LETTER

Reverse Link Capacity of Orthogonal Multi-Code DS-CDMA with Multiple Connections

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SUMMARY DS-CDMA provides a flexible support for the low-to-high bit rate of multimedia services upon a specific user's request. A simple capacity expression is derived for a power-controlled reverse link of orthogonal multi-code DS-CDMA with multiple connections. It is found that an orthogonal multi-code user having multiple connections is equivalent to a single connection user, but with a spreading factor reduced by a factor of the total number of parallel codes and a required signal energy per symbol-to-interference plus noise power spectrum density ratio which is the average taken over multiple connections. Furthermore, the use of antenna diversity is found equivalent to the use of higher spreading factor increased by a factor of the number of antennas.

key words: *orthogonal multi-code system, DS-CDMA, reverse link capacity, mobile communication system*

1. Introduction

In multimedia wireless communications, a flexible support for the low-to-high bit rate of multimedia services upon a specific user's request is required. Direct sequence code division multiple access (DS-CDMA) is well suited for this purpose. High-rate CDMA is implemented by assigning one mother root code of a subtree of maximum length orthogonal code [1], or equivalently, using the maximum length codes in parallel [2]. The former is called the orthogonal variable spreading factor (OVSF) approach and the latter the orthogonal multi-code approach. DS-CDMA can allow multiple connections (with different data rates and qualities) per user. There may be two ways to realize multiple connections. Data sequences of multiple connections are channel-coded using different channel codes, depending on their requested communication quality and type of data traffic (e.g., continuous traffic or packet traffic). In the OVSF approach, channel-coded data sequences associated with multiple connections are time-multiplexed into a single, high rate data sequence to be spread by a single OVSF code. On the other hand, in the orthogonal multi-code approach, the channel-coded sequence of each connection is serial-to-parallel converted into multiple data sequences, each being spread by different orthogonal spreading code with identical spreading factor. The orthogonal multi-code approach has the following features:

(a) Multiple connections with different data rates and communication qualities can be realized without changing symbol timing.
(b) Multi-rate transmission can be realized on each connection without changing symbol timing. (Of course, the OVSF approach can be applied to each connection, but this is not considered here.)
(c) Transmit power control (TPC) can be independent for each connection.
(d) One of the disadvantages of the multi-code approach is the large peak-to-average power ratio.

In this letter, we consider orthogonal multi-code DS-CDMA with multiple connections per user and derive an expression for the reverse link capacity.

2. Analysis

A model of mobile transmitter with multi-code and multiple connections is shown in Fig. 1. At the n-th mobile user transmitter, the high speed data sequence of the i-th connection is serial-to-parallel converted to $R_{n,i}$ low speed parallel data sequences to be spread by $R_{n,i}$ orthogonal spreading code sequences. The total number of spreading codes is $\sum_i R_{n,i}$. When the spreading chip rate and the spreading factor of each spreading code are taken to be $1/T_c$ and G, respectively, the symbol length of parallel data sequences is $T = GT_c$, which is the same for all users. A frequency-selective Rayleigh channel with a uniform power delay profile having L resolvable propagation paths is assumed. At the base station, J-branch antenna diversity reception and L-finger rake reception, both using maximal ratio combining [3], are assumed.

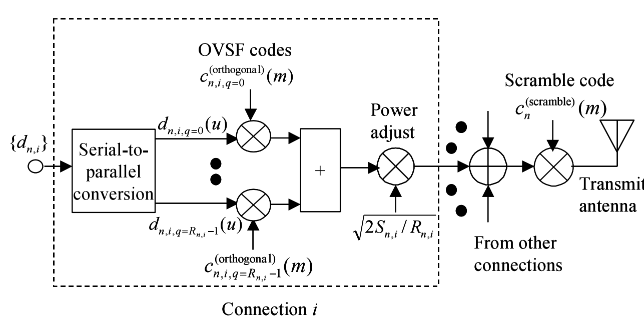

Fig. 1 A model of mobile transmitter with multi-code and multiple connections (discrete time representation is used).

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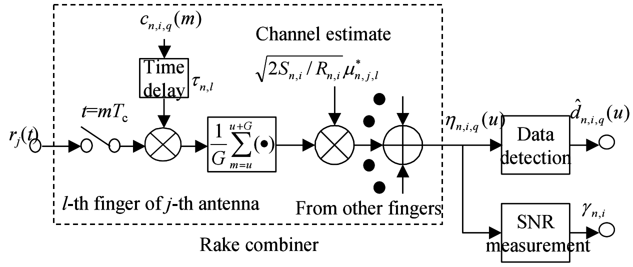

Fig. 2 Rake combiner for i-th connection of n-th user.

The received signal on the j-th antenna can be expressed as

$$r_j(t) = \sum_{n=0}^{C-1}\sum_{l=0}^{L-1}\mu_{n,j,l}(t)s_n(t-\tau_{n,l}) + w_j(t), \tag{1}$$

where C is the number of simultaneously transmitting users, $s_n(t)$ is the n-th user's spread signal, $\mu_{n,j,l}(t)$ and $\tau_{n,l}$ are the complex gain and time delay of the l-th propagation path for the n-th user seen on the j-th antenna (time delay is assumed identical for all antennas), respectively, and $w_j(t)$ is the additive white Gaussian noise with power spectrum density N_0. $s_n(t)$ is expressed in the equivalent lowpass representation as

$$s_n(t) = \sum_i \sqrt{2S_{n,i}/R_{n,i}}\left(\sum_{q=0}^{R_{n,i}-1}\sum_{m=-\infty}^{\infty} d_{n,i,q}(u) \cdot c_{n,i,q}(m)h(t-mT_c)\right). \tag{2}$$

In the above equation, $S_{n,i}$ is the total average signal power of the i-th connection, $\{d_{n,i,q}(u)\}$ is the complex-valued data symbol sequence with $\mathrm{E}[|d_{n,i,q}(u)|^2] = 1$ and $u = \lfloor m/G \rfloor$, where E[.] represents ensemble average operation and $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x. $c_{n,i,q}(m) = c^{(\text{orthogonal})}_{n,i,q}(m-uG)c^{(\text{scramble})}_n(m)$ with $|c_{n,i,q}(m)| = 1$, where $c^{(\text{orthogonal})}_{n,i,q}(m)$ and $c^{(\text{scramble})}_n(m)$ are the orthogonal spreading chip sequence with G-chip repetition period and the scramble sequence, respectively. $h(t)$ is the impulse response of the overall (transmit plus receive) chip pulse-shaping filter and T_c is the chip duration.

Without loss of generality, the i-th connection of the n-th user is considered. The received interference on each antenna is the sum of self-interference (due to inter-path interference) and other-user interference and can be approximated as a Gaussian process. Thus, the sum of interference and background noise can be treated as an equivalent Gaussian noise. Since the bit error rate performance of data transmission can be determined by the signal-to-interference plus noise power ratio (SINR) after rake combining, it is assumed that TPC is based on SINR after rake combining and independently performed on each connection. In the l-th finger of the L-finger rake combiner, the signal received at the j-th antenna $r_j(t)$ is multiplied by the delayed replica of the spreading sequence $c_{n,i,q}(m)$ synchronized to the l-th path time delay, and integrated over one parallel data symbol duration $T = GT_c$ to get the l-th finger output (sum of the data symbol component $\sqrt{2S_{n,i}/R_{n,i}}\mu_{n,j,l}d_{n,i,q}(u)$ and interference plus noise component). Hereafter, time dependency of the path gain is dropped for the sake of simplicity. Assuming perfect channel estimation, the l-th finger output is multiplied by the complex conjugate of $\sqrt{2S_{n,i}/R_{n,i}}\mu_{n,j,l}$ to be coherently combined with other finger outputs to produce the rake combiner output $\eta_{n,i,q}(u)$, which is the decision variable for the recovered data $\hat{d}_{n,i,q}(u)$.

Remember that transmitted data sequences $\{d_{n,i,q}(u)\}$ are independent for all n, i, q and that the spreading sequences $\{c_{n,i,q}(m)\}$ of the same user are orthogonal, but those among different users are independent due to different scramble sequences. Furthermore, remember that own-user interference is produced only due to inter-path interference (no interference among different connections is produced within each path). It can be shown that the signal component $\eta^{(s)}_{n,i,q}(u)$ and noise (plus interference) power $N_{n,i,q}(u)$ at the rake combiner output are given by

$$\begin{cases} \eta^{(s)}_{n,i,q}(u) = 2\dfrac{S_{n,i}}{R_{n,i}}\left(\displaystyle\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2\right)d_{n,i,q}(u) \\ N_{n,i,q}(u) = 2\dfrac{S_{n,i}}{R_{n,i}}\displaystyle\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2 \\ \quad \cdot\left[\dfrac{N_0}{T} + \dfrac{\xi}{G}\left(\displaystyle\sum_{k=0}^{C-1}S_k\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2 - S_n|\mu_{n,j,l}|^2\right)\right] \end{cases}, \tag{3}$$

where $S_n = \sum_i S_{n,i}$ is the total received signal power of the n-th user and ξ is the interference suppression factor [4] due to the time-asynchronous assumption of spatially distributed users; $\xi = 1-\rho/4$ for the raised cosine Nyquist chip filter with a roll-off factor ρ and 2/3 for the square chip pulse shaping.

With SINR-based TPC, the total desired signal power (over $R_{n,i}$ codes), $S_{n,i}\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2$, after rake combining for the i-th connection is kept at $\hat{S}_{n,i}$ so that the SINR after rake combining meets the target. It should be noticed that, due to SINR-based TPC, the value of $\hat{S}_{n,i}$ varies according to variations in the average interference power. Equation (3) becomes

$$\begin{cases} \eta^{(s)}_{n,i,q}(u) = \dfrac{2\hat{S}_{n,i}}{R_{n,i}} d_{n,i,q}(u) \\ N_{n,i,q}(u) = \dfrac{2\hat{S}_{n,i}}{R_{n,i}T} \\ \quad \cdot \left[N_0 + \xi \displaystyle\sum_{k=0}^{C-1} \hat{S}_k T_c \left(\dfrac{\sum_{l=0}^{L-1} |\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{k,j,l}|^2} \right) \right. \\ \quad \left. -\xi \hat{S}_n T_c \dfrac{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^4}{\left(\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2 \right)^2} \right] \end{cases} \quad (4)$$

The average signal power of the i-th connection is the same for all q ($q = 0 \sim R_{n,i} - 1$). The average noise power is the ensemble average of $N_{n,i,q}$. Therefore, SINR after rake combining can be expressed as

$$\begin{aligned} SINR &= \frac{\mathrm{E}[|\eta^{(s)}_{n,i,q}(u)|^2]}{\mathrm{E}[N_{n,i,q}(u)]} = 2\gamma_{n,i} \\ &= 2(G/R_{n,i})\hat{S}_{n,i}T_c \Bigg/ \Bigg\{ N_0 + \xi \sum_{k=0}^{C-1} \hat{S}_k T_c \\ &\quad \cdot \mathrm{E}\left[\sum_{l=0}^{L-1} \left(\frac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{k,j,l}|^2} \right) \right] \\ &\quad -\xi \hat{S}_n T_c \mathrm{E}\left[\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} \left(\frac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2} \right)^2 \right] \Bigg\} \end{aligned} \quad (5)$$

for all q. In Eq. (5), $\gamma_{n,i}$ represents *the signal energy per data symbol-to-interference plus noise power spectrum density ratio*. We have from Appendix

$$\begin{cases} \mathrm{E}\left[\displaystyle\sum_{l=0}^{L-1} \left(\dfrac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{k,j,l}|^2} \right) \right] \\ \quad = L\mathrm{E}\left[\dfrac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{k,j,l}|^2} \right] = \dfrac{1}{J} \\ \mathrm{E}\left[\displaystyle\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} \left(\dfrac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2} \right)^2 \right] \\ \quad = JL\mathrm{E}\left[\left(\dfrac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1} |\mu_{n,j,l}|^2} \right)^2 \right] = \dfrac{2}{JL+1} \end{cases} \quad (6)$$

Substituting Eq. (6) into Eq. (5) gives

$$\gamma_{n,i} = \frac{(G/R_{n,i})\hat{S}_{n,i}T_c}{N_0 + \dfrac{\xi}{J}\left(\displaystyle\sum_{k=0}^{C-1} \hat{S}_k T_c - \dfrac{2J}{JL+1}\hat{S}_n T_c \right)}. \quad (7)$$

With SINR-based TPC, $\gamma_{n,i}$ is kept at the required value $\hat{\gamma}_{n,i}$. Since $\hat{S}_n = \sum_i \hat{S}_{n,i}$ and the denominator of Eq. (7) is the same for all i,

$$\sum_i R_{n,i}\hat{\gamma}_{n,i} = \frac{G\hat{S}_n T_c}{N_0 + \xi\left(\dfrac{1}{J}\displaystyle\sum_{k=0}^{C-1} \hat{S}_k T_c - \dfrac{2}{JL+1}\hat{S}_n T_c \right)}. \quad (8)$$

Letting

$$\begin{cases} \bar{\gamma}_n = \dfrac{\sum_i R_{n,i}\hat{\gamma}_{n,i}}{\sum_i R_{n,i}} \\ \bar{G}_n = \dfrac{G}{\sum_i R_{n,i}} \end{cases} \quad (9)$$

we have from Eq. (8)

$$\hat{S}_n T_c = \frac{\bar{\gamma}_n}{\bar{G}_n + \dfrac{2\xi}{JL+1}\bar{\gamma}_n} \left[N_0 + \frac{\xi}{J}\sum_{n=0}^{C-1} \hat{S}_n T_c \right], \quad (10)$$

where $\bar{\gamma}_n$ represents the average (over multiple connections) required signal energy per symbol-to-interference plus background noise power spectrum density ratio and $\bar{G}_n$ represents the spreading factor equivalent to a single user. Taking the summation of Eq. (10) with respect to n, we obtain

$$\frac{\sum_{n=0}^{C-1} \hat{S}_n T_c}{N_0} = \frac{\displaystyle\sum_{n=0}^{C-1} \frac{\bar{\gamma}_n}{\bar{G}_n + (2\xi/(JL+1))\bar{\gamma}_n}}{1 - \dfrac{\xi}{J}\displaystyle\sum_{n=0}^{C-1} \frac{\bar{\gamma}_n}{\bar{G}_n + (2\xi/(JL+1))\bar{\gamma}_n}}. \quad (11)$$

If admission control is introduced so that the interference plus noise power spectrum density, $N_0 + (1/J)\sum_{n=0}^{C-1} \hat{S}_n T_c$, seen on each diversity antenna is kept below αN_0 (α is called the interference rise factor), the maximum number $C_{\max}$ of simultaneously communicating users is found from

$$\frac{1}{J}\sum_{n=0}^{C_{\max}-1} \frac{\bar{\gamma}_n}{\bar{G}_n/\xi + \bar{\gamma}_n 2/(JL+1)} = \frac{\alpha-1}{\alpha-1+1/\xi}. \quad (12)$$

It is seen from Eq. (12) that an orthogonal multi-code user with multiple connections is equivalent to a user of single connection with spreading factor of $\bar{G}_n$ and required signal energy per bit-to-interference plus noise power spectrum density ratio of $\bar{\gamma}_n$. Furthermore, it is seen from Eq. (12) that the use of J-branch antenna diversity is equivalent to J times increase in the spreading factor.

Substituting Eq. (11) into Eq. (10), the total received signal power $\hat{S}_n$ of the n-th user becomes

$$\frac{\hat{S}_n T_c}{N_0} = \frac{\dfrac{\bar{\gamma}_n}{\bar{G}_n + (2\xi/(JL+1))\bar{\gamma}_n}}{1 - \dfrac{\xi}{J}\displaystyle\sum_{n=0}^{C-1} \frac{\bar{\gamma}_n}{\bar{G}_n + (2\xi/(JL+1))\bar{\gamma}_n}} \quad (13)$$

and, from Eqs. (7), (8), and (9), the received signal power per code, $\hat{S}_{n,i}/R_{n,i}$, for the i-th connection is given by

$$\frac{(\hat{S}_{n,i}/R_{n,i})T_c}{N_0} = \frac{\hat{\gamma}_{n,i}}{(G/\bar{G}_n)\bar{\gamma}_n}\frac{\hat{S}_n T_c}{N_0}, \quad (14)$$

which can be used to estimate the required maximum transmit power from a mobile. The required received signal power increases as the number of simultaneously communicating users increases. By introducing the interference rise factor, too large received signal power is avoided. The maximum value of $(\hat{S}_n T_c/N_0)$ is given by

$$\left(\frac{\hat{S}_n T_c}{N_0}\right)_{\max} = \frac{\bar{\gamma}_n}{\bar{G}_n/\xi + \bar{\gamma}_n 2/(JL+1)}(\alpha - 1 + 1/\xi), \quad (15)$$

As a special case, assume the same distributions of bit rates and required qualities for all users ($\bar{G}_n = \bar{G}$ and $\bar{\gamma}_n = \bar{\gamma}$). Then, from Eq. (12), we have

$$\begin{cases} C_{\max} = \left\lfloor J\dfrac{\alpha - 1}{\alpha - 1 + 1/\xi}\left(\dfrac{2}{JL+1} + \dfrac{\bar{G}}{\xi\bar{\gamma}}\right)\right\rfloor \\ \left(\dfrac{\hat{S}_n T_c}{N_0}\right)_{\max} = \dfrac{\bar{\gamma}(\alpha - 1 + 1/\xi)}{\bar{G}/\xi + \bar{\gamma}2/(JL+1)} \end{cases}. \quad (16)$$

For no antenna diversity reception ($J = 1$), a single propagation path ($L = 1$), and $\alpha \to \infty$, Eq. (16) gives the link capacity of single connection per user with the spreading factor of $\bar{G}$ and the required signal energy per bit-to-interference plus noise power spectrum density ratio of $\bar{\gamma}$ [5].

3. Conclusions

The simple expression for the reverse link capacity of the power-controlled orthogonal multi-code DS-CDMA with multiple connections was derived. Once the data rate and required quality of each connection are given, the maximum number of simultaneously communicating users can be conveniently computed from Eq. (12). It was found that:

(a) An orthogonal multi-code user with multiple connections is equivalent to a user of single connection with the spreading factor of $\bar{G}_n$ and the required signal energy per symbol-to-interference plus noise power spectrum density ratio of $\bar{\gamma}_n$, where $\bar{G}_n$ and $\bar{\gamma}_n$ are defined in Eq. (9).

(b) The use of antenna diversity is equivalent to the use of higher spreading factor increased by a factor of the number of antennas.

It should be noted that one of the disadvantages of the multi-code approach is the large peak-to-average power ratio. In this letter, a single-cell system is considered and a uniform power delay profile of the multipath channel is assumed. Capacity analysis for a multi-cell system and other power delay profiles is left for future study.

References

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Appendix

Since we are assuming identical uniform power delay profile seen on all diversity antennas, i.e., $\mathrm{E}[|\mu_{n,j,l}|^2] = 1/L$ and that the fading statistics are identical and independent for all antennas, Eq. (6) becomes

$$\begin{cases} \mathrm{E}\left[\displaystyle\sum_{l=0}^{L-1}\left(\frac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right)\right] \\ \quad = L\mathrm{E}\left[\dfrac{|\mu_{k,0,0}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right] \\ \mathrm{E}\left[\displaystyle\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}\left(\frac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2\right] \\ \quad = JL\mathrm{E}\left[\left(\dfrac{|\mu_{n,0,0}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2\right] \end{cases} \quad (\mathrm{A}\cdot 1)$$

Hence, the problem is rewritten as a general case of finding the first and second moments of t:

$$t = \frac{x_0}{\sum_{m=0}^{JL-1} x_m} = \frac{x_0}{x_0 + y}, \qquad (\mathrm{A}\cdot 2)$$

where $\{x_m\}$ are independently and identically distributed variables having exponential distribution with unity mean and $y = \sum_{m=1}^{JL-1} x_m$. Probability density function (pdf) of y follows a chi-square distribution with $2(JL-1)$-degrees of freedom [3]. Since x_0 and y are independent, the joint pdf of x_0 and y is given by

$$p(x_0, y) = p(x_0)p(y) = \frac{y^{JL-2}}{(JL-2)!} \exp[-x_0 - y]. \qquad (\mathrm{A}\cdot 3)$$

The transformation of $t = x_0/(x_0 + y)$ is introduced. Since $y = x_0(1/t - 1)$, the joint pdf of x_0 and t is obtained from Eq. (A·3) as

$$p(x_0, t) = \frac{1}{(JL-2)!} \frac{(1-t)^{JL-2}}{t^{JL}} x_0^{JL} \exp[-x_0/t], \qquad (\mathrm{A}\cdot 4)$$

from which the pdf of t can be found as

$$p(t) = \int_0^\infty p(x_0, t) dx_0 = (JL-1)(1-t)^{JL-2}. \qquad (\mathrm{A}\cdot 5)$$

Therefore, we have

$$\begin{cases} \mathrm{E}[t] = \dfrac{1}{JL} \\ \mathrm{E}[t^2] = \dfrac{2}{JL(JL+1)} \end{cases}. \qquad (\mathrm{A}\cdot 6)$$

Finally, from Eqs. (A·1) and (A·6), we obtain

$$\begin{cases} \mathrm{E}\left[\sum_{l=0}^{L-1}\left(\dfrac{|\mu_{k,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{k,j,l}|^2}\right)\right] = \dfrac{1}{L} \\ \mathrm{E}\left[\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}\left(\dfrac{|\mu_{n,j,l}|^2}{\sum_{j=0}^{J-1}\sum_{l=0}^{L-1}|\mu_{n,j,l}|^2}\right)^2\right] = \dfrac{2}{JL+1} \end{cases} \qquad (\mathrm{A}\cdot 7)$$