can be neglected. Hence, the signal component $\eta_s(k)$ in the ideal Rake combiner output can be mathematically expressed using an integral form as

$$\begin{aligned} \eta_s(k) &= \int_{-\infty}^{\infty} \eta_s(k,\tau)h^*(\tau)d\tau \\ &\approx \sqrt{2S}d_k N \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^*(\tau)h(\tau')h_c(\tau-\tau')d\tau d\tau'. \end{aligned} \quad (8)$$

On the other hand, the noise component $\eta_n(k,\tau)$ in the Rake finger output $\eta(k,\tau)$ is given by

$$\eta_n(k,\tau) = \sum_{m=kN}^{(k+1)N-1} c_m^* \int_{-\infty}^{\infty} n(\tau')h_R(mT_c+\tau-\tau')d\tau' \quad (9)$$

and thus, the Rake combiner output noise is given by

$$\begin{aligned} \eta_n(k) = \int_{-\infty}^{\infty} n(\tau') \Bigg[\sum_{m=kN}^{(k+1)N-1} c_m^* \int_{-\infty}^{\infty} h^*(\tau) \\ \cdot h_R(mT_c+\tau-\tau')d\tau \Bigg] d\tau'. \quad (10) \end{aligned}$$

Since $0.5E[n(\tau)n*(\tau')] = N_0\delta(\tau-\tau')$ and $h_R(t) = (1/T_c)h_T^*(-t)$, the noise power becomes

$$\begin{aligned} \sigma^2 &= \frac{1}{2}E[|\eta_n(k)|^2] \\ &= N_0 \frac{N}{T_c} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^*(\tau)h(\tau')h_c(\tau-\tau')d\tau d\tau'. \end{aligned} \quad (11)$$

Finally, the following time-domain expression for the SNR γ at the Rake combiner output is obtained:

$$\begin{aligned} \gamma &= \frac{|\eta_s(k)|^2}{\sigma^2} \\ &\approx 2\frac{ST}{N_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_c(\tau-\tau')h(\tau')h^*(\tau)d\tau d\tau', \end{aligned} \quad (12)$$

where $T = NT_c$. Alternatively, the frequency-domain expression is given by

$$\gamma \approx 2\frac{ST}{N_0}\int_{-\infty}^{\infty} H_c(f)|H(f)|^2 df. \quad (13)$$

Since $H_c(f) = H_T(f)H_R(f) = (1/T_c)|H_T(f)|^2$ and $S|H_T(f)|^2/T_c$ gives the transmitted signal power spectrum density,

$$\begin{aligned} &S\int_{-\infty}^{\infty} H_c(f)|H(f)|^2 df \\ &\quad = S\int_{-\infty}^{\infty}\left(\frac{1}{T_c}|H_T(f)|^2\right)|H(f)|^2 df \end{aligned} \quad (14)$$

is the received power of the spectrum-distorted DS-spread signal. Hence, it is understood that the ideal Rake combiner can collect all powers of the received multipath signal components.

Table 1 ITU-R defined Vehicular B propagation channel model.

Tap	Relative delay (ns)	Average gain (dB)
0	0	-2.5
1	300	0
2	8900	-12.8
3	12900	-10.0
4	17100	-25.2
5	20000	-16.0

3. Discussions

First, the effect of spreading bandwidth is discussed using time-domain expression. Equations (12) and (14) clearly show that the Rake combiner output SNR depends on the spreading chip rate (or the spreading bandwidth) through $h_c(t)$ (or $H_c(f)$) and therefore, its statistics does too. Assuming the discrete time delay model of the multipath channel given by Eq. (2), we have

$$\gamma \approx 2\frac{ST}{N_0}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} h_c(\tau_l - \tau_m)\xi_m\xi_l^* \tag{15}$$

from the time-domain expression of Eq. (12). In the following, a rectangular-spectrum chip pulse shaping with bandwidth of $1/T_c$ is assumed as a simple example, i.e., $h_c(t) = \sin(\pi t/T_c)/(\pi t/T_c)$, and how the spreading chip rate affects the SNR statistics is discussed. We consider the ITU-R defined Vehicular B propagation channel model [4] having six Rayleigh faded discrete paths, i.e., l = 0–5 in Eq. (2). The values of average gain $E[|\xi_l|^2]$ and time delay τ_l are listed in Table 1. The cumulative probability distributions of the normalized SNR γ/Γ, where Γ denotes the average SNR, obtained by Monte Carlo simulation, are plotted for $1/T_c = 0.12$, 0.96, 1.92, 3.84 Mchip/s (Mcps) in Fig. 2. As the chip rate increases, the resolution capability of propagation paths can increase and the variations in the achievable SNR after Rake combining can be reduced, thereby improving the transmission performance. This is clearly seen in the figure.

An interesting implication of Eq. (13) is that the Rake combiner output SNR can be found for different chip rates if $|H(f)|^2$ of the multipath channel is measured. For this purpose, we transmit a wideband signal that has a flat power spectrum density over a larger bandwidth than the maximum chip rate and measure its received signal power spectrum shape, $P(f)$, at different time instants (or different receiver locations) by using a spectrum analyzer. Since the measured power spectrum shape is proportional to $|H(f)|^2$, the statistics of the Rake combiner output SNR for different chip rates can be found using Eq. (13).

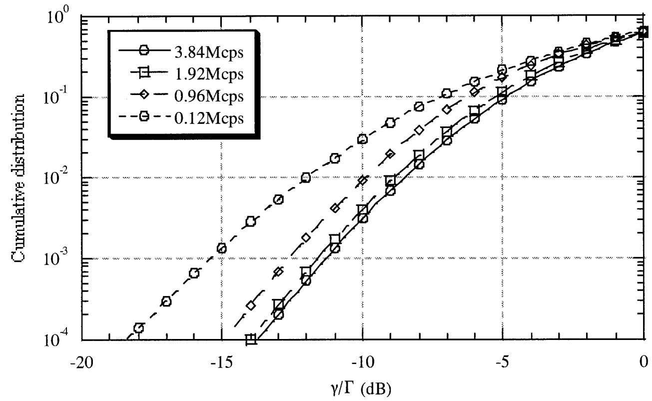

Fig. 2 Computer simulated cumulative distribution function of SNR after Rake combining.

4. Conclusion

In this paper, the frequency- and time-domain expressions for the SNR of an ideal Rake combiner are derived. The derived expressions can be conveniently used for assessing the effect of spreading chip rate (or the spreading bandwidth) on the Rake combining performance. Since, in the analysis, the inter-path interference was ignored (this can be true for a large spreading factor, i.e., $N \gg 1$) and perfect knowledge of multipath channel was assumed, the SNR obtained from the derived SNR expressions is the theoretically achievable maximum one. If the multipath channel model is given, the statistics of the Rake combiner output SNR for an arbitrary spreading chip rate can be found by computer-simulation using the time-domain expression. The frequency-domain expression suggests that the SNR statistics for an arbitrary spreading chip rate in a real propagation channel can be estimated by measuring, using a spectrum analyzer, the power spectrum shape of the received wideband signal having a flat power spectrum density over the maximum spreading bandwidth of interest.

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