LETTER

Comparative Study of Iterative Channel Estimation Schemes for Turbo Decoding with Antenna Diversity Reception in Rayleigh Fading

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SUMMARY Turbo decoding with coherent detection requires accurate channel estimation. In this paper, we consider outer-turbo channel estimation (OTCE), which carries out iterative channel estimation before turbo decoding, and inner-turbo channel estimation (ITCE), which incorporates iterative channel estimation into turbo decoding process. The average bit error rate (BER) performances with OTCE and ITCE in a frequency nonselective Rayleigh fading channel with antenna diversity reception are evaluated by means of computer simulations to be compared. It is found that although ITCE is superior to OTCE, OTCE provides the average BER performance very close to ITCE when dual antenna diversity reception is used.

key words: *turbo decoding, pilot symbol, iterative channel estimation, iterative decoding*

1. Introduction

Turbo coding [1], introduced in 1993 by Berrou et al., is well known for its ability to achieve the bit error rate (BER) performances close to the Shannon limit. In the 3rd generation mobile communications systems called IMT2000 systems, turbo coding is used as the error correction coding for high-speed data communications [2]. Turbo decoder uses as input, the soft decision sample sequence of coherently detected received signal. For coherent detection, the knowledge of the channel gain (or channel estimation) is required. However, in general, coherent detection of the received signal is difficult in a mobile channel characterized by fast multi-path fading. Conventional channel estimation schemes consist of transmitting known pilot symbols with the data symbols; at the receiver, the pilot symbols are extracted and used to estimate the time varying channel gain.

The channel estimation accuracy can be improved by incorporating an iterative channel estimation scheme that uses decision feedback of data symbols [3]. Initial estimation is done by extracting the pilot symbols, these estimated values are used for the coherent detection of the received signals, and then tentative decision is done. The tentative decision results are fedback to the channel estimator where they are used as pilot symbols for re-channel estimation. This iterative process is repeated to improve the channel estimation accuracy.

There may be two ways to combine iterative channel estimation and turbo decoding. One is to carry out iterative channel estimation before turbo decoding. This is called outer-turbo channel estimation (OTCE) in this paper. Another is to incorporate iterative channel estimation into turbo decoding process [4], which is called inner-turbo channel estimation (ITCE) in this paper. An interesting question is which can yield better BER performance when antenna diversity is used, OTCE or ITCE? To the best of authors' knowledge, it is not yet known to which extent the BER performance can be improved with OTCE or ITCE. The objective of this paper is to evaluate the performances of OTCE and ITCE in a frequency nonselective Rayleigh fading channel with antenna diversity reception to give an answer to the above question.

The remainder of this paper is organized as follows. The transmission model is presented in Sect. 2. Section 3 explains the difference between OTCE and ITCE schemes. In Sect. 4, the BER performances achievable by the use of OTCE and ITCE are evaluated by means of computer simulations in a frequency non-selective Rayleigh fading channel with antenna diversity reception and are compared. Section 5 concludes the paper.

2. Transmission Model

At the transmitter side, binary information sequence $\{d_i = \pm 1;\ i = 0 \cdots (N-1)\}$ of length N is encoded by a turbo encoder. The original coding rate is assumed to be 1/3; however, the two parity bit sequences are punctured alternately to increase the coding rate to 1/2. The turbo encoded binary sequence is channel-interleaved and then modulated by binary phase shift keying (BPSK) modulation to be transmitted. Coherent detection at a receiver requires accurate channel estimation. For channel estimation, one pilot symbol is inserted every $(K-1)$ data symbols in the BPSK symbol stream (see Fig. 1), since this provides better tracking ability against fast fading for the given pilot insertion ratio [3], giving the sequence $\{s_k = \pm 1\}$.

The transmitted signal is received at the receiver

Manuscript received August 28, 2002.

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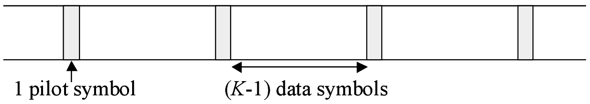


Fig. 1 Pilot insertion.

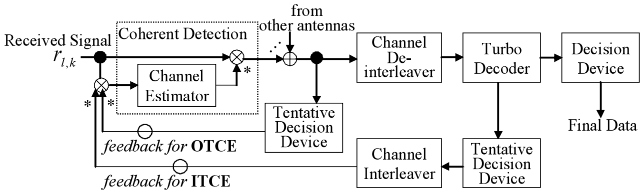


Fig. 2 Receiver model.

by L antennas via a frequency nonselective Rayleigh fading channel. Assuming perfect time synchronization, the received signal is sampled at the symbol rate. The low-pass equivalent of the received signal seen on the lth receive antenna, $l = 0 \cdots (L-1)$, may be expressed as

$$r_{l,k} = \sqrt{2S}\xi_{l,k}s_k + n_{l,k}, \tag{1}$$

where S denotes the average received signal power, s_k is the kth transmitted BPSK symbol, and $\xi_{l,k}$ is the complex channel gain experienced by the kth symbol received by the lth antenna. In Eq. (1), $\{\xi_{l,k}\}$ are independent and identically distributed (i.i.d.) complex Gaussian processes with zero mean and variance $E[|\xi_{l,k}|^2] = 1$, where $E[.]$ denotes the ensemble average operation. $n_{l,k}$ is the noise sample received on the lth antenna characterized by a complex Gaussian variable with zero mean and variance $2\sigma^2 = 2N_0/T$, where $1/T$ is the information bit rate and N_0 is the single-sided power spectral density of the additive white Gaussian noise (AWGN).

Figure 2 illustrates a simplified receiver block diagram with either OTCE or ITCE. There are two feedback loops: one is for OTCE, the other is for ITCE. In OTCE, iterative channel estimation is done prior to turbo decoding. The channel values obtained after the qth iteration is represented as $\hat{\xi}_{l,k}^{(q)}$, where $q = 1 \cdots Q$ ($Q = 1$ implies that iterative channel estimation is not performed). The coherently detected signal sample $z_{l,k}^{(q)}$ associated with the lth antenna after coherent detection may be expressed as

$$z_{l,k}^{(q)} = \mathrm{Re}\left[r_{l,k} \cdot \hat{\xi}_{l,k}^{(q)^*}\right], \tag{2}$$

For turbo decoding with L-branch antenna diversity reception, the optimal combining is the well-known maximal ratio combining (MRC) [5]. The MRC output sequence $\{z_k^{(q)}\}$ can be represented as [6]

$$z_k^{(q)} = \sum_{l=0}^{L-1} z_{l,k}^{(q)}, \tag{3}$$

Tentative data decision is performed on $z_k^{(q)}$ and is fedback to remove the data modulation from the L received signal sample sequences, on each of which re-channel estimation is performed. This is done repeatedly to obtain the final soft decision sample sequence. After removing the pilot symbols, the final soft decision sample sequence $\{z_k^{(Q)}\}$ is fed to the turbo decoder after de-interleaving.

On the other hand, in ITCE, the coherently detected sample sequence $\{z_k^{(1)}\}$ is input to the turbo decoder, where iterative channel estimation is incorporated into iterative turbo decoding. After the $(q-1)$th decoding iteration ($q \geq 2$), re-channel estimation is performed to obtain $\{\hat{\xi}_{l,k}^{(q)}\}$ using the turbo decoder output and the new sequence $\{z_k^{(q)}\}$ of soft decision values is used for the next decoding iteration.

3. Iterative Channel Estimation

In both OTCE and ITCE, the initial channel estimation ($q = 1$) applies the simple average (SA) filter. In this paper, without loss of generality, the pilot symbols are taken to be $\{s_{k=mK} = 1 + j0\}$. The initial channel estimate is given by

$$\hat{\xi}_{l,mK}^{(1)} = \frac{r_{l,mK} + r_{l,(m+1)K}}{2}, \tag{4}$$

which is used as the channel gain estimate in the coherent detection of all $(K-1)$ data symbols between mth and $(m+1)$th pilot symbols.

OTCE: For the iterative channel estimation ($q \geq 2$), the tentatively decided data symbol sequence obtained after the $(q-1)$th iteration is fedback to reverse modulate the received signal sample sequence for removing the data modulation. The tentative decision is made as

$$\hat{s}_k^{(q-1)} = \begin{cases} +1, & \text{if } \mathrm{Re}[z_k^{(q-1)}] \geq 0 \\ -1, & \text{otherwise} \end{cases}, \tag{5}$$

Hence, the instantaneous channel gain estimate $\tilde{\xi}_{l,k}^{(q)}$ is obtained as follows:

$$\tilde{\xi}_{l,k}^{(q)} = r_{l,k}\hat{s}_k^{(q-1)}. \tag{6}$$

Then, moving average (MA) filtering is applied to the sequence $\{\tilde{\xi}_{l,k}^{(q)}\}$ to reduce the noise effect. The channel gain estimate after the qth iteration $\hat{\xi}_{l,k}^{(q)}$ can be expressed as [7]

$$\hat{\xi}_{l,k}^{(q)} = \frac{1}{M}\sum_{m=k-\lfloor (M-1)/2 \rfloor}^{k+\lfloor (M-1)/2 \rfloor} \tilde{\xi}_{l,m}^{(q)}, \tag{7}$$

where M is the MA filter size and $\lfloor x \rfloor$ is the smallest integer larger than or equal to x. This estimated channel gain sequence $\{\hat{\xi}_{l,k}^{(q)}\}$ is used for the coherent detection of the received signal sample sequence. The resulting

MRC sequence $\{z_k^{(q)}\}$ is used for tentative data decision, which is again fed back to assist in the channel estimation. The process is repeated $(Q-1)$ times, i.e., $q = Q$ and the estimated channel gain sequence is utilized for the final coherent detection of the received signals; the final soft value sequence $\{z_k^{(Q)}\}$ is input to the turbo decoder.

ITCE: In ITCE, channel estimation is performed with every turbo decoding iteration. The initial channel gain estimate ($q = 1$) using SA filtering is used for the coherent detection of the received signal sample sequence, which is input to the turbo decoder. After the first turbo decoding iteration ($q \geq 2$), only the systematic bit sequence $\{d_i^{(q-1)}\}$ of the turbo decoder output is fedback. Reverse modulation and MA filtering is applied as in OTCE to estimate the instantaneous channel gain. This channel gain estimate sequence $\{\hat{\xi}_{l,k}^{(q)}\}$ is used for the coherent detection of the received signal sample sequence that is used for the qth iterative turbo decoding. (Note that the received signal samples corresponding to the systematic bits $\{d_i\}$ are only reverse modulated. Although the number of signal samples used in MA filtering with size M is $(M-1)/2$ for turbo coding rate of 1/2, the achievable BER performance was found to be better than that achieved by the use of turbo re-encoding. This is because error propagation can be avoided that is produced if re-encoding is used for feeding back both the systematic and parity bit sequences.)

4. Computer Simulation

The computer simulation conditions are shown in Table 1. The turbo encoder internal interleaver is an S-random interleaver [8] with $S \approx \sqrt{N}$. Since, in ITCE, the systematic bits need to appear periodically such that channel estimation accuracy is homogeneous, the turbo-encoded bit sequence is interleaved with a 45×45-block channel interleaver. Pilot symbol is inserted every $K = 10$ symbols. The frequency nonselective but time selective Rayleigh fading channel is assumed in the simulations.

Table 1 Simulation condition.

Transmitter	Data Length	N=1008 bits
	Turbo Encoder	(7,5)RSC Encoder
		Rate 1/2
		S-random Interleaver
	Channel Interleaver	Block Interleaver(45x45)
	Modulation	BPSK
	Pilot insertion	K=10
Channel Model	Freq.Nonselective Rayleigh fading	
Receiver	Antenna Diversity	L branch MRC
	Channel Estimation	SA(q=1)
		MA(q≧2), M=41
	Turbo Decoder	Log-MAP
		8 iterations

The effect of the number Q of channel estimation iterations on the achievable average BER performance with OTCE is plotted with f_DT as a parameter in Fig. 3, where f_D represents the fading maximum Doppler frequency. The average received signal energy per information bit-to-AWGN power spectrum density ratio, E_b/N_0, is taken to be 2.5 dB. When $Q = 1$, i.e., no iterative channel estimation, OTCE is equivalent to the simple averaging channel estimation (henceforth is represented as OTCE ($Q = 1$)). In both cases of slow fading ($f_DT = 0.001$) and fast fading ($f_DT = 0.01$), as Q increases, the average BER significantly reduces if antenna diversity is used. A large improvement is observed when Q increases from 1 to 2. It is also seen that irrespective of f_DT, almost no further improvement is attained for $Q > 3$. However, to be on the safe side, in the following simulation results shown, unless otherwise stated, $Q = 4$ is used for OTCE.

Figure 4 compares the average BER performance achievable with OTCE ($Q = 4$) and ITCE for $f_DT =$ 0.001 and 0.01. The BER performance improves as the number L of antennas increases. Since, in ITCE, the information sequence estimate obtained after turbo decoding is used for re-channel estimation, ITCE provides better BER performance than OTCE ($Q = 4$) when no antenna diversity is used. However, as L increases, the performance superiority of ITCE against OTCE ($Q = 4$) tends to be lost. When $L = 2$, OTCE ($Q = 4$) provides average BER performance very close to ITCE and when $L = 3$, almost no performance difference between the two is seen.

Figure 5 plots how the fading rate impacts the average BERs achievable with OTCE ($Q = 4$) and ITCE when the average received $E_b/N_0 = 2.5$ dB and $L = 2$. In slow fading channels (e.g., $f_DT < 0.001$), OTCE ($Q = 4$) and ITCE provide similar average

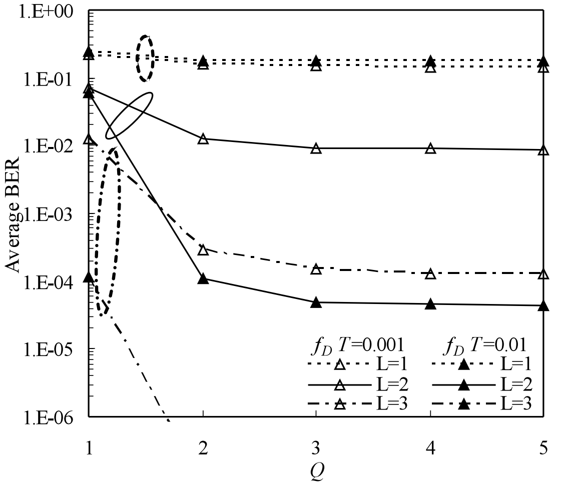
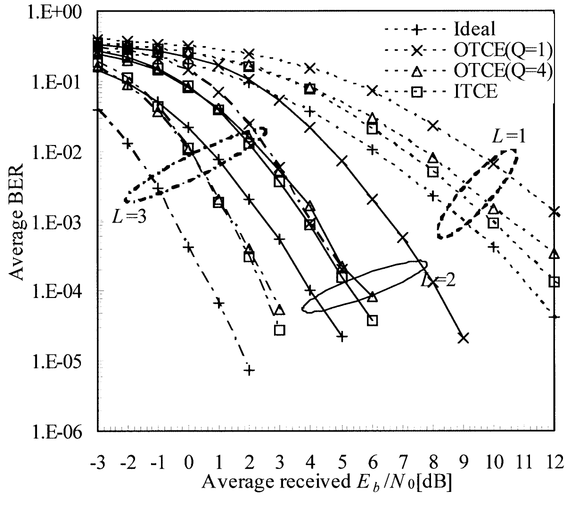
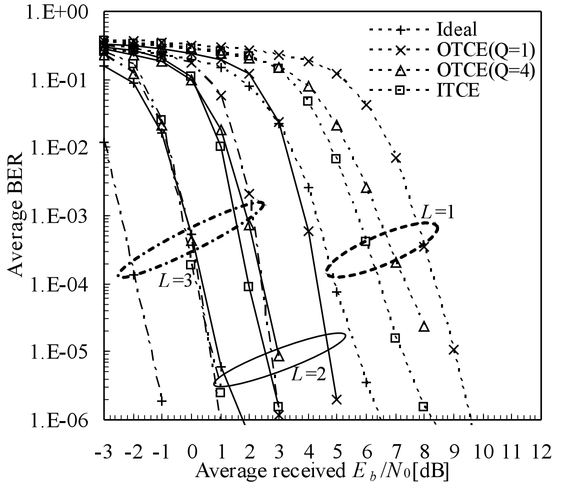


Fig. 3 Effect of the number Q of channel estimation iterations on BER performance with OTCE for the average received $E_b/N_0 = 2.5$ dB.



(a) $f_D T = 0.001$



(b) $f_D T = 0.01$

Fig. 4 Performance comparison of OTCE ($Q = 4$) and ITCE.

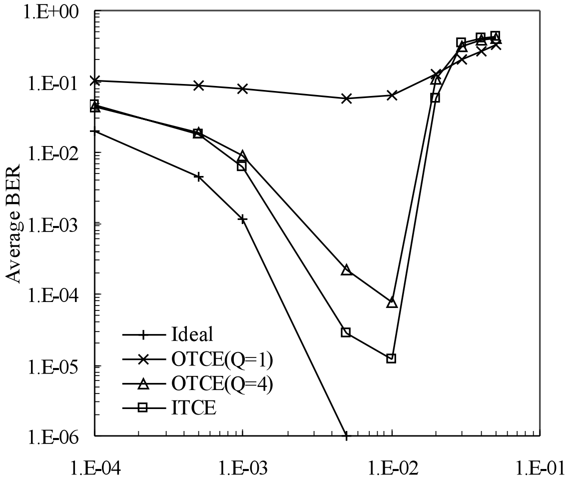


Fig. 5 Impact of fading rate on OTCE ($Q = 4$) and ITCE for the average received $E_b/N_0 = 2.5$ dB and $L = 2$.

BERs. As the fading rate increases, the average BERs reduce to a certain extent since turbo coding performs better in faster fading channels. In ITCE, the information sequence estimate obtained after each turbo decoding iteration is used for re-channel estimation; hence, ITCE provides smaller average BER values than OTCE ($Q = 4$) in a fast fading environment (e.g., $f_D T = 0.01$). (Note that the difference in the required E_b/N_0 values for achieving the average BER $= 10^{-5}$ is very small as seen in Fig. 4 (b)). However, the fading rate increases beyond $f_D T = 0.01$, the average BER abruptly increases because the channel estimator cannot track the fast varying channel.

5. Conclusion

In this paper, we evaluated, by means of computer simulations, the turbo decoding performances achievable with iterative channel estimation in terms of average BER over frequency nonselective Rayleigh fading channels with antenna diversity reception. For the iterative channel estimation using OTCE, the BER performance improves with the increase in the number of iterations. However, it was found that three iterations are sufficient, i.e., $Q = 3$. Although ITCE is superior to OTCE ($Q = 4$), the performance superiority of ITCE against OTCE ($Q = 4$) tends to be lost as the number L of diversity antennas increases. When $L = 2$, OTCE ($Q = 4$) provides the average BER performance very close to ITCE and when $L = 3$, almost no performance difference between the two is seen.

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