LETTER

A Weighted Delay Transmit Diversity System Combined with Antenna Diversity Reception for DS-CDMA Mobile Radio

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SUMMARY In this paper, we study a delay transmit diversity system combined with antenna diversity reception that transmits the time-delayed and weighted versions of the same signal from multiple antennas. At a receiver, multiple receive antennas are used and all delayed signals received on multiple antennas are coherently combined by a Rake receiver. The set of optimum antenna weights for maximizing the received signal-to-noise power ratio (SNR) after Rake combining is theoretically analyzed to show that the optimum solution is to transmit only from the best antenna that has the maximum equivalent channel gain seen after Rake combining. The bit error rate (BER) performance is theoretically analyzed and evaluated by computer simulation. The combined effect of transmit diversity and transmit power control (TPC) is also investigated.

key words: *transmit diversity, transmit power control, fading, DS-CDMA, Rake combining*

1. Introduction

In direct sequence code division multiple access (DS-CDMA), Rake combining is used to improve transmission performance in a frequency selective fading channel. Further performance improvement can be achieved by using antenna diversity. Recently, transmit diversity (TD) has been attracting much attention [1]. Delay TD transmits the same signal from multiple antennas with different time delays [2]. Recently, we modified delay TD and proposed a weighted delay TD, in which the delayed versions of the same signal are weighted and then transmitted from different antennas [3]. It was found [3] that when a single receive antenna is used, the optimum weighted delay TD transmits the signal only from one antenna which has the maximum channel gain. This transmit diversity is called selection combining (SC) TD. SCTD provides superior bit error rate (BER) performance to delay TD in a frequency non-selective fading environment. However, most of multipath channels for high speed data transmission are frequency selective. An interesting question is what is the optimum set of transmit antenna weights when multiple receive antennas and Rake combining are used in a frequency selective fading channel. This paper answers this question.

The remainder of this paper is organized as follows. Section 2 derives the optimum set of antenna weights for the weighted delay TD combined with antenna diversity reception followed by Rake combining. It will be shown that the optimum set of antenna weights is to transmit the signal from the best antenna that has the maximum channel gain seen after Rake combining. Therefore, the diversity is also called SCTD. In Sect. 3, the BER performance analysis is presented. The joint use of SCTD and fast transmit power control (TPC) is considered. In Sect. 4, the computer simulation results are presented. Section 5 draws some conclusions.

2. Optimum Antenna Weights

Figure 1 illustrates the block diagram of the weighted delay transmit diversity system using M transmit antennas and N receive antennas. The data modulated signal waveform $d(t)$ is multiplied by the spreading sequence waveform $c(t)$. M copies of $d(t)c(t)$ are incurred different time delays $\{\tau_m;\ m = 0 \sim M-1\}$ and then, multiplied by weights $\{\alpha_m\}$ for transmission from M antennas. We assume

$$\sum_{m=0}^{M-1} |\alpha_m|^2 = 1, \tag{1}$$

so that the total power is kept constant. We assume an L-path fading channel having a uniform power delay profile and the maximum time delay difference of $\Delta\tau_{\max}$. It is assumed that $0{=}\tau_0 < \tau_1 < \tau_2 < \ldots < \tau_{M-1}$ with $\tau_m - \tau_{m-1} \leq \Delta\tau_{\max}$ for all m and

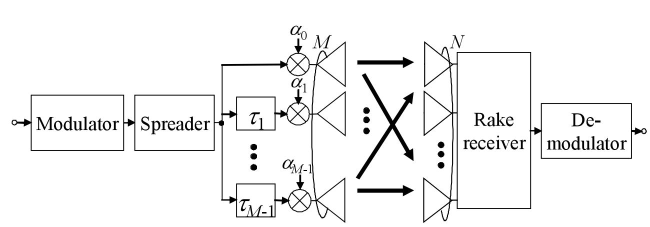

Fig. 1 Weighted delay TD combined with antenna diversity reception followed by Rake combining.

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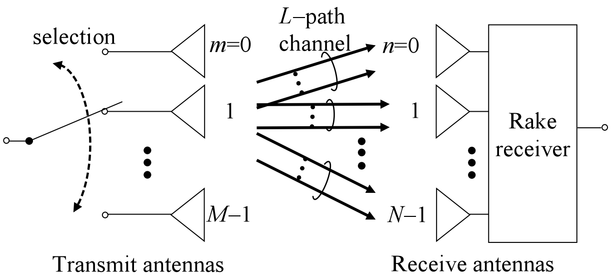

Fig. 2 SCTD.

$\tau_{M-1} + \Delta\tau_{\max} < T$, where T denotes the data symbol length.

The following theoretical analysis assumes that the transmitter has perfect knowledge of the transmit channel gains between M transmit antennas and N receive antennas. It also assumes a very large spreading factor, so that the inter-path interference can be neglected. In this case, the receiver is equivalent to an $N \times L$-branch antenna diversity receiver, with the reduced received signal power per antenna by a factor of L, in a frequency non-selective fading channel. Therefore, the received signal energy per bit-to-the noise power spectrum density ratio E_b/N_0, γ_R, after coherent Rake combining can be given by

$$\gamma_R = \gamma_T \sum_{m=0}^{M-1} |\alpha_m|^2 \left(\sum_{n=0}^{N-1} \sum_{l=0}^{L-1} \left| g_{mn}^{(l)} \right|^2 \right), \tag{2}$$

where γ_T is transmit E_b/N_0, $g_{mn}^{(l)}$ is the l-th path gain between m-th transmit antenna and n-th receive antenna with $E[|g_{mn}^{(l)}|^2] = 1/L$ for all m and n ($E[.]$ denotes the ensemble average operation). From Eq. (2), let

$$h_m = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} \left| g_{mn}^{(l)} \right|^2. \tag{3}$$

h_m can be viewed as the equivalent channel gain between the m-th transmit antenna and the Rake combiner output. Therefore, we can apply the theory we developed in Ref. [3]. The optimum set of antenna weights $\{\hat{\alpha}_m\}$ that maximizes γ_R and the maximum value of γ_R can be respectively given by [3]

$$\begin{cases} \hat{\alpha}_m = \delta_{mi} \\ \gamma_R = \gamma_T h_i \quad \text{if } h_i = \max\limits_{m=\{0,1,\cdots,M-1\}} \{h_m\} \end{cases}, \tag{4}$$

where δ_{mi} is Kronecker's delta function. The optimum set of antenna weights is to transmit only from the best antenna that has the maximum equivalent channel gain. Hence, hereafter in this paper, this transmit diversity is called SCTD as in [3] (see Fig. 2). The time delays $\{\tau_m\}$ do not need to be added at the transmitter.

3. Theoretical Analysis

The average BER performance and the required average transmit E_b/N_0 are theoretically analyzed.

3.1 BER Performance

Since h_i is the random variable, the received E_b/N_0, γ_R, after Rake combining also becomes a random variable. The probability density function (pdf) $p(\gamma_R)$ is given by (see Appendix A)

$$p(\gamma_R) = \frac{M\gamma_R^{NL-1}}{(NL-1)!(\gamma_T/L)^{NL}} \exp\left(-\frac{\gamma_R}{\gamma_T/L}\right) \times \left\{ 1 - \exp\left(-\frac{\gamma_R}{\gamma_T/L}\right) \sum_{q=0}^{NL-1} \frac{1}{q!} \left(\frac{\gamma_R}{\gamma_T/L}\right)^q \right\}^{M-1}. \tag{5}$$

Using Eq. (5), the average BER $P_{b,SCTD}\,(\gamma_T)$ is given by

$$\begin{aligned} P_{b,SCTD}(\gamma_T) &= \int_0^\infty \frac{1}{2} erfc\sqrt{\gamma_R}\, p(\gamma_R) d\gamma_R \\ &= \frac{1}{(NL-1)!} \sum_{m=1}^{M} \binom{M}{m} \frac{(-1)^{m+1}(m-1)!}{m^{NL-1}} \\ &\quad \times \sum_{\{a_q;q=0,1,\cdots,NL-1\}\in\Omega(m)} \frac{(NL+A(a_q)-1)!}{m^{A(a_q)}} B(a_q) \\ &\quad \times \frac{1}{2} \left[1 - \frac{1}{\sqrt{1+mL/\gamma_T}} \right. \\ &\quad \left. \times \sum_{j=0}^{NL+A(a_q)-1} \frac{\binom{2j}{j}}{\{4(1+\gamma_T/(mL))\}^j} \right] \\ &\approx \frac{(2MNL-1)!!}{2^{MNL+1}\{(NL)!\}^M} \left(\frac{L}{\gamma_T}\right)^{MNL}, \\ &\quad \text{for } \gamma_T \gg 1, \end{aligned} \tag{6}$$

where

$$\begin{cases} \binom{M}{m} = \dfrac{M!}{m!(M-m)!} \\ A(a_q) = \displaystyle\sum_{q=0}^{NL-1} q a_q \\ B(a_q) = \dfrac{1}{\prod_{q=0}^{NL-1} a_q!(q!)^{a_q}}, \ \{a_q\} = 0,1,2,\cdots \end{cases} \tag{7}$$

Ω(m) represents the set of combinations $\{a_0, a_1, \cdots, a_{NL-1}\}$ which satisfies $\sum_{q=0}^{NL-1} a_q = m-1$,and

$$erfc(y) = \frac{2}{\sqrt{\pi}} \int_y^\infty \exp(-t^2) dt$$

is the complimentary error function. On the other hand, the average BER $P_{b,delayTD}\,(\gamma_T)$ of delay TD is given by [4]

$$\begin{aligned}
&P_{b,delayTD}(\gamma_T)\\
&=\frac{1}{2}\left[1-\frac{1}{\sqrt{1+ML/\gamma_T}}\right.\\
&\qquad\left.\times\sum_{j=0}^{MNL-1}\frac{\binom{2j}{j}}{\{4\,(1+\gamma_T/(ML))\}^j}\right]\\
&\approx\frac{(2MNL-1)!!}{2^{MNL+1}(MNL)!}\left(\frac{ML}{\gamma_T}\right)^{MNL},\ \text{for } \gamma_T\gg 1,
\end{aligned}\tag{8}$$

since the delay TD system is equivalent to an $M\times N\times L$-branch antenna diversity receiver using maximum ratio combining (MRC) with the reduced received signal power per antenna by a factor of $M\times L$. Therefore, the gain G_T in the required γ_T of SCTD over the delay TD approaches

$$G_T=10\log\left(M\left[\frac{\{(NL)!\}^M}{(MNL)!}\right]^{1/MNL}\right)\ \text{dB}\tag{9}$$

for $\gamma_T\to\infty$. The value of asymptotic G_T is plotted as a function of L in Fig. 3. The gain decreases as L increases for the given M and N. Although the gain decreases as N increases, gain of 0.7 dB can be obtained when $M=N=L$=2.

3.2 Joint Use of SCTD and Fast TPC

The joint use of SCTD and fast TPC is considered. Ideal fast TPC is assumed such that transmit power S is controlled to keep γ_R=γ_{tpc}. The transmit E_b/N_0, γ_T, for SCTD can be obtained using Eqs. (3) and (4). For delay TD, it can be obtained using the fact that the delay TD is equivalent to MRC receive diversity with the reduced received signal power per antenna by a factor of $M\times L$. We have

$$\gamma_T=S\left(\frac{T}{2N_0}\right)
=\begin{cases}
\gamma_{tpc}\Big/\left\{\max\limits_{m=\{0,1,\cdots,M-1\}}\left(\sum\limits_{n=0}^{N-1}\sum\limits_{l=0}^{L-1}\left|g_{mn}^{(l)}\right|^2\right)\right\}, & \\
\quad\text{for SCTD} & \\
\gamma_{tpc}\Big/\left(\dfrac{1}{ML}\sum\limits_{m=0}^{M-1}\sum\limits_{n=0}^{N-1}\sum\limits_{l=0}^{L-1}\left|g_{mn}^{(l)}\right|^2\right), & \\
\quad\text{for delay TD.} &
\end{cases}\tag{10}$$

γ_T is a random variable. Its pdf $p\,(\gamma_T)$ is given by (see Appendix B)

$$p(\gamma_T)
=\begin{cases}
\dfrac{1}{\gamma_{tpc}}\dfrac{MN}{(NL)!}\left(\dfrac{L}{\gamma_T/\gamma_{tpc}}\right)^{NL+1}\exp\left(-\dfrac{L}{\gamma_T/\gamma_{tpc}}\right)\\
\quad\times\left\{1-\exp\left(-\dfrac{L}{\gamma_T/\gamma_{tpc}}\right)\right.\\
\qquad\left.\times\sum\limits_{q=0}^{NL-1}\dfrac{1}{q!}\left(\dfrac{L}{\gamma_T/\gamma_{tpc}}\right)^q\right\}^{M-1},\\
\quad\text{for SCTD}\\
\dfrac{1}{\gamma_{tpc}}\dfrac{N}{(MNL)!}\left(\dfrac{ML}{\gamma_T/\gamma_{tpc}}\right)^{NL+1}\exp\left(-\dfrac{ML}{\gamma_T/\gamma_{tpc}}\right),\\
\quad\text{for delay TD}
\end{cases}\tag{11}$$

When $M\times N\times L$=1, $E[\gamma_T]$=∞. When $M\times N\times L\neq$1, the average γ_T, $E[\gamma_T]$, is given by

$$E[\gamma_T]
=\begin{cases}
\begin{cases}\gamma_{tpc}D(M), \text{ if } NL=1\\ \gamma_{tpc}F(M,N,L), \text{ if } NL\geq 2\end{cases}, & \text{for SCTD}\\
\gamma_{tpc}\dfrac{ML}{MNL-1}, \text{ for delay TD}
\end{cases}\tag{12}$$

where $A(a_q)$ and $B(a_q)$ are given by Eq. (7), and $D(M)$ and $F(M,L,N)$ are given by

$$D(M)
=\begin{cases}
M\ln\left[\dfrac{2^{\binom{M-1}{1}}4^{\binom{M-1}{3}}\cdots M^{\binom{M-1}{M-1}}}{1^{\binom{M-1}{0}}3^{\binom{M-1}{2}}\cdots(M-1)^{\binom{M-1}{M-2}}}\right],\\
\quad\text{if } \ M=2,4,\cdots\\
M\ln\left[\dfrac{2^{\binom{M-1}{1}}4^{\binom{M-1}{3}}\cdots(M-1)^{\binom{M-1}{M-2}}}{1^{\binom{M-1}{0}}3^{\binom{M-1}{2}}\cdots(M-2)^{\binom{M-1}{M-3}}M^{\binom{M-1}{M-1}}}\right],\\
\quad\text{if } \ M=3,5,\cdots
\end{cases}\tag{13a}$$

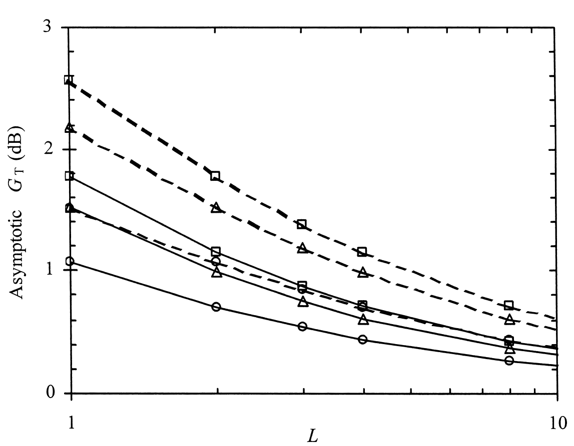

Fig. 3 Asymptotic G_T without TPC.

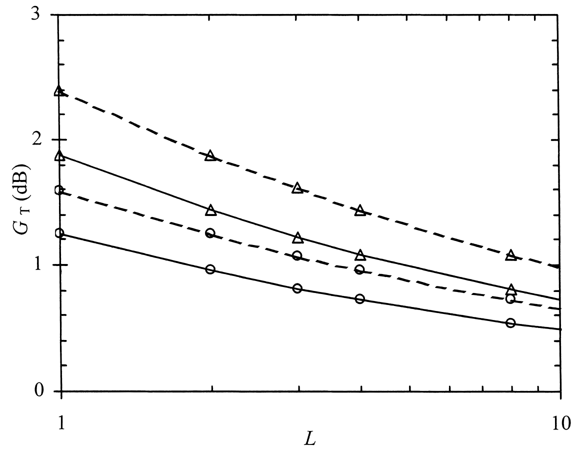

Fig. 4 G_T with fast TPC.

and

$$F(M,N,L) = \frac{L}{(NL-1)!}\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m+1}(m-1)!}{m^{NL-2}} \times \sum_{\{a_q;q=0,1,\cdots,NL-1\}\in\Omega(m)}\frac{(NL+A(a_q)-2)!}{m^{A(a_q)}}B(a_q). \tag{13b}$$

Since γ_R is kept at the target value γ_{tpc}, the BER $P_b(\gamma_{tpc})$ is given by

$$P_b(\gamma_{tpc}) = \frac{1}{2}erfc\sqrt{\gamma_{tpc}}, \tag{14}$$

which can be rewritten as a function of $E[\gamma_T]$. From Eq. (12), the SCTD gain G_T over delay TD to achieve a certain BER is given by

$$G_T = 10\log\left\{\frac{ML}{(MNL-1)F(M,N,L)}\right\}\text{ dB}. \tag{15}$$

Figure 4 plots the value of G_T as a function of L. Similar to the case without TPC, the gain decreases as L increases. The value of G_T is slightly larger with TPC than without TPC.

4. Computer Simulation

Simulation parameters are listed in Table 1. We assume quaternary phase shift keying (QPSK) data modulation and SCTD with two transmit antennas (M=2) and two receive antennas (N=2). Since the transmit power is assumed to be updated only by the limited amount of Δ_{tpc}=1 dB every T_{tpc}=64 symbols, the fast and deep power drops due to fading cannot be fully regulated and hence the average BER performance may degrade compared to the ideal TPC case. Perfect knowledge of transmit channel gains is assumed. In the simulation, the interpath interference due to asynchronism among different paths is simulated although it is neglected in the analysis.

Table 1 Simulation parameters.

Propagation channel		Frequency selective Rayleigh fading (L-path)
Data modem.	Modulation	QPSK
	Demodulation	Ideal coherent detection
Spreading factor		64
TPC	Period T_{tpc}	64 symbols
	Step size Δ_{tpc}	1 dB
	Delay	T_{tpc}
Transmit diversity	Channel estimation	Ideal
	Diversity scheme	Ideal M=2-antenna SCTD
Receiver diversity		N=2-antenna+L-finger Rake combining based on MRC

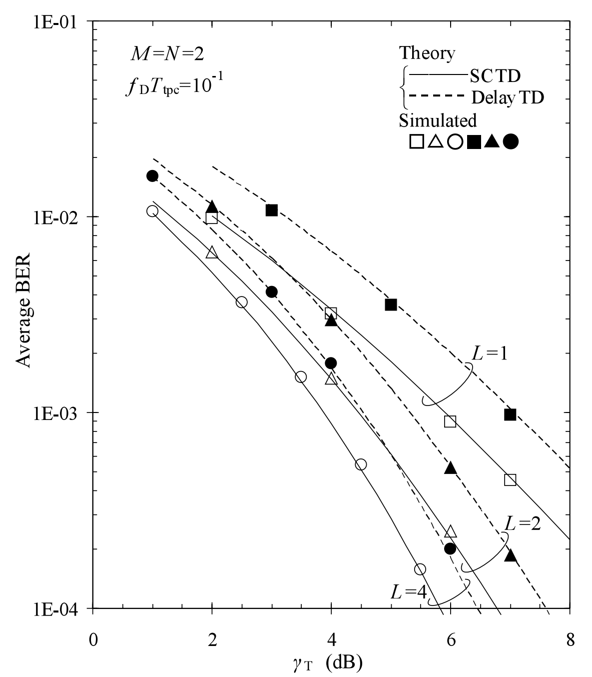

Fig. 5 BER performance without fast TPC.

Figure 5 plots the theoretical and simulated average BER performances for $f_D T_{tpc}$=10^{-1} without TPC as a function of γ_T. A fairly good agreement between theory and simulation is seen. SCTD provides better BER performance than delay TD. The SCTD gain over delay TD in the required average γ_T for BER=10^{-3} is 1.1 dB, 0.9 dB and 0.6 dB with L=1, 2 and 4, respectively.

Figure 6 plots the theoretical and simulated average BER performances with fast TPC as a function of the average γ_T for various maximum Doppler frequency $f_D T_{tpc}$ when L=2. Similar to the case without TPC, the SCTD can achieve better performance than the delay TD. It is seen that when $f_D T_{tpc}$=10^{-1}, the

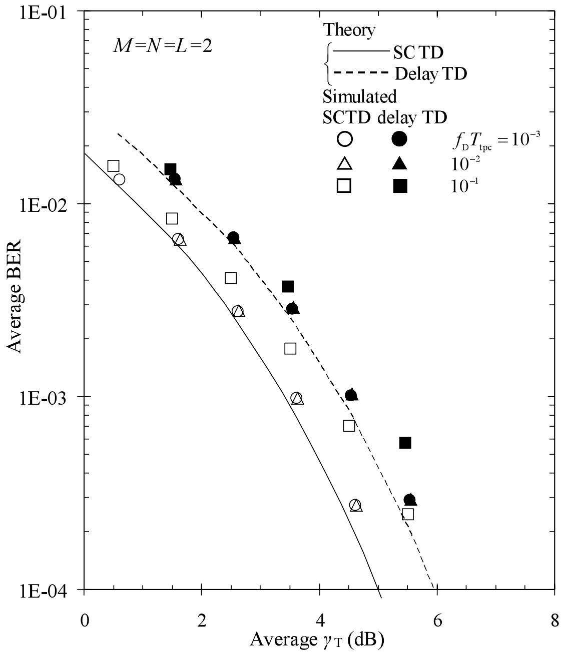

Fig. 6 BER performance with fast TPC.

simulated BER performances of both transmit diversity schemes are worse than the ideal TPC case. This is because TPC cannot track the fast channel gain variations. However, the SCTD gain over delay TD of 1.0 dB is still obtained.

5. Conclusions

In this paper, we considered the weighted delay antenna transmit diversity combined with antenna diversity reception for DS-CDMA mobile radio. The optimum set of antenna weights was found to transmit only from the best antenna that has the maximum equivalent channel gain after Rake combining. This diversity has been called SCTD. The joint effect of SCTD and fast TPC was theoretically analyzed and evaluated by computer simulation to show superiority of SCTD over delay TD.

In this paper, it was assumed that the transmitter has the perfect knowledge of transmit channel gains. In practice, however, the transmitter needs to estimate transmit channel gains from the received signals and thus the BER performance may degrade. This is left as an interesting future study.

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Appendix A: Pdf of Received E_bN_0 of SCTD

For the uniform power delay profile, $E[|g_{mn}^{(l)}(t)|^2] = 1/L$. Since the cumulative distribution function of h_i is given by

$$P(h_i) = \left[1 - e^{-Lh_i} \sum_{q=0}^{NL-1} \frac{(Lh_i)^q}{q!}\right]^M, \qquad \text{(A·1)}$$

pdf of h_i is given by

$$p(h_i) = \frac{\partial P(h_i)}{\partial h_i} = \frac{ML(Lh_i)^{NL-1}}{(NL-1)!} \cdot e^{-Lh_i} \left\{1 - e^{-Lh_i} \sum_{q=0}^{NL-1} \frac{(Lh_i)^q}{q!}\right\}^{M-1} \qquad \text{(A·2)}$$

and the pdf $p(\gamma_R)$ of the received E_b/N_0, γ_R, is given by

$$\begin{aligned} p(\gamma_R) &= p(h_i)/\gamma_T \\ &= \frac{M\gamma_R^{NL-1}}{(NL-1)!(\gamma_T/L)^{NL}} \exp\left(-\frac{\gamma_R}{\gamma_T/L}\right) \\ &\times \left\{1 - \exp\left(-\frac{\gamma_R}{\gamma_T/L}\right) \sum_{q=0}^{NL-1} \frac{1}{q!} \left(\frac{\gamma_R}{\gamma_T/L}\right)^q\right\}^{M-1}. \end{aligned} \qquad \text{(A·3)}$$

Appendix B: Pdf of Transmit E_b/N_0 of SCTD with Fast TPC

Assume fast TPC, $\gamma_{tpc}=\gamma_R$. Since $\gamma_R=\gamma_T h_i$, we have $\gamma_T=\gamma_R/h_i=\gamma_{tpc}/h_i$. Therefore, the pdf $p(\gamma_T)$ is given by

$$\begin{aligned} p(\gamma_T) &= p(h_i)\frac{h_i^2}{\gamma_{tpc}} \\ &= \frac{MN}{(NL)!}\left(\frac{L}{\gamma_T/\gamma_{tpc}}\right)^{NL+1} \exp\left(-\frac{L}{\gamma_T/\gamma_{tpc}}\right) \\ &\times \left\{1 - \exp\left(-\frac{L}{\gamma_T/\gamma_{tpc}}\right) \sum_{q=0}^{NL-1} \frac{1}{q!} \left(\frac{L}{\gamma_T/\gamma_{tpc}}\right)\right\}^{M-1}. \end{aligned} \qquad \text{(A·4)}$$