LETTER

Uplink Link Capacity of DS-CDMA Packet Mobile Communications with Rake Combining and Transmit Power Control

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SUMMARY Without transmit power control (TPC) and Rake combining, the uplink capacity of a direct sequence code division multiple access (DS-CDMA) packet mobile communication system significantly degrades due to the near-far problem and multipath fading. In this letter, assuming a single cell system with an interference-limited channel, the impact of the joint use of Rake combining and TPC on the uplink capacity is evaluated by computer simulation. Slow TPC is found to give a link capacity larger than fast TPC. This is because, with slow TPC, the received signal power variations due to fading remain intact and this results in a larger capture effect.

key words: *DS-CDMA, packet communication, transmit power control, Rake combining, link capacity, capture effect*

1. Introduction

In a packet mobile communication system, a packet with larger power can be received correctly even when the packets collide. This is known as the capture effect [1], [4]. An interesting question is whether the capture effect can be obtained when packet transmission is combined with direct sequence code division multiple access (DS-CDMA), in which multiple users transmit their packets using the same carrier frequency. Packets transmitted by different users suffer from different path loss, shadowing loss and multipath fading and received by a base station with significantly different power from others. This produces too large multi-access interference (MAI) and may offset the capture effect. Transmit power control (TPC) and Rake combining can reduce the variations in the received signal power and then reduce the MAI. However, the capture effect may be lost when TPC and Rake combining are jointly used. It is interesting to see how the joint use of TPC and Rake combining impacts the link capacity of a DS-CDMA packet mobile communication system. There are two types of TPC: fast TPC and slow TPC. In Ref. [2], the impact of control delay in the fast TPC on the packet link capacity is theoretically investigated. In Ref. [3], the link capacity with fast TPC but without Rake combining is theoretically analyzed. However, to the best of authors' knowledge, how the joint use of TPC and Rake combining impacts the packet link capacity has not been fully understood.

The remainder of this letter is organized as follows. Section 2 presents formulations of the throughput and outage probability. Section 3 derives expressions for signal-to-interference plus noise power ratio (SINR) assuming a single cell system with an interference-limited channel. Section 4 evaluates the link capacity by computer simulation. Section 5 gives some conclusions.

2. Throughput and Outage Probability

Outage occurs if the transmission quality drops below the required quality of services (QoS), i.e., packet throughput and delay. The link capacity is defined as the maximum number of active users that satisfies the allowable outage probability (Q_{allow}) and Q_{allow} is defined as the maximum outage probability that satisfies the system requirement. In a packet communication system, automatic repeat request (ARQ) is used. Assuming infinite number of retransmissions (infinite delay is allowed before successful transmission of a packet), the throughput S is given by

$$S = 1 - p\,(K, \lambda)\,, \tag{1}$$

where $p(K, \lambda)$ is the average packet error rate with K and λ being the number of active users and the packet occurrence rate, respectively. The outage occurs only if throughput is less than the required value. The outage probability Q is given by

$$Q = \mathrm{Prob}[S < S_{req}], \tag{2}$$

where S_{req} is the required throughput which is defined as the minimum throughput that satisfies the required QoS.

As K increases, the MAI increases and therefore, the outage probability increases. We assume that the occurrence rate of original packets is the same for all active users and is denoted by λ_0. The packet occurrence rate λ is $(1 + \text{packet retransmission rate}) \times \lambda_0$. When TPC is used, since packet errors occur equally likely for all active users, λ is given by

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$$\lambda = \frac{\lambda_0}{1 - p(K, \lambda)}. \quad (3)$$

From Eqs. (1) and (3), we have

$$S = \lambda_0/\lambda. \quad (4)$$

There exist $K-1$ active interfering users. Assuming that the original and retransmitted packets are randomly produced, $p(K, \lambda)$ can be computed using

$$p(K, \lambda) = \sum_{k=0}^{K-1} p_k \cdot \binom{K-1}{k} \lambda^k (1-\lambda)^{K-k-1}, \quad (5)$$

where p_k is the conditional packet error rate when k interfering packets are received and $\binom{K-1}{k}$ is the binomial coefficient. Assuming a slotted packet transmission and a block fading (the fading stays almost constant over a packet), p_k may be given by

$$p_k = 1 - [1 - p_b(\gamma_k)]^N, \quad (6)$$

where $p_b\ (\gamma_k)$ is the bit error rate (BER) with γ_k being the received SINR when k interfering packets are received and N is the number of bits in a packet. Assuming coherent BPSK data modulation and that the sum of colliding packets is approximated as a Gaussian process, $p_b(\gamma_k)$ is given by [5]

$$p_b(\gamma_k) = \frac{1}{2} erfc \sqrt{\frac{\gamma_k}{2}}, \quad (7)$$

where $erfc(x) = (2/\sqrt{\pi}) \int_x^\infty e^{-y^2} dy$ is the complimentary error function. The SINR expressions to compute the BER are derived in Sect. 3.

Solving Eq. (3) theoretically is quite difficult if not impossible. We resort to an iterative computation. Let $g(\lambda)=\lambda_0/[1-p(K,\lambda)]$. The iterative computation is as follows.

Step 1: Set K=0.
Step 2: Increase the value of K by one and set $\lambda = \lambda_0$ and compute $g(\lambda)$.
Step 3: Set λ=$g(\lambda)$ and compute $g(\lambda)$.
Step 4: Repeat step 3 until the difference between previous and new values of λ is within a given bound. In our computation, the bound is taken to be 10^{-4}.
Step 5: Obtain the throughput S using Eq. (4).
Step 6: Repeat step 2 to step 5 until $S < S_{req}$ is reached. The maximum number of K which satisfies $S \geq S_{req}$ is the link capacity C.

If TPC is not used, packets transmitted from different users reach the base station with different powers due to different path loss, shadowing loss and multipath fading. Therefore, throughputs are different for different users and must be treated as random variables. No analytical method is available to find the throughput. We resort to Monte Carlo simulation. The Monte Carlo simulation is performed as follows.

Step 1: Set K=0.
Step 2: Increase the value of K by one and set users' locations and generate path loss, shadowing loss and multipath fading associated with each user.
Step 3: Generate original packets with the occurrence rate of λ_0.
Step 4: Generate packet errors according to the BER computed using SINR γ associated with each user's packet received at the base station (Eqs. (6), (7), and (10) are used).
Step 5: If packet error occurs in step 4, retransmit the packet of that user.
Step 6: Repeat step 3 to step 5 to obtain the packet error rate of each user.
Step 7: Change locations of all users and repeat step 3 to step 6 to compute the probability distribution of the throughput S. The probability of $S \leq S_{req}$ represents the outage probability Q.
Step 8: Repeat step 2 to step 7 until $Q \geq Q_{allow}$. The maximum number of K that satisfies $Q < Q_{allow}$ is the link capacity C.

3. SINR Expressions

We assume a single cell system with an interference-limited channel and ideal TPC based on the received signal power. A frequency selective block fading channel having L discrete paths is assumed. Below, expressions for γ_k to compute the packet error rate p_k using Eqs. (6) and (7) are derived for no TPC, fast TPC and slow TPC.

(a) No TPC
Assuming the interference-limited channel, the received signal $r(t)$ may be expressed, using equivalent baseband representation, as [6]

$$r(t) = \sum_{i=0}^{K-1} \sqrt{2A_i}\beta_i \left(\sum_{l=0}^{L-1} \xi_{i,l} d_i(t-\tau_{i,l}) c_i(t-\tau_{i,l}) \right), \quad (8)$$

where i denotes the user index, A_i is the average received power and β_i (=0 or 1) represents the transmission state; "β_i=1" represents transmission of a packet and "β_i=0" otherwise. $d_i(t)$ and $c_i(t)$ are the data-modulated waveform and the spreading chip waveform, respectively. $\xi_{i,l}$ and $\tau_{i,l}$ are the complex path gain and the time delay of the lth path, respectively. Time dependency of the path gain is dropped for the sake of simplicity. It is assumed that $E[\sum_{l=0}^{L-1} |\xi_{i,l}|^2] = 1$, where $E[\cdot]$ is the ensemble average operation. A_i is given by

$$A_i = P_i r_i^{-\alpha} 10^{-\eta_i/10}, \quad (9)$$

where P_i, r_i, α, and η_i are the transmit power, the distance from the base station normalized by the cell radius, the path loss exponent, and the shadowing loss [dB], respectively. The received faded signal is resolved into L path components by L matched filters (Rake fingers) for coherent Rake combining based on maximal ratio combining (MRC) methods [5]. Ideal channel estimation is assumed. Without loss of generality, the 0th user is considered as the desired user.

In the lth Rake finger, the received signal $r(t)$ is multiplied by the delayed replica of the spreading waveform $c_0(t)$ synchronized to the lth path time delay, and integrated over one data symbol duration to get the lth finger output. Then, the lth finger output is multiplied by the complex conjugate of $\xi_{i,l}$ and coherently combined with other finger outputs to produce the Rake combiner output. The received interference is the sum of self-interference (due to inter-path interference) and other-user interference and is approximated as a white Gaussian process. The received SINR γ after Rake combining is therefore, given by

$$\gamma = 2SF\frac{P_0 r_0^{-\alpha} 10^{-\eta_k/10}\left(\sum_{l=0}^{L-1}|\xi_{0,l}|^2\right)^2}{\left(\begin{array}{l}P_0 r_0^{-\alpha}10^{-\frac{\eta_0}{10}}\sum_{l=0}^{L-1}\sum_{\substack{m=0\\m\neq l}}^{L-1}\left|\xi_{0,m}\xi_{0,l}^*\right|^2\\+\sum_{i=1}^{K-1}\beta_i\left(P_i r_i^{-\alpha}10^{-\eta_i/10}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\left|\xi_{i,m}\xi_{0,l}^*\right|^2\right)\end{array}\right)}. \tag{10}$$

(b) Fast TPC
The transmit power of each mobile station is controlled so that the received instantaneous signal power is kept at the prescribed target value for all users. With fast TPC, the ith user's transmit signal power P_i becomes

$$P_i = \frac{P_{target}}{r_i^{-\alpha}10^{-\eta_i/10}\sum_{l=0}^{L-1}|\xi_{i,l}|^2}, \tag{11}$$

where P_{target} is the target received signal power. The 0th user's SINR γ with fast TPC is given by

$$\gamma = \frac{2SF\left(\sum_{l=0}^{L-1}|\xi_{0,l}|^2\right)^2}{\left(\begin{array}{l}\sum_{l=0}^{L-1}\sum_{\substack{m=0\\m\neq l}}^{L-1}\left|\xi_{0,m}\xi_{0,l}^*\right|^2\\+\left(\sum_{l=0}^{L-1}|\xi_{0,l}|^2\right)\sum_{i=1}^{K-1}\beta_i\left(\frac{\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\left|\xi_{i,m}\xi_{0,l}^*\right|^2}{\sum_{l=0}^{L-1}|\xi_{i,l}|^2}\right)\end{array}\right)}. \tag{12}$$

(c) Slow TPC
The transmit power of each user is controlled so that the power variations due to path loss and shadowing loss are completely removed but those due to fading remain intact. The ith user's average transmit power P_i becomes

$$P_i = \frac{\bar{P}_{target}}{r_i^{-\alpha}10^{-\eta_i/10}}, \tag{13}$$

where $\bar{P}_{target}$ is the target average signal power. The 0th user's SINR γ with slow TPC, is given by

$$\gamma = \frac{2SF\left(\sum_{l=0}^{L-1}|\xi_{0,l}|^2\right)^2}{\sum_{l=0}^{L-1}\sum_{\substack{m=0\\m\neq l}}^{L-1}\left|\xi_{0,m}\xi_{0,l}^*\right|^2+\sum_{i=1}^{K-1}\beta_i\left(\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\left|\xi_{i,m}\xi_{0,l}^*\right|^2\right)}. \tag{14}$$

So far, SINR expressions without TPC, with fast TPC and with slow TPC have been developed. γ_k in Eq. (6) is given by Eqs. (10), (12) and (14) with

$$k = \sum_{i=1}^{K-1}\beta_i, \tag{15}$$

i.e., when k interfering packets are received.

4. Computer Simulation

The uplink capacity is evaluated by the Monte-Carlo simulation. Table 1 shows the simulation parameters.

Figure 1 plots the uplink capacity normalized by the spreading factor SF as a function of the number L of propagation paths. When L=1, since no Rake combining effect is expected, too large MAI is produced. This offsets the capture effect and none of the colliding packets survives. However, as L increases, the packet

Table 1 Simulation parameters.

User distribution		Uniform
Propagation channel	Fading	Block Rayleigh
	Number of paths	L=1~16
	Path loss exponent	α=3.5
	Standard deviation of shadowing	7dB
Packet transmission	Data-mod.& demod.	Coherent BPSK
	Spreading factor	SF=1~512
	Packet length	N=512 bits
	Original packet occurrence rate	λ_0=0.05
QoS	Required throughput	S_{req}=0.9
	Allowable outage probability	Q_{allow}=0.1

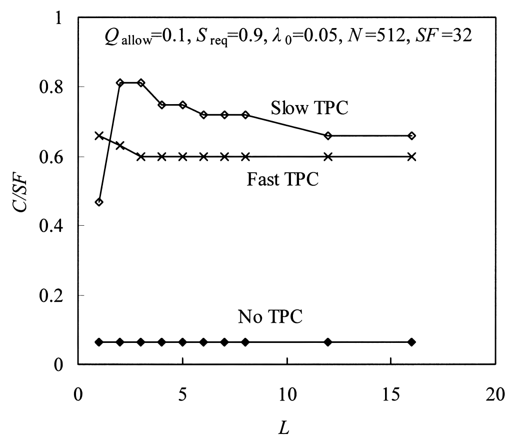

Fig. 1 Normalized link capacity C/SF as a function of L.

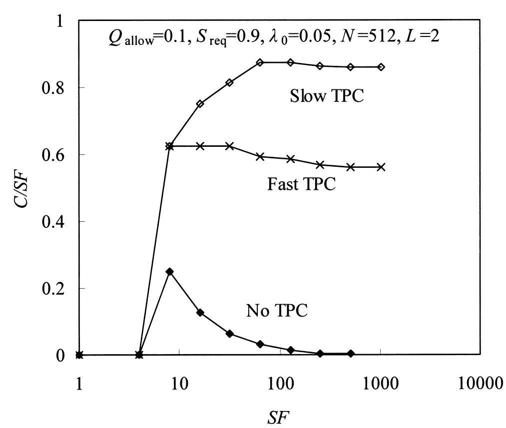

Fig. 2 Normalized link capacity C/SF as a function of spreading factor SF.

power variations reduce due to increased Rake combining effect and hence the probability of packet survival increases. As L increases, the normalized capacity increases with the slow TPC and maximizes when L=3. Beyond L=3, however, the normalized capacity decreases and approaches that with fast TPC.

On the other hand, in the case of fast TPC, the normalized link capacity is almost constant because all packets are received with the same power. The reason for slightly larger normalized link capacity for small L is due to less interpath interference produced on the desired packet. The link capacity without TPC is much smaller than with TPC due to the near-far problem.

Figure 2 plots the normalized link capacity C/SF as a function of spreading factor when L=2. Below $SF \leq 4$, only a single user can be accommodated, i.e., $C/SF \approx 0$. However, as the value of SF increases beyond 4, the normalized link capacity starts to increase. For $SF > 8$, the normalized link capacity with fast TPC becomes almost constant, while that with slow TPC still continues to increase owing to the capture effect.

5. Conclusions

This letter evaluated by computer simulation the impact of TPC and Rake combining on the uplink capacity in a DS-CDMA packet mobile communication system. It was found that the slow TPC provides larger link capacity than the fast TPC owing to the capture effect if more than one propagation path are present ($L > 1$). However, when only a single path is present (L=1), the slow TPC provides smaller capacity than the fast TPC because the large MAI offsets the capture effect. In this letter, the single cell system was considered. An extension to multiple cell system is left for further study.

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