LETTER

Capacity Expressions for Power-Controlled Multi-Class DS-CDMA Reverse Link with Antenna Diversity and Rake Combining

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SUMMARY A DS-CDMA mobile communication system accommodating multi-class users is considered. The number of supportable users depends on the distributions of data rate and required communication quality among users. Simple expressions for the reverse link capacity with transmit power control, antenna diversity, and rake combining, are derived for a single-cell system and a multi-cell system.
key words: *DS-CDMA, transmit power control, antenna diversity, rake combining*

1. Introduction

A flexible support for the low-to-high bit rate of multimedia services is required in recent wireless communications. Direct sequence code division multiple access (DS-CDMA) is well suited for this purpose. In a DS-CDMA mobile communication system, transmit power control (TPC) is an indispensable technique to avoid the well-known near-far problem and to reduce the adverse effect of multipath fading [1], [2]. Since the DS-CDMA links are interference-limited, the link capacity can be maximized if the TPC is based on the signal-to-average interference plus background noise power ratio (SINR) [3], [4]. The DS-CDMA reverse link capacity with SINR-based TPC in a single-cell system is theoretically analyzed for the multi-rate users case in [5]; however, antenna diversity and rake combining are not considered. In [4], a numerical computation method of the link capacity of the power controlled reverse link with antenna diversity and rake combining is presented for a multi-cell system, that requires the iterative computation of the mean and variance of the other-cell interference. In this paper, simple capacity expressions for the power controlled multi-class DS-CDMA reverse link with antenna diversity and rake combining are derived. The term "multi-class" means that there are different combinations of data rate and required quality offered in the communication service. Both a single-cell system and a multi-cell system are considered. To avoid too large transmit power with TPC, limitation to the interference power spectrum density is taken into account. The numerical results are graphically presented and compared with those of [4]. The derived capacity and received signal power expressions provide a good understanding of the roles of antenna diversity and rake combining when SINR-based TPC is used.

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2. Analysis

2.1 Single-Cell System

It is assumed that the C users are communicating with a base station with different data rates and different required qualities but with the same spreading chip rate, $1/T_c$. Since the received interference on each antenna at the base station is the contributions from many users, it can be approximated as a Gaussian process and thus, can be combined with the background noise characterized by additive white Gaussian noise (AWGN). Therefore, the bit error rate (BER) performance can be determined by signal energy per bit-to-average interference plus background noise power spectrum density ratio. A frequency selective multipath channel with a uniform power delay profile having L resolvable propagation paths is considered. J-branch antenna diversity reception and L-finger rake reception, both using maximal ratio combining (MRC) [6], [7] are assumed.

Assuming coherent binary phase shift keying (BPSK) data modulation and spreading modulation, the n-th user's spread signal component, which is propagated along with the l-th path and received on the j-th antenna at the base station, may be expressed using the equivalent baseband representation as

$$s_{n,j,l}(t)=\sqrt{2S_n}\mu_{n,j,l}(t)d_n(t-\tau_{n,j,l})c_n(t-\tau_{n,j,l}), \tag{1}$$

where S_n is the average total signal power received on J antennas, $d_n(t)$ and $c_n(t)$ are the BPSK data modulated waveform and the spreading waveform, respectively, $\mu_{n,j,l}(t)$ and $\tau_{n,j,l}$ are the zero-mean complex-valued path gain having unity variance and the time delay of the l-th propagation path seen on the j-th antenna at the base station, respectively ($\tau_{n,j,l}$ may be the same for all j). In the l-th finger of the L-finger rake combiner, the signal received on the j-th antenna is multiplied by the delayed replica of the spreading waveform synchronized to the l-th path time delay, and integrated over one bit duration to get the l-th finger output. Hereafter, time dependency of the path gain is dropped for the sake of simplicity. Assuming perfect channel estimation, the l-th finger output is multiplied by the complex conjugate of $\sqrt{2S_n}\mu_{n,j,l}$ to be coherently combined with other finger outputs to produce the rake combiner output. Denoting the bit length and spreading factor of the n-th user as T_n and G_n, respectively, with $n = 0 \sim (C-1)$, the signal power $P_{S,n}$ and the interference plus background noise power $P_{I+N,n}$ at the rake combiner output for the n-th user is given by

$$\begin{aligned} P_{S,n} &= \left(2S_n \sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2\right)^2 \\ P_{I+N,n} &= \frac{2S_n}{T_n}\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2 \\ &\cdot \left[N_0 + \xi\left(\sum_{k=0}^{C-1}\sum_{q=0}^{L-1}\,_{k\neq n \text{ and } q\neq l} S_k T_c |\mu_{k,j,q}|^2\right)\right] \\ &= \frac{2S_n}{T_n}\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2 \\ &\cdot \left[N_0 + \xi\left(\sum_{k=0}^{C-1}\sum_{l=0}^{L-1} S_k T_c |\mu_{k,j,l}|^2 - S_n T_c |\mu_{n,j,l}|^2\right)\right], \end{aligned} \tag{2}$$

where ξ is the interference suppression factor [8] due to asynchronous assumption of spatially distributed users; $\xi = 1-\beta/4$ for the raised cosine Nyquist chip filter with a roll-off factor β and 2/3 for the square chip pulse shaping. Denoting the instantaneous total received signal power on J antennas as $\hat{S}_n = S_n \sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2$, Eq. (2) becomes

$$\begin{aligned} P_S &= 4\hat{S}_n^2 \\ P_{I+N} &= \frac{2\hat{S}_n}{T_n}\left[N_0 + \xi\sum_{k=0}^{C-1}\hat{S}_k T_c \sum_{l=0}^{L-1} \cdot \left(\frac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right)\right] \\ &\quad -2\xi\hat{S}_n^2\frac{T_c}{T_n}\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}\left(\frac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2. \end{aligned} \tag{3}$$

With SINR-based TPC, the value of $\hat{S}_n$ fluctuates according to variations in the average interference power. The average interference plus background noise power is the ensemble average of P_{I+N}. Denoting the ensemble average operation over the fading statistics by $E[.]$, the SINR at the rake combiner output can be expressed as

$$SINR = \frac{P_{S,n}}{E[P_{I+N,n}]} = 2\gamma_n \tag{4a}$$

with

$$\gamma_n = \frac{\hat{S}_n T_n}{\begin{aligned} &N_0 + \xi\sum_{k=0}^{C-1}\hat{S}_k T_c \sum_{l=0}^{L-1} E\left[\frac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right] \\ &-\xi\hat{S}_n T_c \sum_{j=0}^{J-1}\sum_{l=0}^{L-1} E\left[\left(\frac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2\right]\end{aligned}}, \tag{4b}$$

where γ_n represents the signal energy per bit-to-average interference plus background noise power spectrum density ratio, which determines the BER. Assuming a uniform power delay profile, i.e., $E[|\mu_{k,j,l}|^2] = 1/L$ for all l, $E[(|\mu_{k,j,l}|^2/\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2)]$ and $E[(|\mu_{n,j,l}|^2/\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2)^2]$ are respectively the same for all j and l and we have [4], [12]

$$\begin{aligned} E\left[\left(\frac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right)\right] &= \frac{1}{JL} \\ E\left[\left(\frac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2\right] &= \frac{2}{JL(JL+1)}. \end{aligned} \tag{5}$$

Substituting Eq. (5) into Eq. (4), we obtain

$$\gamma_n = \frac{\hat{S}_n T_n}{N_0 + \frac{\xi}{J}\sum_{k=0}^{C-1}\hat{S}_k T_c - \frac{2\xi}{JL+1}\hat{S}_n T_c}. \tag{6}$$

With SINR-based TPC, γ_n is kept at the target value $\gamma_{0,n}$. Hence,

$$\hat{S}_n T_c = \frac{\gamma_{0,n}}{G_n + \frac{2\xi}{JL+1}\gamma_{0,n}}\left[N_0 + \frac{\xi}{J}\sum_{n=0}^{C-1}\hat{S}_n T_c\right], \tag{7}$$

where $G_n = T_n/T_c$ is the spreading factor. Remembering that $[N_0 + (\xi/J)\sum_{n=0}^{C-1}\hat{S}_n T_c]$ of Eq. (7) is the same for all n and by summing up Eq. (7) for all n, we get

$$\sum_{n=0}^{C-1}\hat{S}_n T_c = N_0 \frac{\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n + (2\xi/(JL+1))\gamma_{0,n}}}{1 - \frac{\xi}{J}\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n + (2\xi/(JL+1))\gamma_{0,n}}}. \tag{8}$$

Finally, substituting the above into Eq. (7), we obtain

$$\hat{S}_n T_c = \frac{\gamma_{0,n}}{G_n + \dfrac{2\xi}{JL+1}\gamma_{0,n}} \cdot \frac{N_0}{1 - \dfrac{\xi}{J}\displaystyle\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n + (2\xi/(JL+1))\gamma_{0,n}}}. \tag{9}$$

If the interference plus background noise power spectrum density, $N_0 + (1/J)\sum_{n=0}^{C-1}\hat{S}_n T_c$, seen on each antenna is limited to αN_0, the supportable number C of users is upper bounded, from Eq. (8), by

$$\xi\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n+(2\xi/(JL+1))\gamma_{0,n}} \le J\frac{\xi(\alpha-1)}{\xi(\alpha-1)+1}. \tag{10}$$

This clearly shows that the use of J-branch antenna diversity can increase the link capacity by J times. Substituting Eq. (10) into Eq. (9), the received signal power of the n-th user is upper bounded by

$$\hat{S}_n T_c \le \{\xi(\alpha-1)+1\}N_0\frac{\gamma_{0,n}}{G_n+(2\xi/(JL+1))\gamma_{0,n}}, \tag{11}$$

from which the maximum transmit power can be computed for the given propagation parameters (path loss exponent and shadowing loss standard deviation [9]). Equations (10) and (11) are the general expressions with antenna diversity and rake combining.

2.2 Multi-Cell System

It is assumed that users are uniformly distributed over an entire area. The number of users per cell in communication is C. Since the other-cell interference is the contributions from many users, it is approximated as a Gaussian process as in the single-cell system. The statistics of other-cell interference seen at each base station are equally likely. The average other-cell interference power is the same for all cells, and is f times larger than the own-cell total received signal power, i.e., $\sum_{n=0}^{C-1}\hat{S}_n$. The value of f depends on the path loss exponent and shadowing loss standard deviation [10]. As discussed in [4], f needs to be modified when TPC is employed since TPC varies the transmit power in order to keep $S_n\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2 = \hat{S}_n$. Therefore, the average interference power increases by a factor of $E[1/\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2] = L/(JL-1)$ for a uniform power delay profile [4, Eq. (43)]. Letting $f_{tpc} = fL/(JL-1)$, the received signal energy per bit-to-average interference plus background noise power spectrum density ratio, γ_n, can be given by

$$\gamma_n = \frac{\hat{S}_n T_n}{N_0 + \left(\dfrac{\xi}{J}\displaystyle\sum_{k=0}^{C-1}\hat{S}_k T_c - \dfrac{2\xi}{JL+1}\hat{S}_n T_c\right) + f_{tpc}\cdot\dfrac{\xi}{J}\left(\displaystyle\sum_{k=0}^{C-1}\hat{S}_k T_c\right)}. \tag{12}$$

The second and third terms of the denominator of Eq. (12) are respectively the interference contributions from the own cell and other cells. Similarly to the single-cell case, by letting $\gamma_n = \gamma_{0,n}$, we get

$$\hat{S}_n T_c = \gamma_{0,n}\frac{N_0 + (1+f_{tpc})(\xi/J)\displaystyle\sum_{n=0}^{C-1}\hat{S}_n T_c}{G_n + (2\xi/(JL+1))\gamma_{0,n}}, \tag{13}$$

from which we obtain

$$\sum_{n=0}^{C-1}\hat{S}_n T_c = \left(N_0 + (1+f_{tpc})\frac{\xi}{J}\sum_{n=0}^{C-1}\hat{S}_n T_c\right) \cdot \sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n + (2\xi/(JL+1))\gamma_{0,n}}. \tag{14}$$

Hence, the total received signal power spectrum density with SINR-based TPC is given by

$$\sum_{n=0}^{C-1}\hat{S}_n T_c = N_0 \cdot \frac{\displaystyle\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n + (2\xi(JL+1))\gamma_{0,n}}}{1-(1+f_{tpc})\dfrac{\xi}{J}\displaystyle\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n+(2\xi/(JL+1))\gamma_{0,n}}}. \tag{15}$$

Substituting the above into Eq. (13) leads to

$$\hat{S}_n T_c = \frac{\gamma_{0,n}}{G_n + \dfrac{2\xi}{JL+1}\gamma_{0,n}} \cdot \frac{N_0}{1-(1+f_{tpc})\dfrac{\xi}{J}\displaystyle\sum_{n=0}^{C-1}\frac{\gamma_{0,n}}{G_n+(2\xi/(JL+1))\gamma_{0,n}}}. \tag{16}$$

Assume that the average interference plus background noise power spectrum density experienced on each diversity antenna is limited to αN_0, i.e.,

$$N_0 + \frac{1+f_{tpc}}{J}\left(\sum_{n=0}^{C-1}\hat{S}_n T_c\right) \le \alpha N_0. \tag{17}$$

From Eq. (17), we have $\sum_{n=0}^{C-1}\hat{S}_n T_c \le N_0 J(\alpha-1)/(1+f_{tpc})$. Substituting this into Eq. (15) leads to

$$\xi \sum_{n=0}^{C-1} \frac{\gamma_{0,n}}{G_n + (2\xi/(JL+1))\gamma_{0,n}} \leq \frac{J}{1+f_{tpc}} \frac{\xi(\alpha-1)}{\xi(\alpha-1)+1}. \quad (18)$$

Comparison of Eqs. (10) and (18) shows that the link capacity of multi-cell case is simply reduced by a factor of $(1+f_{tpc})$ from that of the single-cell case. It can be shown from Eq. (16) that the maximum received signal power is identical to Eq. (11).

Let's consider a single-cell system ($f_{tpc}=0$) with a single propagation path ($L=1$), no antenna diversity ($J=1$) and the same bit rate for all users ($G_n = G$). Assuming $G \gg \xi\gamma_{0,n}$ and $\alpha \to \infty$, we have from Eq. (18)

$$\frac{\xi}{G} \sum_{n=0}^{C-1} \gamma_{0,n} = 1, \quad (19)$$

which can also be found from [5]. If $\gamma_{0,n} = \gamma_0$ for all n and $\alpha \to \infty$, we have the well-known result [11]:

$$C = 1 + \frac{G}{\xi\gamma_0}. \quad (20)$$

3. Numerical Computation and Discussion

Assume that the bit rate and the target value of the signal energy per bit-to-average interference plus background noise power spectrum density ratio are respectively the same for all users, i.e., $G_n = G$ and $\gamma_{0,n} = \gamma_0$ for all n. Since $f_{tpc} = fL(JL-1)$ for a uniform power delay profile, we obtain from Eqs. (16) and (18)

$$\frac{C_{\max}}{G} = \frac{1}{\xi\gamma_0}\left(1 + \frac{2\xi}{JL+1}\frac{\gamma_0}{G}\right)\frac{J}{1+fL/(JL-1)} \cdot \frac{\xi(\alpha-1)}{\xi(\alpha-1)+1}$$

$$\frac{\hat{S}_{\max}T_c}{N_0} = \{\xi(\alpha-1)+1\}\frac{\gamma_0}{G+2\xi\gamma_0/(JL+1)}, \quad (21)$$

where $C_{\max}$ and $\hat{S}_{\max}$ are the maximum allowable number of communicating users and the maximum value (required when $C = C_{\max}$) of each user's total received signal power (the sum of signal powers received on J antennas), respectively. We assume the path loss exponent $\mu = 4$, the shadowing standard deviation $\sigma = 8$, $\xi = 2/3$ (i.e., the square chip pulse shaping), $G = 128$ and $\gamma_0 = 1.92$ (2.84 dB) for all users as assumed in [4]. In [4], the total received signal power is limited to $\hat{S}_{\max}T_c/N_0 = 0$ dB. In our case, the rise of the interference power spectrum density is limited by controlling the value of α, and $\hat{S}_{\max}T_c/N_0 = 0$ dB corresponds to $\alpha \approx 100$. Note that $f = 0$ for a single-cell system and 0.659 for a multi-cell system (f equals $M(\mu, \sigma)$ of [4] and is 0.659 for the path loss exponent $\mu = 4$ and shadowing loss standard deviation $\sigma = 8$). The normalized capacities for the single and multi-cell systems with $\alpha \to \infty$ are plotted in Fig. 1 as a function of J with L as a parameter. As was expected, the achievable capacity is almost linearly proportionate to the number J of antennas. In the case of single-cell system, the capacity is almost independent of L. For the multi-cell system case, however, the capacity increases as L increases from 1 to 2. Additional gain in the capacity becomes smaller as L increases beyond $L = 2$. The capacities for no diversity ($J = 1$) are listed in Table 1. The results are almost identical to [4].

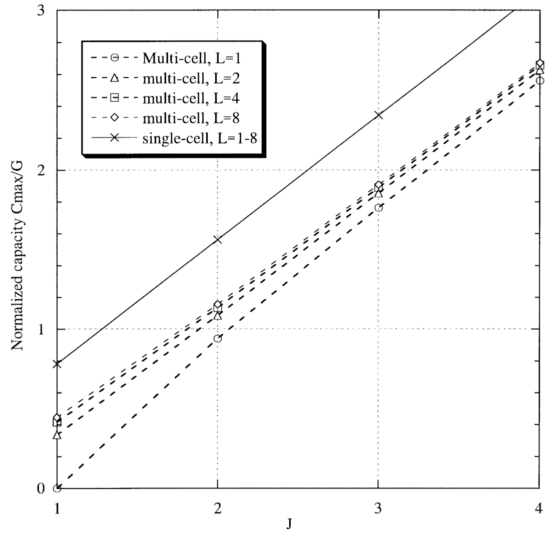

Fig. 1 Normalized capacities of a single-cell system and a multi-cell system.

Table 1 Link capacities of a single-cell system and a multi-cell system with no antenna diversity ($J = 1$).

L	1	2	4	8
Single	0.78	0.78	0.78	0.78
Multiple	0	0.34	0.42	0.45
Multiple/Single	0	0.43	0.53	0.57

The value of α for achieving 90% of $C_{\max,\alpha\to\infty}$ becomes $\alpha = 14.5$ (11.6 dB) since

$$\frac{C_{\max}}{G} \Big/ \left(\frac{C_{\max}}{G}\right)_{\alpha\to\infty} = \frac{2(\alpha-1)}{2\alpha+1}. \quad (22)$$

It is found from Eq. (21) that the received signal energy per chip-to-background noise power spectrum density ratio $\hat{S}_{\max}T_c/N_0$ rises to approximately 0.15 (−8.24 dB) almost irrespectively of J and L.

Next, we consider two classes of data rate users, $G_0 = 128$ and $G_1 = 32$, but requiring the same $\gamma_0 = 1.92$. Using Eq. (18), the relation between C_0 and C_1 is plotted in Fig. 2 for various values of L when $J = 1$ and

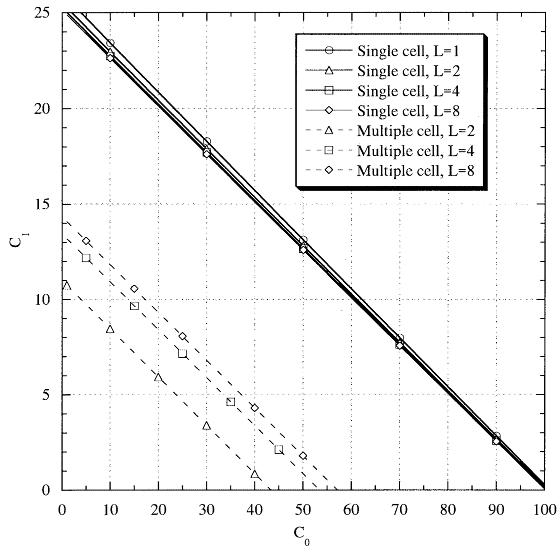

Fig. 2 Effect of number L of paths on capacities of a single-cell system and a multi-cell system. Two classes of data rate users, $J = 1$, and $\alpha \to \infty$.

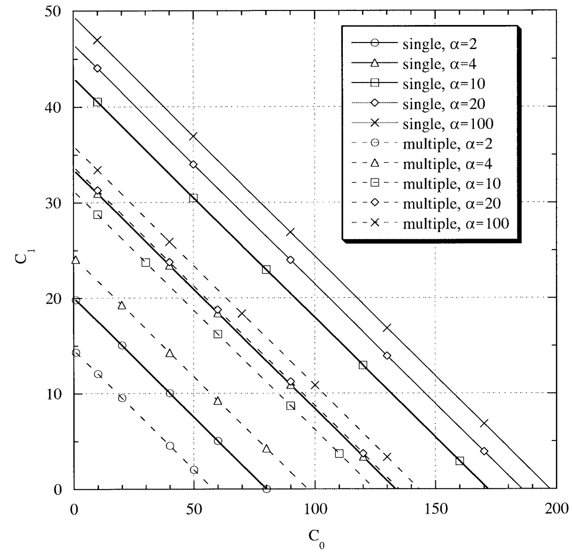

Fig. 4 Effect of α on capacities of a single-cell system and a multi-cell system. Two classes of data rate users, $J = 2$, and $L = 4$.

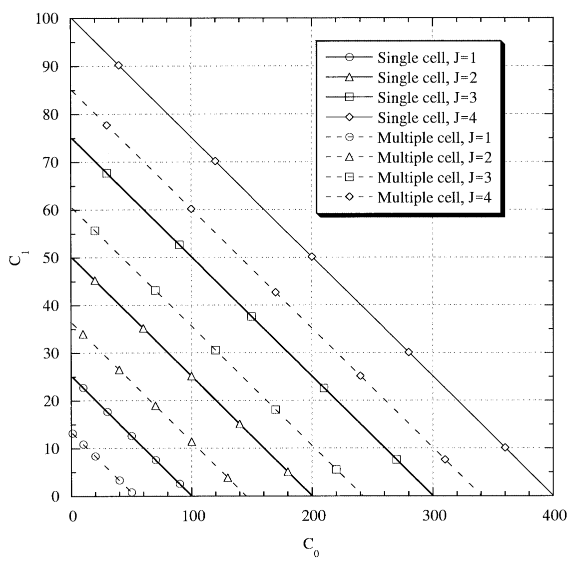

Fig. 3 Effect of number J of antennas on capacities of a single-cell system and a multi-cell system. Two classes of data rate users, $L = 4$, and $\alpha \to \infty$.

$\alpha \to \infty$. The results for various values of J are plotted in Fig. 3 when $L = 4$ and $\alpha \to \infty$. Finally, the results for various values of α are plotted in Fig. 4 when $J = 2$ and $L = 4$.

4. Conclusions

Simple capacity expressions for the power controlled multi-class DS-CDMA reverse link with antenna diversity and rake combining were derived for a single-cell system and a multi-cell system. A uniform power delay profile of the propagation channel was assumed. To avoid too large transmit power with TPC, limitation to the interference power spectrum density was considered. The derived capacity expressions allow to include the distributions of data rate and required quality among users with the same chip rate. Using the derived capacity expressions, the reverse link capacities were computed and compared with those of [4] to find that almost identical results are obtained. Despite of the assumption of uniform power delay profile, the derived capacity expressions provide a good understanding of the roles of antenna diversity and rake combining when SINR-based TPC is used. In this paper, channel coding is not considered. However, the obtained expressions can be applied to a coded DS-CDMA system by replacing G_n and $\gamma_{0,n}$ with the processing gain and the target value of γ_n after channel decoding, respectively. An extension of the analysis to other power delay profile shapes of the multipath channel is left as a future study.

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