PAPER

Effect of Transmit Power Limitation in Power Controlled DS-CDMA

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SUMMARY In DS-CDMA mobile communications systems, transmit power control (TPC) is an indispensable technique on the reverse (mobile-to-base) links to minimize the received signal power variations produced by multipath fading, shadowing, and distance dependent path loss. However, a large transmit power is sometimes required with TPC. This is an undesirable burden for a mobile station because the transmit power amplifier must have a fairly wide range of linearity. Furthermore, in the case of cellular systems, a large interference is produced to other cells, thereby reducing reverse link capacity. In this paper, we study the effect of the mobile transmitter power limitation on the transmission performance and the required transmit power that is directly related to the other cell interference.

key words: *DS-CDMA, transmit power control, fading*

1. Introduction

In direct sequence code division multiple access (DS-CDMA) mobile communications systems, a large power difference occurs among different users' signals transmitted from different distances from a base station and accordingly, a large multi-access interference (MAI) is produced on the reverse link. This is because a non-zero cross-correlation exists between spreading code sequences. This is known as the near-far problem [1], [2]. In addition to this, fast power variations exist due to multipath fading as well as shadowing [3]. To avoid a large MAI produced by distance dependent path loss, shadowing and multipath fading, transmit power control (TPC) is necessary [4]. With fast TPC, the base station measures the received signal power-to-interference plus background noise power ratio (SINR) every T_{tpc} seconds and transmits the TPC command to a mobile station in order to raise (lower) the transmit power by Δ_{tpc} dB if the measured SINR is below (above) the TPC target [5], [6]. To minimize the received signal power variations due to multipath fading, a large mobile transmit power is sometimes requested by TPC. This is an undesirable burden to the mobile transmitter linear power amplifier. Furthermore, in the case of cellular systems, large interference is given to other cells, thereby reducing the reverse link capacity. In this paper, we study the effect of the mobile transmitter power limitation on the transmission performance and the required transmit power that is directly related to other cell interference.

The remainder of this paper is organized as follows. Sect. 2 presents the transmission system model and theoretically analyzes the average bit error rate (BER) performance when TPC is used together with antenna diversity reception. In Sect. 3, the theoretical analysis is confirmed by computer simulations. Sect. 4 concludes the paper.

2. Analysis

2.1 Transmission System Model

The transmission system model is illustrated in Fig. 1. The propagation channel is a frequency selective Rayleigh fading channel and is assumed to have L discrete paths, each following independent and identically distributed Rayleigh fading distribution (i.e., uniform power delay profile). In a real propagation channel, the distance dependent path loss and shadowing loss are present. Since they are much slower compared to multipath fading, they are neglected in the analysis of transmit power limitation effect and in the computer simulations. The transmit power limitation in the presence of the distance dependent path loss and shadowing loss is discussed in Sect. 3.3. The base station receiver assumes M-antenna diversity reception using maximal ratio combining (MRC) and L-finger rake combining [7]. Although, in the computer simulations, despreading and rake combining operations are performed, the theoretical analysis neglects the inter-path interference (IPI) since large spreading factor is assumed.

In this paper, it is assumed that a large number of users are in communication and that the transmit power control is stable and the received signal powers of all users are identical and independently distributed. Under such an assumption, it is well known that the MAI can be well approximated as a complex Gaussian process due to the central limit theorem. So, we use this approximation [8], [9] and combine the MAI with the background noise process to treat as an equivalent Gaussian noise process. By introducing the equivalent noise power spectrum density η_0, the transmission performance under the multi-user environment can be analyzed assuming a single-user environment but with

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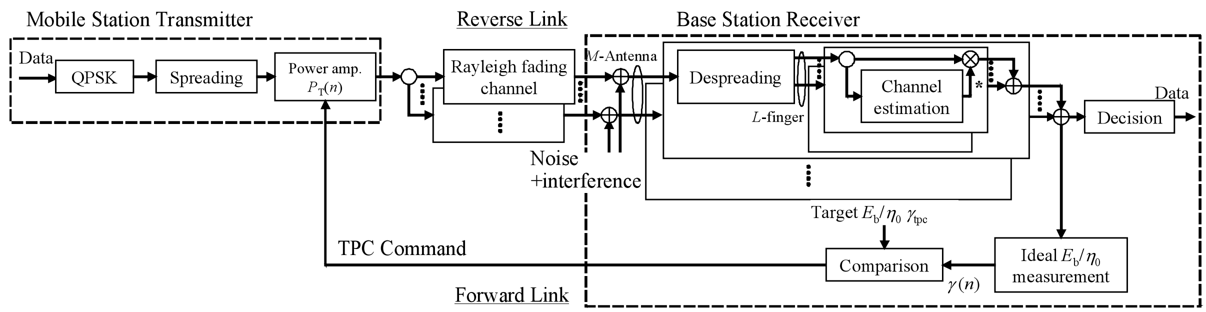


Fig. 1 Transmission system model. M-antenna diversity using MRC and L-finger rake combining.

the replacement the background Gaussian noise by the equivalent Gaussian noise. Throughout this paper, we use the signal energy per bit-to-equivalent noise power spectrum density ratio E_b/η_0.

In this paper, quaternary phase shift keying (QPSK) data modulation is assumed so SINR=$2E_b/\eta_0$ and η_0 is assumed to be identical for all antennas.

2.2 Transmit Power Control

Denoting the mobile transmit power as $P_T(t)$, the received signal power $P_{R,m,l}(t)$ of the lth path associated with the mth antenna is given by

$$P_{R,m,l}(t) = P_T(t) g_{m,l}(t), \tag{1}$$

where $g_{m,l}(t)$ is the channel power gain of the lth path associated with the mth antenna and $E[\sum_{l=0}^{L-1} g_{m,l}(t)] = 1$ with $E[.]$ representing the ensemble average operation. Assuming ideal channel estimation for M-antenna diversity and L-finger rake combining, the resultant E_b/η_0, $\gamma_R(t)$, after antenna diversity and rake combining becomes

$$\begin{aligned}\gamma_R(t) &= \left(\frac{T}{2\eta_0}\right) \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} P_{R,m,l}(t) \\ &= P_T(t) \left(\frac{T}{2\eta_0}\right) g_{MRC}(t),\end{aligned} \tag{2}$$

where T is the QPSK symbol length and

$$g_{MRC}(t) = \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} g_{m,l}(t) \tag{3}$$

can be considered as an equivalent channel gain seen after antenna diversity and rake combining.

The base station measures the received E_b/η_0, $\gamma_R(t)$, and compares it with the target E_b/η_0, γ_{tpc}, in order to generate the TPC command (however, in a practical transmit power control system, the signal-to-interference plus noise power ratio (SIR) is measured instead of E_b/η_0 and is compared to the target SIR to generate the TPC command [6]). Assuming that the E_b/η_0 measurement is ideal and the TPC command is received by the mobile station without error, the mobile transmit power becomes, from Eq. (2),

$$P_T(t) = \left(\frac{2\eta_0}{T}\right) \frac{\gamma_{tpc}}{g_{MRC}(t)}. \tag{4}$$

In the above equation, continuous time representation is used for TPC operation. However, in practice, the TPC measurement is done only every T_{tpc} seconds (TPC period) and the mobile transmit power is raised or lowered by Δ_{tpc} dB (TPC step size).

2.3 Average BER

The average BER expression when limiting the peak transmit power is derived to see how the transmit power limitation affects the achievable BER performance. Eq. (4) shows that sometimes excessively large transmit power is required when the equivalent channel power gain $g_{MRC}(t)$ fades. Let the transmit E_b/η_0, $\gamma_T(t)$, be defined as

$$\gamma_T(t) = \left(\frac{T}{2\eta_0}\right) P_T(t) = \frac{\gamma_{tpc}}{g_{MRC}(t)}. \tag{5}$$

We introduce the transmit power limitation so that

$$\gamma_T(t) \leq \alpha\gamma_{tpc}, \tag{6}$$

i.e., $\gamma_T(t)$ is limited to the peak $\gamma_T = \alpha\gamma_{tpc}$, where α (> 0) is called the transmit power limitation factor. Hence, $\gamma_T(t)$ becomes

$$\gamma_T(t) = \begin{cases} \gamma_{tpc}/g_{MRC}(t), & \text{if } g_{MRC}(t) \geq 1/\alpha \\ \alpha\gamma_{tpc}, & \text{otherwise} \end{cases}. \tag{7}$$

The transmit power of each user may change according to the change in the value of α. As previously mentioned, this paper assumes a large number of users and a stable transmit power control. Under such an assumption, the sum of MAI and the background noise can be treated as the equivalent Gaussian noise process due to the central limit theorem. The probability density function (pdf), $p(\gamma_T)$, of $\gamma_T = \gamma_T(t)$ can be obtained as (see Appendix A)

$$p(\gamma_T)=\begin{cases}\dfrac{(L\gamma_{tpc}/\gamma_T)^{ML}}{(ML-1)!\gamma_T}\exp(-L\gamma_{tpc}/\gamma_T), \\ \quad \text{if} \quad \gamma_T\le\alpha\gamma_{tpc} \\ \left\{1-\exp(-L/\alpha)\displaystyle\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}\right\} \\ \quad \cdot\delta(\gamma_T-\alpha\gamma_{tpc}), \\ \quad\quad \text{otherwise}\end{cases}. \quad (8)$$

The average γ_T becomes

$$E[\gamma_T]=\int_0^\infty\gamma_T p(\gamma_T)d\gamma_T=\gamma_{tpc}D(M,L,\alpha) \quad (9)$$

with

$$D(M,L,\alpha)=\begin{cases}-E_i(-1/\alpha)+\alpha\{1-\exp(-1/\alpha)\}, \\ \quad \text{if } ML=1 \\ \dfrac{L\exp(-L/\alpha)}{ML-1}\displaystyle\sum_{k=0}^{ML-2}\frac{(L/\alpha)^k}{k!} \\ \quad +\alpha\left\{1-\exp(-L/\alpha)\displaystyle\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}\right\}, \\ \quad \text{if} \quad ML\ge 2\end{cases}, \quad (10)$$

where $E_i(-x)=-\int_x^\infty t^{-1}\exp(-t)dt$ is the exponent integral, which is given by [8]

$$E_i(-x)=\ln x+\mu+\sum_{n=1}^{\infty}\frac{(-x)^n}{n\cdot n!}<0$$

with μ (= 0.57721...) being the Euler's constant. When $ML=1$,

$$E[\gamma_T]\approx\gamma_{tpc}[\ln\alpha+0.42279]\approx\gamma_{tpc}\ln\alpha \quad (11)$$

for $\alpha\gg 1$ and the value of $E[\gamma_T]$ approaches infinity when $\alpha\to\infty$. When $ML\ge 2$, however, we have the finite average γ_T:

$$E[\gamma_T]=\frac{L}{ML-1}\gamma_{tpc} \quad \text{as } \alpha\to\infty. \quad (12)$$

The average BER is derived below. From Eq. (5), the received $E_b/\eta_0(t)$, $\gamma_R(t)$, at the base station is given by

$$\begin{aligned}\gamma_R(t)&=\gamma_T(t)g_{MRC}(t)\\&=\begin{cases}\gamma_{tpc}, & \text{if } \gamma_T\le\alpha\gamma_{tpc}\\ \alpha\gamma_{tpc}g_{MRC}(t), & \text{otherwise}\end{cases}.\end{aligned} \quad (13)$$

Remember that the condition $\gamma_T\le\alpha\gamma_{tpc}$ is equivalent to $g_{MRC}(t)\ge 1/\alpha$. Using Eq. (13), the pdf $p(\gamma_R)$ of γ_R can be derived as (see Appendix B)

$$p(\gamma_R)=\begin{cases}\dfrac{(L\gamma_R/\alpha\gamma_{tpc})^{ML-1}}{(ML-1)!(\alpha\gamma_{tpc}/L)}\exp(-L\gamma_R/\alpha\gamma_{tpc}), \\ \quad \text{if} \quad \gamma_R<\gamma_{tpc} \\ \exp(-L/\alpha)\displaystyle\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}\delta(\gamma_R-\gamma_{tpc}), \\ \quad \text{otherwise}\end{cases}. \quad (14)$$

Then, the average BER $P_b(\gamma_{tpc})$ can be obtained as

$$\begin{aligned}P_b(\gamma_{tpc})&=\int_0^\infty\frac{1}{2}\operatorname{erfc}\sqrt{\gamma_R}\,p(\gamma_R)d\gamma_R\\&=\frac{1}{2}\left[1-\frac{1-\operatorname{erfc}\sqrt{\gamma_{tpc}+L/\alpha}}{\sqrt{1+(L/\alpha)/\gamma_{tpc}}}\sum_{k=0}^{ML-1}\binom{2k}{k}\right.\\&\quad\left.\cdot\frac{1}{\{4(1+\gamma_{tpc}/(L/\alpha))\}^k}\right]+\sqrt{\frac{\gamma_{tpc}}{\pi}}\\&\quad\cdot\exp\{-(\gamma_{tpc}+L/\alpha)\}\sum_{k=1}^{ML-1}\binom{2k}{k}\\&\quad\cdot\frac{1}{\{4(1+\gamma_{tpc}/(L/\alpha))\}^k}\sum_{j=0}^{k-1}\frac{\{2(\gamma_{tpc}+L/\alpha)\}^j}{(2j+1)!!}.\end{aligned} \quad (15)$$

Note that when $ML=1$, $\sum_{k=1}^{0}(.)=0$. In particular, when $ML=1$, 2, Eq. (15) becomes

$$P_b(\gamma_{tpc})=\begin{cases}\dfrac{1}{2}\left[1-\sqrt{\dfrac{\alpha\gamma_{tpc}}{1+\alpha\gamma_{tpc}}}(1-\operatorname{erfc}\sqrt{\gamma_{tpc}+1/\alpha})\right], \\ \quad \text{if } ML=1 \\ \dfrac{1}{2}\left[1-\dfrac{1+(3/2)/(\alpha\gamma_{tpc}/L)}{\{1+1/(\alpha\gamma_{tpc}/L)\}^{3/2}}\right. \\ \quad \left.\cdot(1-\operatorname{erfc}\sqrt{\gamma_{tpc}+L/\alpha})\right] \\ \quad +\dfrac{1}{2}\sqrt{\dfrac{\gamma_{tpc}}{\pi}}\dfrac{\exp\{-(\gamma_{tpc}+L/\alpha)\}}{1+\alpha\gamma_{tpc}/L}, \\ \quad \text{if } ML=2\end{cases}. \quad (16)$$

When $\alpha\to\infty$, Eq. (15) reduces to the AWGN channel case:

$$P_b(\gamma_{tpc})=\frac{1}{2}\operatorname{erfc}\sqrt{\gamma_{tpc}}, \quad (17)$$

where $\operatorname{erfc}(x)=(2/\sqrt{\pi})\int_x^\infty\exp(-t^2)dt$ is the complementary error function.

Using the relationship of the average transmit E_b/η_0 (average γ_T) and TPC target E_b/η_0 (γ_{tpc}) given by Eq. (9), we can obtain the average BER performance as a function of the average γ_T. The results for the frequency nonselective Rayleigh fading case ($L = 1$) are plotted in Figs. 2–7. In these figures, the computer simulation results are also plotted (see Sect. 3).

3. Computer Simulation

Table 1 summarizes the computer simulation condition. Spreading modulation assumes binary shift keying (BPSK) with the spreading factor (SF) of 64. In each simulation run, 10 billion QPSK symbols were transmitted to measure the average BER, the average γ_T, and the peak γ_T for various values of γ_{tpc} and α.

3.1 Frequency Nonselective Channel ($L = 1$)

No antenna diversity reception case ($M = 1$):
Figure 2 plots the measured average BER performance as a function of the peak γ_T with α as parameter for

Table 1 Simulation condition.

Data modem.	Modulation	QPSK
	Demodulation	Ideal coherent detection
TPC	Period T_{tpc}	64 symbols
	Step size Δ_{tpc}	1dB
Fading channel		L-path Rayleigh fading
Spreading/ despreading	Spread factor	64
	Matched filter	Ideal
Rake combining		L-finger coherent Rake combining
Antenna diversity		M-antenna MRC

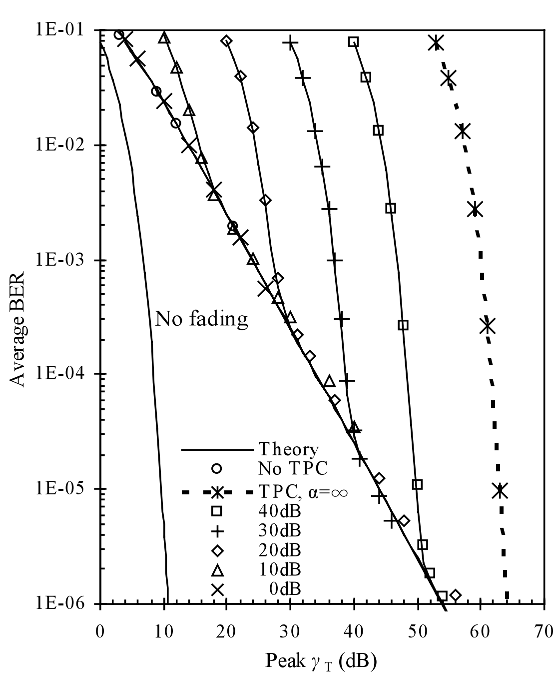


Fig. 2 Average BER performance as a function of the peak γ_T with α as a parameter for $M = 1$, $L = 1$, and $f_D T_{tpc} = 10^{-4}$.

the normalized maximum Doppler frequency $f_D T_{tpc} = 10^{-4}$. It can be clearly seen from Fig. 2 that the peak transmit power can be significantly lowered compared to no transmit power limitation (i.e., $\alpha \to \infty$) for achieving a certain BER. Without transmit power limitation, theoretically the peak $\gamma_T \to \infty$. However, in practice, since the transmit power is updated every T_{tpc} seconds by only Δ_{tpc} dB, the value of γ_T cannot be infinite even for the slow fading case of $f_D T_{tpc} = 10^{-4}$.

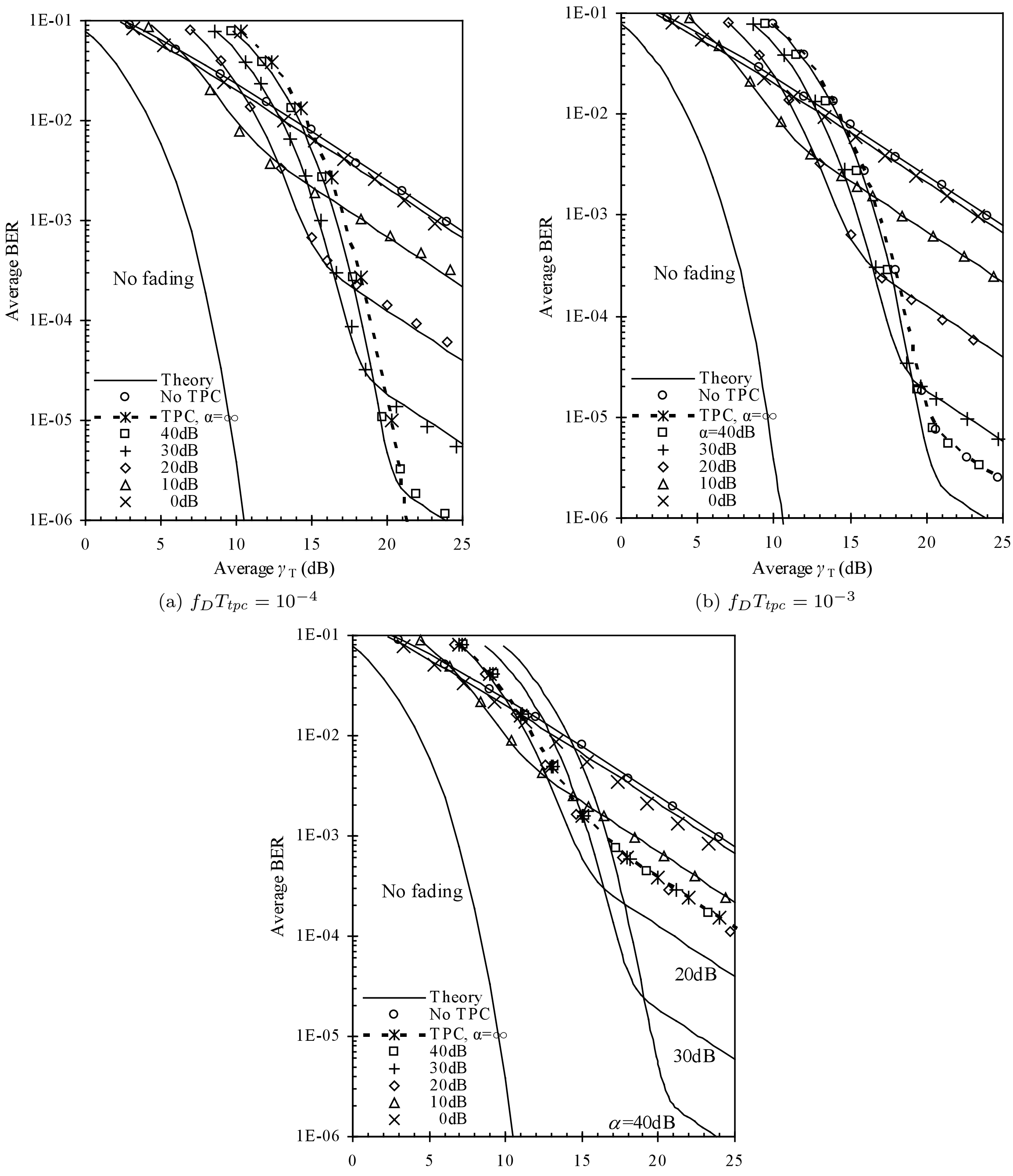
The transmit power is also an important factor for a cellular system since it represents the interference power to other cells. Hence, the average BER performance is evaluated as a function of the average γ_T, which is plotted with α as parameter in Fig. 3. For comparison, the BER performance without fading given by [7]

$$P_b(\gamma_T) = \frac{1}{2} \operatorname{erfc} \sqrt{M\gamma_T} \tag{18}$$

and the one with no TPC (see Appendix C) are also plotted. Good agreement is seen between the theoretical and measured results except for no transmit power limitation. It can be seen that for the given average γ_T, the average BER reduces first as the value of α increases, but increases for α beyond a certain value. This is more evident for slower fading. As a consequence, for achieving a comparatively large average BER, e.g., BER $= 10^{-2}$ or 10^{-3}, the required average transmit power can be significantly reduced by introducing the transmit power limitation compared to the no transmit power limitation case, i.e., $\alpha \to \infty$.

The effect of α on the required average γ_T and γ_{tpc} for achieving a certain BER is plotted in Fig. 4. There exits an optimum α that minimizes the required average γ_T. For BER $= 10^{-2}$, the optimum α is 9 dB. With this optimum α, the value of the required γ_{tpc} and the required average γ_T become about 6 dB and 10 dB, respectively. When $f_D T_{tpc} = 10^{-4}$, about 5 dB transmit power reduction can be achieved from the no transmit power limitation case. The optimum α becomes larger for smaller BER. As a summary, limiting the peak transmit power can reduce the required average transmit power, thereby reducing the interference power to other cells in the case of a cellular system.

Antenna diversity reception case ($M = 2$):
To compare the combined effect of antenna diversity reception and TPC, Fig. 5 plots the measured average BER performance as a function of the peak γ_T with α as parameter for the normalized maximum Doppler frequency $f_D T_{tpc} = 10^{-4}$. Comparison between Figs. 2 and 5 shows that the required peak γ_T for achieving a certain BER can be significantly reduced compared to the no diversity case. This is because the variations in the equivalent channel power gain seen after antenna diversity reception becomes less severe and consequently, the probability of the requested transmit γ_T being above the allowable peak $\alpha\gamma_{tpc}$ (see Eq. (6)) be-



(a) $f_D T_{tpc} = 10^{-4}$

(b) $f_D T_{tpc} = 10^{-3}$

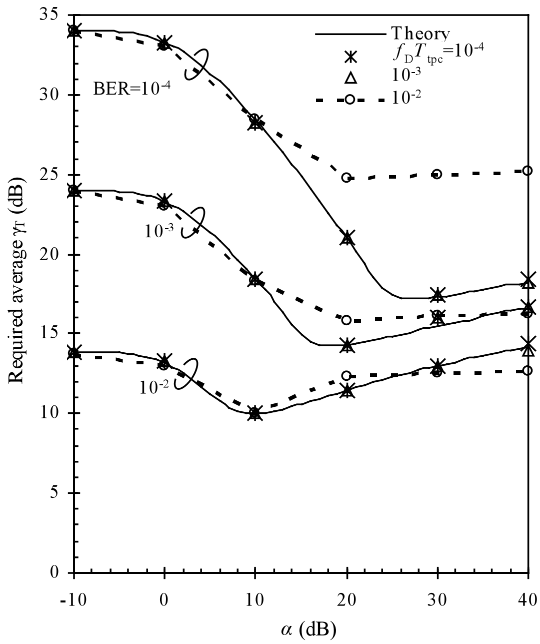
(c) $f_D T_{tpc} = 10^{-2}$

Fig. 3 Average BER performance as a function of average γ_T for $M = 1$ and $L = 1$.

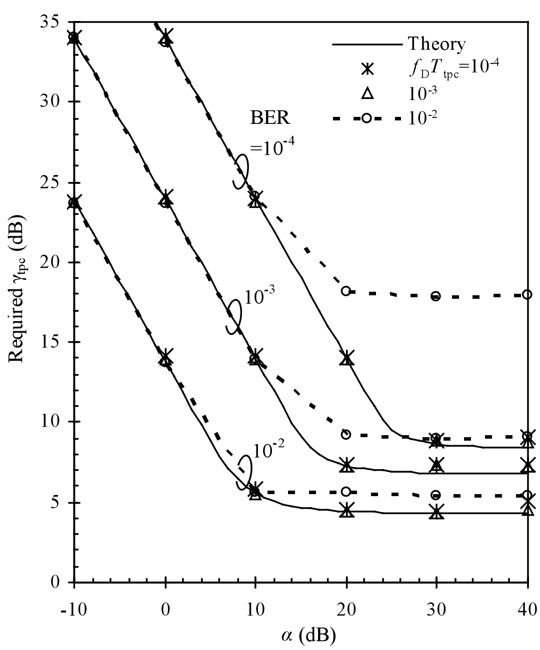
comes smaller.

Figure 6 plots the average BER performance as a function of average γ_T with α as parameter for $M = 2$ and $L = 1$. For comparison, also plotted are the BER performance without fading given by Eq. (18) and the average BER performance with no TPC and $M = 2$-antenna diversity (see Appendix C). Figure 6 is compared to Fig. 3 of no antenna diversity reception. As the value of α becomes larger, the BER performance continuously improves and approaches that of the no fading case by about 3 dB. It can be understood from Eq. (6) that increasing the value of α means increasing the peak transmit power in the transmitter linear power amplifiers. Hence, we want to use an α value as small as possible.

How the value of α affects the required average γ_T and γ_{tpc} for achieving a certain BER is plotted in Fig. 7, which can be compared to Fig. 4. It was seen from Fig. 4 that when antenna diversity is not used, the optimum α exists. However, this is not the case when antenna di-



(a) Required average γ_T



(b) Required γ_{tpc}

Fig. 4 Required average γ_T and γ_{tpc} as a function of α for $M = 1$ and $L = 1$.

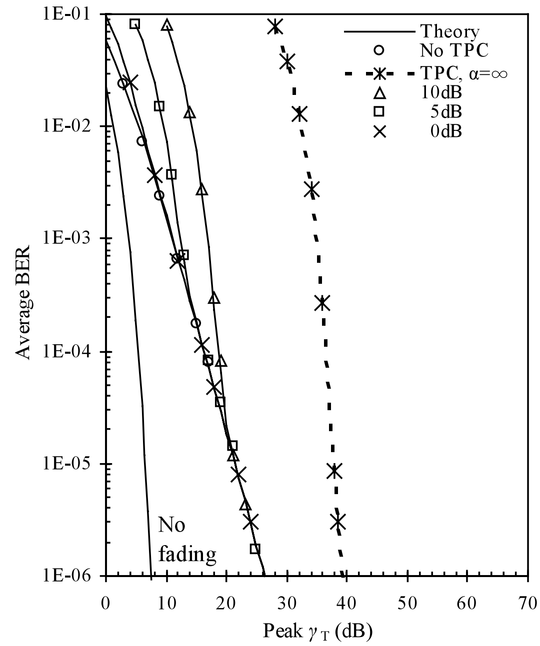


Fig. 5 Average BER performance as a function of the peak γ_T with α as a parameter for $M = 2$, $L = 1$, and $f_D T_{tpc} = 10^{-4}$.

versity is used. It can be seen in Fig. 7 that if the value of α is not set to be too small, the required average γ_T remains almost constant, which is much smaller than the no diversity case. For BER $= 10^{-2}$, the value of α can be set as small as 0 dB, which is smaller by about 9 dB than the no diversity case. With optimum α, the value of the required average γ_T becomes as small as about 4 dB, which is smaller by about 6 dB than the no diversity case, and the value of the required γ_{tpc} can be set about 6 dB without degrading the transmission performance. This suggests that antenna diversity reception can reduce both the required α and average γ_T, thereby significantly lowering the peak transmit power necessary for linear transmit power amplifiers.

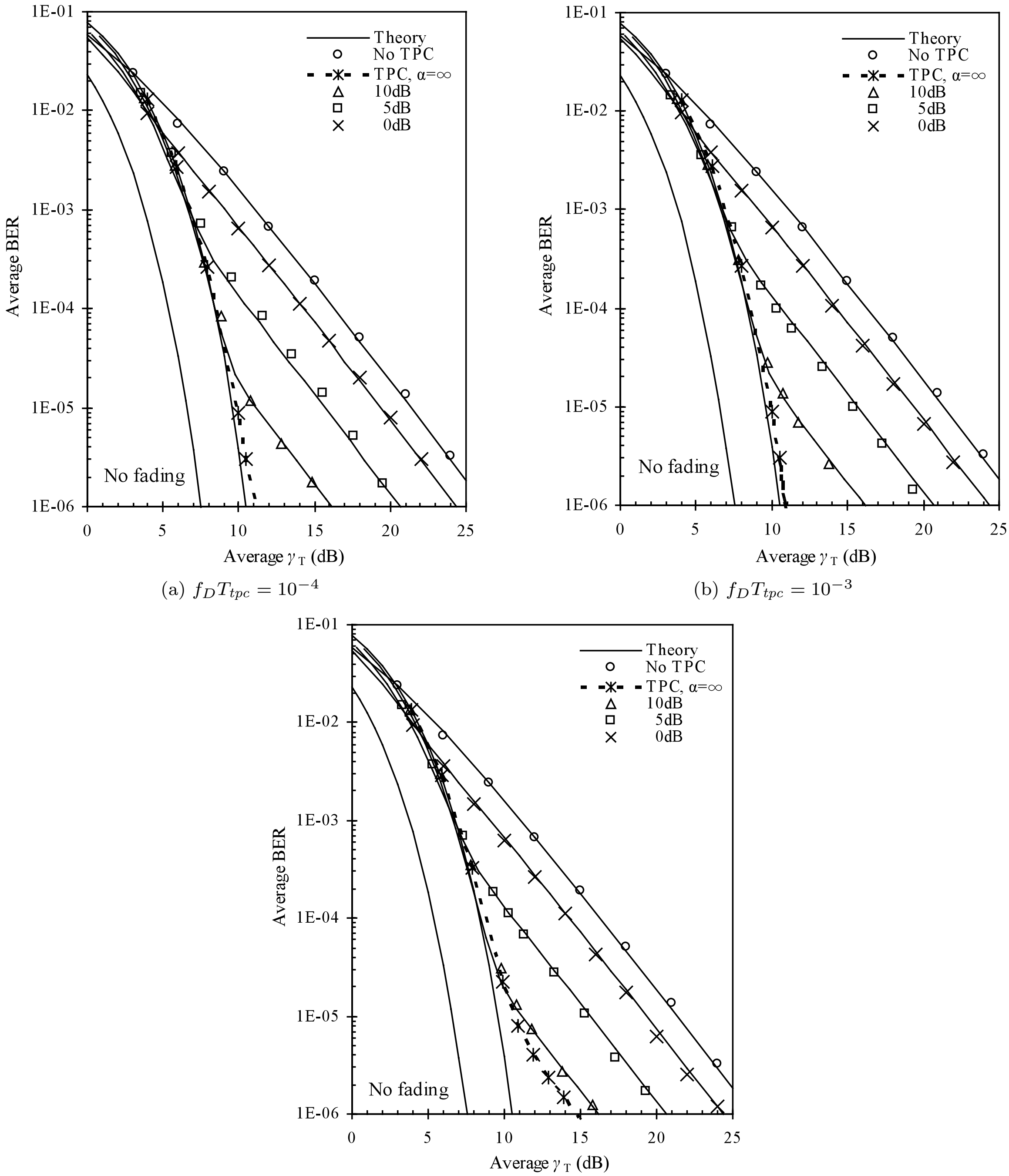
3.2 Frequency Selective Channel ($L = 2$)

When the channel is frequency selective, rake combining can be used. L-finger rake combining is equivalent to L-antenna diversity and can improve the BER performance significantly. Figure 8 plots the average BER performance as a function of average γ_T for $M = 1$ and $L = 2$. The average BER performance with $M = 1$ and $L = 2$ is just 3 dB inferior to that with $M = 2$ and $L = 1$. This is because the average received signal power per path is $1/L$ of the total signal power. This can be generalized to any combination of M and L for the given value of ML. It can be understood from Eq. (15) that the BER performance with M-antenna diversity and L-finger rake combining in a frequency selective fading channel just degrades by $10 \log_{10} L$ dB compared to that with ML-antenna diversity in a frequency nonselective fading channel.

3.3 Considerations on γ_{tpc}, Average Transmit Power, and Peak Transmit Power

Setting γ_{tpc}:

The optimum α that minimizes the average γ_T (or minimizes MAI) and the TPC target E_b/η_0, γ_{tpc}, for that optimun α can be found from Figs. 4 and 7. In a practical TPC system, the outer loop control [10], [11] can



(a) $f_D T_{tpc} = 10^{-4}$

(b) $f_D T_{tpc} = 10^{-3}$

(c) $f_D T_{tpc} = 10^{-2}$

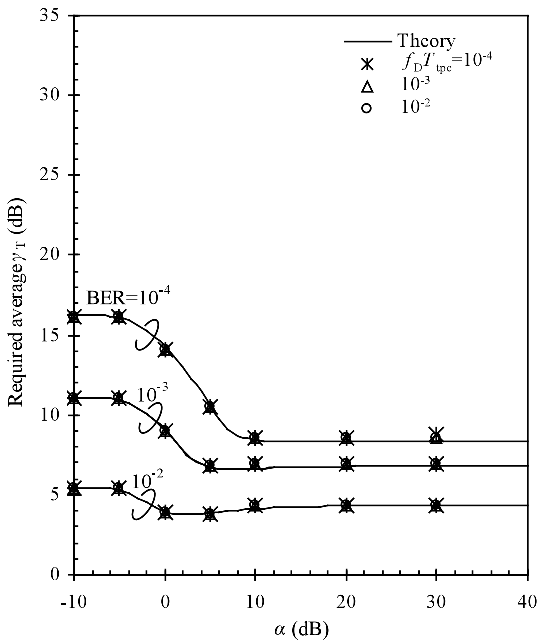
Fig. 6 Average BER performance as a function of average γ_T for $M = 2$ and $L = 1$.

be introduced to adaptively set the value of γ_{tpc} when the optimum α is used (however, note that since the BER is difficult to measure in practice, the frame error rate is measured).

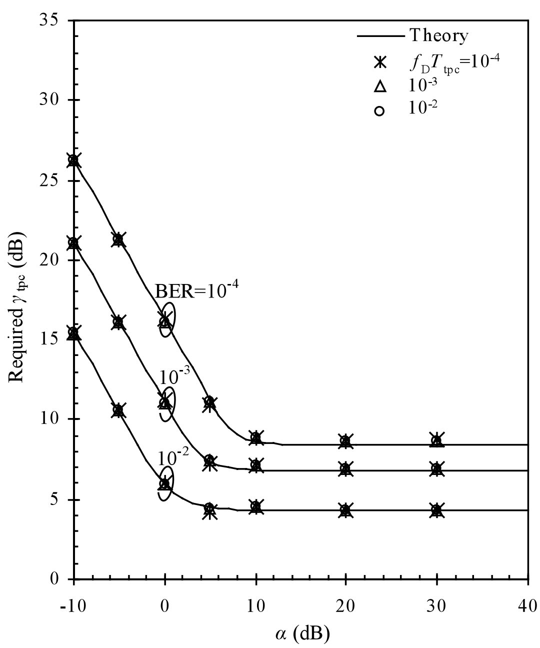
Average transmit power under multi-user environment: The transmit signal power is controlled so that the received energy per bit-to-equivalent noise power spectrum density ratio, E_b/η_0, becomes the target E_b/η_0, γ_{tpc}. Assuming that C users per cell are in communication, the equivalent noise power spectrum density η_0 is approximately given by

$$\eta_0 \approx N_0 + (C/SF)E[E_b][1 + f \cdot \lambda(M, L, \alpha)], \quad (19)$$

where N_0 represents the background noise power spectrum density and the second term represents the sum of other cell MAI and own cell MAI power spectrum densities with f being the other cell-to-own cell MAI power ratio introduced in [1]. The value of f given in [1] takes into account the effects of distance dependent path loss and shadowing loss, but neglects the effects



(a) Required average γ_T



(b) Required γ_{tpc}

Fig. 7 Required average γ_T and γ_{tpc} as a function of α for $M = 2$, $L = 1$.

of antenna diversity reception and fast TPC. To reflect the effects of antenna diversity reception and fast TPC, $\lambda(M, L, \alpha)$ is introduced following to [9], [12]–[14]. In our case, $\lambda(M, L, \alpha)$ can be obtained from

$$\lambda(M, L, \alpha) = E[\gamma_T]/E[\gamma_R], \tag{20}$$

where $E[\gamma_T]$ is given by Eq. (9). We average Eq. (13) using its pdf found in Eq. (14) to obtain

$$E[\gamma_R] = \gamma_{tpc} A(M, L, \alpha), \tag{21}$$

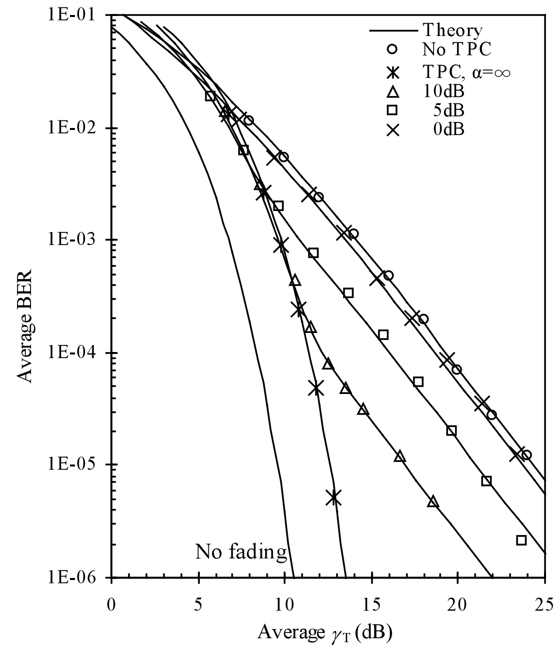


Fig. 8 Average BER performance as a function of average γ_T for $M = 1$, $L = 2$, and $f_D T_{tpc} = 10^{-4}$.

where

$$\begin{aligned} &A(M, L, \alpha) \\ &\quad = \alpha M \left\{ 1 - \exp(-L/\alpha) \sum_{k=0}^{ML} \frac{(L/\alpha)^k}{k!} \right\} \\ &\qquad + \exp(-L/\alpha) \sum_{k=0}^{ML-1} \frac{(L/\alpha)^k}{k!}. \end{aligned} \tag{22}$$

Hence, using Eqs. (9) and (21), $\lambda(M, L, \alpha)$ is represented as

$$\lambda(M, L, \alpha) = D(M, L, \alpha)/A(M, L, \alpha). \tag{23}$$

$A(M, L, \alpha)$ and $\lambda(M, L, \alpha)$ are plotted in Fig. 9 for $M = L = 1$ and $ML = 2$ as a function of α.

Since $E[\gamma_R] = E[E_b]/\eta_0$, where η_0 is given by Eq. (19), and equating it with Eq. (21), we obtain the average received signal energy per bit-to-background noise power spectrum density ratio E_b/N_0 as

$$\frac{E[E_b]}{N_0} \approx \frac{\gamma_{tpc} A(M, L, \alpha)}{1 - \left(\frac{C}{SF}\right) \gamma_{tpc} A(M, L, \alpha)\{1 + f \cdot \lambda(M, L, \alpha)\}}, \tag{24}$$

to which the average received E_b/N_0 with fast TPC converges. From Eq. (24), the average transmit power can be computed.

Peak transmit power:

So far, the distance dependent path loss and shadowing

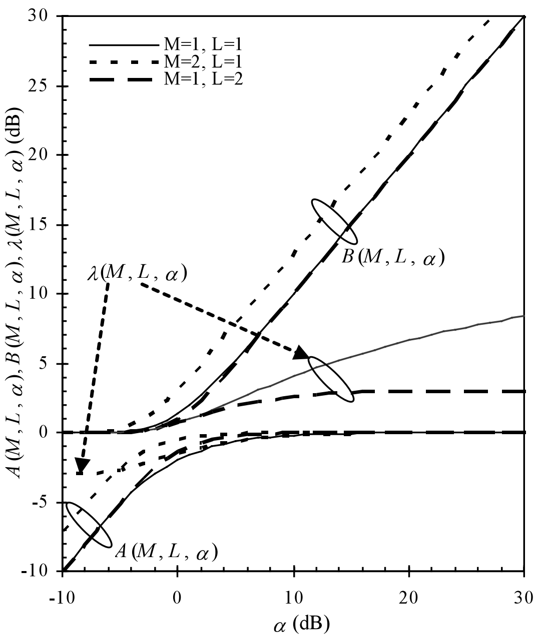


Fig. 9 $A(M,L,\alpha)$, $B(M,L,\alpha)$, and $\lambda(M,L,\alpha)$.

loss were neglected and the peak γ_T was set to $\alpha\gamma_{tpc}$ as shown in Eq. (7). In a real propagation channel, however, since both losses are present, the peak γ_T needs to be varied according to the movement of mobile station and is given by

$$\text{peak}\,\gamma_T = \alpha\gamma_{tpc}\{10^{\xi/10}r^{\beta}\} \tag{25}$$

and Eq. (7) can be modified as

$$\gamma_T = \begin{cases} \dfrac{\gamma_{tpc}}{g_{MRC}}\{10^{\xi/10}r^{\beta}\}, & \text{if } g_{MRC}(t) \geq 1/\alpha \\ \alpha\gamma_{tpc}\{10^{\xi/10}r^{\beta}\}, & \text{otherwise} \end{cases}, \tag{26}$$

where, r represents the distance between the base and mobile stations, and β the distance dependent path loss exponent ξ represents the shadowing loss in dB and is characterized by a zero-mean Gaussian variable with standard deviation σ. For simplicity, time dependency has been omitted here. The mobile station must know the optimum α and the value of $\gamma_{tpc}\{10^{\xi/10}r^{\beta}\}$. The former can be found from Figs. 4 and 7, but the latter is unknown at the mobile station. However, the peak γ_T, or equivalently the peak transmit power (peak P_T), can be set at the mobile station without knowing the value of $\gamma_{tpc}\{10^{\xi/10}r^{\beta}\}$. This is described below.

Since $P_T = (2\eta_0/T)\gamma_T$, the short-term average of $1/P_T$, $\overline{(1/P_T)}$, is obtained from Eqs. (8) and (26), as

$$\overline{\left(\frac{1}{P_T}\right)} = \frac{1}{\alpha\gamma_{tpc}\{10^{\xi/10}r^{\beta}\}}B(M,L,\alpha)\left(\frac{T}{2\eta_0}\right), \tag{27}$$

where

$$B(M,L,\alpha) = \left\{\frac{(L/\alpha)^{ML}}{(ML)!} + (\alpha M - 1)\sum_{k=0}^{ML}\frac{(L/\alpha)^k}{k!}\right\} \cdot \exp(-L/\alpha) + 1. \tag{28}$$

(Note that the averaging time is chosen so that the power variations due to multipath fading can be averaged out but those due to distance dependent path loss and shadowing loss remain almost unchanged). Then, since $P_T = (2\eta_0/T)\gamma_T$ and from Eqs. (25) and (27), we obtain the following simple relationship:

$$\text{peak}\,P_T \times \overline{(1/P_T)} = B(M,L,\alpha), \tag{29}$$

The mobile station can compute the value of $B(M,L,\alpha)$ using Eq. (28) since M and α are the known system parameters and since the value of L of reverse link (mobile-to-base) can be taken to be the same as that of forward link (base-to-mobile). $B(M,L,\alpha)$ is plotted in Fig. 9 for $M = L = 1$ and $ML = 2$ as a function of α.

We can use the following procedure for updating the peak transmit power at the mobile station. The optimum value of α is denoted by α_{opt}.

Step 1: at the beginning of peak transmit power limitation, set the peak transmit power, peak P_T, to an arbitrary value, e.g., the maximum available transmit power.

Step 2: measure the short-term average of $1/P_T$, $\overline{(1/P_T)}$.

Step 3: compute the value of α, $\hat{\alpha}$, which satisfies Eq. (29).

Step 4: Update the peak P_T such that $(\alpha_{opt}/\hat{\alpha})$ peak $P_T \rightarrow$ peak P_T and repeat step 2 to step 4.

In this way, the transmit peak power is updated every short-term averaging period, e.g., 10 sec.

4. Conclusion

In this paper, we studied, theoretically and by computer simulations, the effect of limiting the mobile transmitter power with TPC on the average BER performance and the required transmit power. In the analysis, the transmit E_b/η_0 is limited to α times the TPC target γ_{tpc} (note that $\alpha \rightarrow \infty$ represents no transmit power limitation). The results obtained by theoretical analysis and computer simulations encourage the introduction of the transmit power limitation. We found the following:

(a) No antenna diversity reception case: for a relatively large average BER, e.g., BER $= 10^{-2}$ or 10^{-3}, the required average transmit power can be significantly reduced by introducing the transmit power limitation and using the optimum α, compared to the no transmit power limitation case, i.e., $\alpha \to \infty$. When $\Delta_{tpc} = 1$ dB and $f_D T_{tpc} = 10^{-4}$, the optimum α is 9 dB for BER $= 10^{-2}$ and the required average γ_T becomes about 10 dB, which is about 5 dB smaller compared to the no power limitation case.

(b) Antenna diversity reception case: the equivalent channel gain variations become less severe and thus, the introduction of transmit power limitation cannot reduce the average transmit power. However, since α as small as 0 dB can be used, the peak transmit power necessary for linear transmit power amplifiers can be significantly lowered.

(c) For a frequency selective channel, rake combining can be used instead of or jointly with antenna diversity. Therefore, the introduction of transmit power limitation is to lower the peak transmit power without degrading the transmission performance.

In this paper, we also discussed the average transmit power under multi-user environment and presented the updating procedure for the peak transmit power at the mobile station in the presence of the distance dependent path loss and shadowing loss. In this paper, channel coding was not considered. This was the reason why we discussed the effect of transmit power limitation at the relatively large average BER of 10^{-2} or 10^{-3}. Combined effect of antenna diversity, channel coding, and TPC with transmit power limitation is left as an interesting future study.

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Appendix A: Derivation of pdf of γ_T

When $g_{MRC}(t) \geq 1/\alpha$, $\gamma_T(t) \leq \alpha\gamma_{tpc}$ and ideal power control is performed. Applying the variable transformation $\gamma_{tpc} = \gamma_T g_{MRC}$, the pdf of γ_T is obtained from

$$p(\gamma_T) = p(g_{MRC})|J|, \qquad (\text{A}\cdot 1)$$

where $p(g_{MRC})$ is the pdf of g_{MRC} [7]:

$$p(g_{MRC}) = \frac{L^{ML}}{(ML-1)!} g_{MRC}^{ML-1} \exp(-L g_{MRC}) \qquad (\text{A}\cdot 2)$$

and J denotes the Jacobian of variable transformation:

$$J = \frac{\partial g_{MRC}}{\partial \gamma_T} = -\frac{\gamma_{tpc}}{\gamma_T^2}. \qquad (\text{A}\cdot 3)$$

Hence, we have

$$p(\gamma_T) = \frac{(L\gamma_{tpc}/\gamma_T)^{ML}}{(ML-1)!\gamma_T} \exp(-L\gamma_{tpc}/\gamma_T), \quad \text{if} \quad g_{MRC} \geq 1/\alpha. \qquad (\text{A}\cdot 4)$$

On the other hand, when $g_{MRC}(t) < 1/\alpha$, the transmit power is kept at its allowable peak, i.e., $\gamma_T(t) = \alpha\gamma_{tpc}$, and therefore, the pdf of γ_T is given by $\text{Prob}[g_{MRC} < 1/\alpha]\delta(\gamma_T - \alpha\gamma_{tpc})$, where

$$\begin{aligned} &\text{Prob}[g_{MRC} < 1/\alpha] \\ &\quad = \int_0^{1/\alpha} p(g_{MRC}) dg_{MRC} \\ &\quad = 1 - \exp(-L/\alpha) \sum_{k=0}^{ML-1} \frac{(L/\alpha)^k}{k!}. \end{aligned} \qquad (\text{A}\cdot 5)$$

Finally, we obtain

$$p(\gamma_T) = \begin{cases} \dfrac{(L\gamma_{tpc}/\gamma_T)^{ML}}{(ML-1)!\gamma_T}\exp(-L\gamma_{tpc}/\gamma_T), \\ \quad \text{if } \gamma_T \le \alpha\gamma_{tpc} \\ \left\{1-\displaystyle\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}\exp(-L/\alpha)\right\} \\ \cdot\delta(\gamma_T-\alpha\gamma_{tpc}), \\ \quad \text{otherwise} \end{cases}. \quad (\text{A}\cdot 6)$$

Appendix B: Derivation of the pdf of γ_R

When $g_{MRC}(t) \ge 1/\alpha$, ideal power control is performed and the received E_b/η_0, $\gamma_R(t)$, is kept at it's target, i.e., $\gamma_R(t) = \gamma_{tpc}$. Therefore, the pdf of γ_R is given by $\text{Prob}[g_{MRC}(t) \ge 1/\alpha]\delta(\gamma_R - \gamma_{tpc})$, where

$$\begin{aligned}\text{Prob}\,[g_{MRC} \ge 1/\alpha] &= \int_{1/\alpha}^{\infty} p(g_{MRC})dg_{MRC} \\ &= \exp(-L/\alpha)\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}. \quad (\text{A}\cdot 7)\end{aligned}$$

On the other hand, when $g_{MRC}(t) < 1/\alpha$, $\gamma_T(t)$ is kept at its allowable peak $\gamma_T(t) = \alpha\gamma_{tpc}$ and hence $\gamma_R(t)$ varies following the channel variation. Applying the variable transformation $\gamma_R = \alpha\gamma_{tpc}g_{MRC}$, the pdf of γ_R is obtained from

$$p(\gamma_R) = p(g_{MRC})|J|, \quad (\text{A}\cdot 8)$$

where $p(g_{MRC})$ is given by Eq. (A· 2) and

$$J = \frac{\partial g_{MRC}}{\partial \gamma_R} = \frac{1}{\alpha\gamma_{tpc}}. \quad (\text{A}\cdot 9)$$

We have

$$p(\gamma_R) = \frac{(L\gamma_R/\alpha\gamma_{tpc})^{ML-1}}{(ML-1)!(\alpha\gamma_{tpc}/L)}\exp(-L\gamma_R/\alpha\gamma_{tpc}), \quad \text{if} \quad g_{MRC} < 1/\alpha. \quad (\text{A}\cdot 10)$$

Hence, we obtain

$$p(\gamma_R) = \begin{cases} \dfrac{(L\gamma_R/\alpha\gamma_{tpc})^{ML-1}}{(ML-1)!(\alpha\gamma_{tpc}/L)}\exp(-L\gamma_R/\alpha\gamma_{tpc}), \\ \quad \text{if} \quad \gamma_R < \gamma_{tpc} \\ \exp(-L/\alpha)\displaystyle\sum_{k=0}^{ML-1}\frac{(L/\alpha)^k}{k!}\delta(\gamma_R-\gamma_{tpc}), \\ \quad \text{otherwise} \end{cases}. \quad (\text{A}\cdot 11)$$

Appendix C: BER without TPC but with M-antenna Diversity and L-finger Rake Combining

When M-antenna diversity and L-finger rake combining are both based on MRC, the average BER $P_b(\Gamma)$ in a Rayleigh fading environment is given by [7]

$$\begin{cases} P_b(\Gamma) = \dfrac{1}{2}\left[1-\dfrac{\mu}{\sqrt{2-\mu^2}}\displaystyle\sum_{k=0}^{ML-1}\binom{2k}{k} \cdot\left(\frac{1-\mu^2}{4-2\mu^2}\right)^k\right], \\ \mu = \sqrt{\dfrac{2(\Gamma/L)}{2(\Gamma/L)+1}} \end{cases}, \quad (\text{A}\cdot 12)$$

where Γ is the average received E_b/η_0 per antenna and equal to the transmit E_b/η_0 in this paper. In particular, when $M = L = 1$ (no diversity and no rake combining) and when $ML = 2$, Eq. (A· 12) reduces to

$$P_b(\Gamma) = \begin{cases} \dfrac{1}{2}\left(1-\sqrt{\dfrac{\Gamma}{1+\Gamma}}\right), & ML=1 \\ \dfrac{1}{2}\left[1-\dfrac{1+(3/2)/(\Gamma/L)}{\{1+1/(\Gamma/L)\}^{3/2}}\right], & ML=2 \end{cases}. \quad (\text{A}\cdot 13)$$

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