

PAPER

# Theoretical Analysis of MC-CDMA Forward Link Performance in the Presence of Pure Impulsive Interference

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**SUMMARY** In this paper, expressions are derived for the bit error rate (BER) of the multicarrier-CDMA (MC-CDMA) downlink in the presence of pure impulsive interference and a frequency-selective fading and the BER performance is numerically evaluated by a Monte-Carlo simulation method. Minimum mean square error combining (MMSEC) and orthogonal restoration combining (ORC) are considered for frequency-domain equalization. The joint weight of antenna diversity reception using maximal ratio combining (MRC) and frequency equalization combining is derived. The MC-CDMA transmission performance in the presence of pure impulsive interference is compared with that of DS-CDMA transmission.

**key words:** *impulsive interference, MC-CDMA, OFDM-CDMA, antenna diversity*

## 1. Introduction

Recently, multicarrier code division multiple access (MC-CDMA) (or sometimes called OFDM-CDMA) has been attracting much attention as a promising wireless access technique for a wideband downlink (base-to-mobile) transmission in a frequency selective fading channel [1]–[5]. In MC-CDMA, different user's transmitting data symbol is spread over a number of orthogonal sub-carriers using a different orthogonal spreading sequence defined in the frequency-domain, while in direct sequence CDMA (DS-CDMA) the spreading sequence defined in the time domain is used. Most practical receivers are designed to be optimal or near optimal against Gaussian noise. However, when the radio bandwidth becomes wider in order to meet the demands for higher data rate transmissions, the transmission performance may be seriously impacted by impulsive noises caused by vehicles, power lines, electrical equipment, etc.

Many literatures treating the impacts of impulsive noise on the transmission performance can be found for DS-CDMA, time division multiple access (TDMA) and OFDM. The bit error rate (BER) performances of DS-CDMA in the presence of the Middleton's class-A interference [6] and the  $\varepsilon$ -mixture noise were ana-

lyzed in [7] and [8], respectively. Recently, the impact of pure impulsive noise in DS-CDMA was theoretically analyzed by the authors [9]. The BER performance of TDMA with antenna diversity reception in the presence of the  $\varepsilon$ -mixture noise was theoretically analyzed in [10] to reveal that antenna diversity using linear combining cannot effectively reduce the impacts of impulsive interference. The BER performance analysis of OFDM in the presence of the pure impulsive interference and the  $\varepsilon$ -mixture noise can be found in [11]. However, to the best of authors' knowledge, the BER performance of MC-CDMA in the presence of pure impulsive interference has not been fully understood.

In mobile communications, a user may reach the vicinity of an interference source where the impulsive interference may seriously degrade the downlink transmission performance. Motivated by the above, this paper analyzes the BER performance of MC-CDMA downlink transmission in the presence of pure impulsive interference and a frequency-selective fading. A downlink transmission system model using MC-CDMA is presented in Sect. 2 and BER expressions are derived in Sect. 3. Then, in Sect. 4, the numerical computation based on Monte-Carlo method is performed to evaluate the BER performance. Furthermore, the MC-CDMA transmission performance is compared with the DS-CDMA transmission performance. Section 5 draws some conclusions.

## 2. Down Link Transmission System Model

Figure 1 illustrates the structure of MC-CDMA downlink transmission system. At a base station, binary data to be transmitted is transformed into quaternary phase shift keying (QPSK)-modulated symbol sequence. In this paper, it is assumed that the number of orthogonal sub-carriers equals the spreading factor ( $SF$ ) and that  $K$  users are in communication. The  $q$ th data symbol  $d_k(q)$  of the  $k$ th user,  $k = 0, \dots, K - 1$ , is spread over  $SF$  orthogonal sub-carriers using a  $SF$ -length orthogonal spreading sequence  $\{c_k(i); i = 0, \dots, SF - 1\}$ . All  $i$ th sub-carrier components of  $K$  users are combined and then, multiplied by the  $j$ th chip of a scramble sequence  $\{c_{PN}(j)\}$  to obtain the  $i$ th composite sub-carrier component of MC-CDMA signal,

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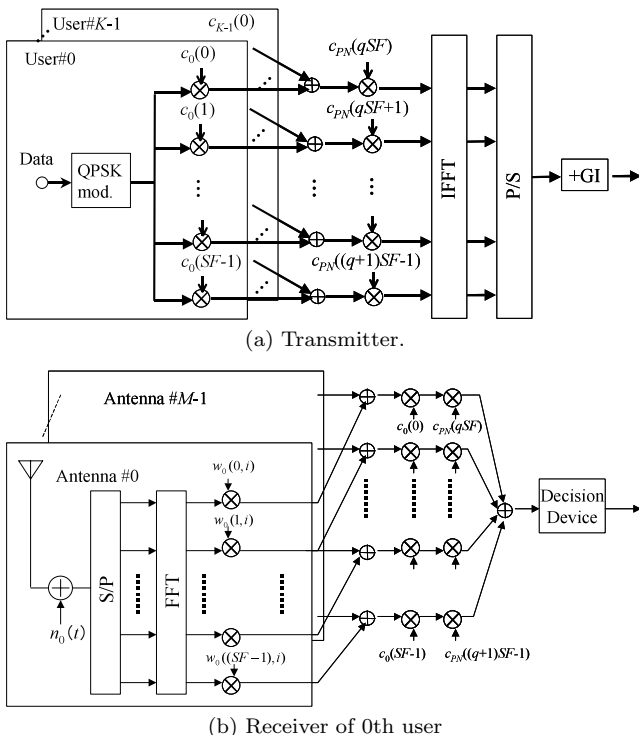


Fig. 1 Structure of MC-CDMA downlink transmission system.

where  $j = qSF + i$ . The scramble sequence is a binary long pseudo noise (PN) sequence whose repetition period is much longer than  $SF$ .

The sum of  $SF$  composite sub-carrier components forms the MC-CDMA waveform of length  $T_s$ , where  $T_s$  is the effective symbol length. This process is performed using the inverse fast Fourier transform (IFFT). Finally, a small time portion  $T_g$  of the MC-CDMA waveform is copied and added as a guard interval (GI) to form the transmitting MC-CDMA symbol of length  $T = T_s + T_g$ . The MC-CDMA signal waveform can be expressed using the equivalent baseband representation as

$$s(t) = \sum_{q=-\infty}^{\infty} g(t - qT) \sqrt{\frac{2S}{SF}} \sum_{i=0}^{SF-1} u(q, i) \cdot \exp\left(j2\pi \frac{i}{T_s}(t - qT)\right), \quad (1)$$

where  $g(t)$  is the transmit pulse waveform given by

$$g(t) = \begin{cases} 1 & -T_g \leq t \leq T_s \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

$S$  is the average signal power per user and  $u(q, i)$  is the  $i$ th composite sub-carrier component for the  $q$ th MC-CDMA symbol and is expressed as

$$u(q, i) = c_{PN}(qSF + i) \sum_{k=0}^{K-1} d_k(q) c_k(i), \quad (3)$$

with  $c_k(i) = \{-1, 1\}$  and  $c_{PN}(i) = \{-1, 1\}$ .

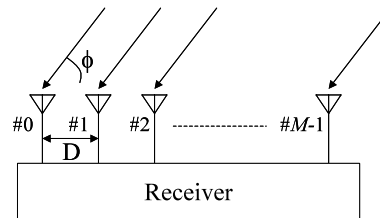


Fig. 2 Antenna arrangement.

$M$ -branch antenna diversity reception using maximal ratio combining (MRC) [12] is considered at the mobile station. Figure 2 illustrates the antenna arrangement with antenna separation of  $D$ . It is assumed that the source of an impulsive interference is located in line of sight, but, sufficiently far from the mobile receiver of interest and hence, it arrives as a plane wave with the same incident angle to all the receiver antennas. The multipath channel is assumed to be a frequency-selective fading channel having  $L$  discrete paths with different time delays. In this paper, we assume that  $L$  paths are subjected to independent Rayleigh fading and furthermore that the antenna separation  $D$  is sufficiently large (e.g.,  $D \approx \lambda/2$ , where  $\lambda$  is the carrier wavelength, for the antenna diversity reception at the mobile station [15]) to obtain independent fading on all antennas. The channel impulse response  $h_m(\tau, t)$  observed by the  $m$ th antenna at time  $t$  may be represented as

$$h_m(\tau, t) = \sum_{l=0}^{L-1} \xi_{m,l}(t) \delta(\tau - \tau_l), \quad (4)$$

where  $\xi_{m,l}(t)$  and  $\tau_l$  are respectively the complex channel gain and the time delay of the  $l$ th path with  $\sum_{l=0}^{L-1} E[|\xi_{m,l}(t)|^2] = 1$  for all  $m$ , where  $E[\cdot]$  represents ensemble average operation.  $\{\xi_{m,l}(t); m = 0, \dots, M-1, l = 0, \dots, L-1\}$  are independent and identically distributed zero-mean complex Gaussian processes. It is assumed that  $h_m(\tau, t)$  exists only over an interval of  $0 \leq \tau \leq T_g$ , i.e.,  $\{\tau_l\}_{\max} - \{\tau_l\}_{\min} \leq T_g$ . The channel transfer function  $H_m(f, t)$  is the Fourier transform of  $h_m(\tau, t)$  with respect to  $\tau$ . Hence, we have

$$H_m(f, t) = \sum_{l=0}^{L-1} \xi_{m,l}(t) \exp(-j2\pi f \tau_l). \quad (5)$$

Assuming that the complex channel gain  $\xi_{m,l}(t)$  varies very slowly and remains constant over an interval of  $T_s$ , the equivalent baseband representation of the  $i$ th received composite sub-carrier component  $\tilde{u}_m(q, i)$  is given by

$$\tilde{u}_m(q, i) = \sqrt{\frac{2S}{SF}} H_m(i/T_s, qT) u(q, i) + \tilde{I}_m(q, i) + \tilde{n}_m(q, i), \quad (6)$$

where  $\tilde{n}_m(q, i)$  is the zero-mean complex Gaussian variable with a variance of  $2N_0/T_s$  owing to the additive

white Gaussian noise (AWGN);  $N_0$  is the single sided power spectrum density. Assuming that a single impulse has occurred in  $qT < t < qT + T_s$ ,  $\tilde{I}_m(q, i)$  is the impulsive noise, given by

$$\tilde{I}_m(q, i) = \frac{A}{T_s} \exp\left(j2\pi\left(m\frac{D}{\lambda}\cos\phi - \frac{t_n - qT}{T_s}i\right)\right), \quad (7)$$

where  $A$  is the area of impulse,  $\phi$  is the incident angle to  $M$  antennas,  $\lambda$  is the carrier wavelength,  $D$  is the antenna separation and  $t_n$  is the occurrence instant of the  $n$ th impulse. We are assuming a pure impulsive interference model [9], [13], [14]. The pure impulse has an infinite amplitude, resulting in infinite energy. However, since the FFT process is equivalent to an integration filter with the bandwidth of  $1/T_s$ , the band-limited impulsive interference has a finite amplitude, which is  $A/T_s$  as seen from Eq. (7).

After removing the GI, the received signal on each antenna is decomposed into  $SF$  sub-carrier components by FFT processing. Since  $H_m(f, t)$  varies over the signal bandwidth due to frequency selective fading, the orthogonal property among  $K$  users is destroyed to some extent and thus, multi user interference (MUI) is produced. Hence, each received composite sub-carrier component needs to be equalized when despreading. This is called frequency equalization combining. In this paper, ORC [2] and MMSEC [3], [4] are considered. The ORC completely removes MUI, but produces the noise enhancement, while the MMSEC minimizes the sum of background noise and MUI. The MMSEC reduces to the ORC when the background noise can be neglected. Without loss of generality, we consider the 0th user's  $q$ th QPSK symbol  $d_0(q)$  is to be detected. The  $M \times SF$   $i$ th sub-carrier components  $\{\tilde{u}_m(q, i); m = 0, \dots, M-1, i = 0, \dots, SF-1\}$  are multiplied with the joint weights  $w_m(q, i)$ , the scramble sequence  $\{c_{PN}(qSF+i)\}$ , and the orthogonal spreading sequence  $\{c_0(i)\}$ , and then summed up. Therefore the decision variable  $\hat{d}_0(q)$  for the symbol  $d_0(q)$  is expressed as

$$\begin{aligned} \hat{d}_0(q) &= \left(\frac{2}{\sqrt{2N_0/T_s}}\right) \sqrt{\frac{1}{SF}\left(\frac{E_s}{N_0}\right)} \\ &\times \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} \tilde{u}_m(q, i) w_m(q, i) \\ &\times c_{PN}(qSF+i) c_0(i) \\ &= \sqrt{2}\mu d_0(q) + z, \end{aligned} \quad (8)$$

where  $\tilde{u}_m(q, i)$  is given by Eq. (6) and

$$\mu = \frac{\sqrt{2}}{SF} \left(\frac{E_s}{N_0}\right) \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} w_m(q, i) H_m(i/T_s, qT) \quad (9a)$$

$$z = MUI + N_{impulse} + N_{AWGN} \quad (9b)$$

with

$$w_m(q, i) = \begin{cases} \frac{H_m^*(i/T_s, qT)}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2 + \left(\frac{K}{SF}\frac{E_s}{N_0}\right)^{-1}} & \text{for MMSEC} \\ \frac{H_m^*(i/T_s, qT)}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2} & \text{for ORC} \end{cases} \quad (10a)$$

$$\begin{aligned} MUI &= \frac{2}{SF} \left(\frac{E_s}{N_0}\right) \sum_{k=1}^{K-1} d_k(q) \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} c_k(i) c_0(i) \\ &\times w_m(q, i) H_m(i/T_s, qT), \end{aligned} \quad (10b)$$

$$\begin{aligned} N_{impulse} &= 2 \left(\frac{A/T_s}{\sqrt{2N_0/T_s}}\right) \sqrt{\frac{1}{SF}\left(\frac{E_s}{N_0}\right)} \\ &\times \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} c_0(i) c_{PN}(qSF+i) \\ &\times w_m(q, i) \exp\left(j2\pi\left(m\frac{D}{\lambda}\cos\phi - \frac{t_n - qT}{T_s}i\right)\right) \end{aligned} \quad (10c)$$

$$\begin{aligned} N_{AWGN} &= \left(\frac{2}{\sqrt{2N_0/T_s}}\right) \sqrt{\frac{1}{SF}\left(\frac{E_s}{N_0}\right)} \\ &\times \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} c_0(i) c_{PN}(qSF+i) \\ &\times w_m(q, i) \tilde{n}_m(q, i) \end{aligned} \quad (10d)$$

and  $E_s (=ST_s)$  is the effective symbol energy. The first term of Eq. (8) is the desired signal component and the second term  $z$  is the sum of the MUI, noise due to impulsive interference, and noise due to AWGN. For derivation of Eq. (10a), see Appendix A.

### 3. BER Expressions for MC-CDMA with MMSEC and ORC

Assuming that the four QPSK data symbols are transmitted equally likely, transmission of  $d_0(q) = (1+j)/\sqrt{2}$  (representing (1, 1) transmission) is considered without loss of generality. The conditional probability of error of the first bit for the given  $\{\xi_{m,l}; m = 0, \dots, M-1, l = 0, \dots, L-1\}$  can be obtained once the statistical property of  $\text{Re}[\hat{d}_0(q)]$  is known, where  $\text{Re}[x]$  is the real part of the complex-valued variable  $x$  and  $\xi_{m,l} = \xi_{m,l}(0)$  for simplicity. Substituting  $d_0(q) = (1+j)/\sqrt{2}$  into Eq. (8) gives

$$\text{Re}[\hat{d}_0(q)|d_0(q) = (1+j)/\sqrt{2}, \{\xi_{m,l}\}] = \mu + \text{Re}[z]. \quad (11)$$

Remembering that the orthogonal spreading sequences

are used, each user's interference before despreading is represented as the sum of the average interference over  $SF$  sub-carriers and the difference from the average interference. Due to the use of orthogonal spreading sequences, the contribution of the average interference to the  $MUI$  value becomes zero and only the differential interference contributes to the  $MUI$  value. Assuming that many users are in communication and  $SF \gg 1$ ,  $MUI$  can be approximated as a zero-mean complex Gaussian variable from the central limit theorem. Since  $c_{PN}(i)$  is the scramble sequence having a period much longer than  $SF$ ,  $c_0(i)c_{PN}(qSF+i)$  can be treated as a random binary sequence. Therefore, from the central limit theorem, the impulsive interference component  $N_{impulse}$  of Eq. (10c) is well approximated as a zero-mean complex Gaussian variable. Since  $\tilde{n}_m(q, i)$  is a zero mean complex Gaussian variable with the variance of  $2N_0/T_s$ ,  $N_{AWGN}$  of Eq. (10d) is a zero mean complex Gaussian variable. Consequently,  $z$  in Eq. (8) can be approximated as a zero-mean complex Gaussian variable with the variance of  $2\sigma^2$ , and the conditional decision error probability for the given  $\{\xi_{m,l}\}$  can be expressed as

$$\begin{aligned} \Pr \left( \operatorname{Re}[\hat{d}_0(q)|d_0(q) = \frac{1+j}{\sqrt{2}}, \{\xi_{m,l}\}] < 0 \right) \\ = \frac{1}{2} \operatorname{erfc} \left( \frac{\mu}{\sqrt{2\sigma^2}} \right), \end{aligned} \quad (12)$$

where  $\operatorname{erfc}(x) = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$  and,  $\mu$  and  $2\sigma^2$  are given by

$$\mu = \frac{\sqrt{2}}{SF} \left( \frac{E_s}{N_0} \right) \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} w_m(q, i) H_m(i/T_s, qT), \quad (13a)$$

$$2\sigma^2 = 2\sigma_{MUI}^2 + 2\sigma_{impulse}^2 + 2\sigma_{AWGN}^2, \quad (13b)$$

for MMSEC, and

$$\mu = \sqrt{2} \frac{E_s}{N_0}, \quad (14a)$$

$$2\sigma^2 = 2\sigma_{impulse}^2 + 2\sigma_{AWGN}^2, \quad (14b)$$

for ORC, where  $2\sigma_{MUI}^2$ ,  $2\sigma_{impulse}^2$  and  $2\sigma_{AWGN}^2$  are the variances of the MUI component, the impulsive interference component and the AWGN component, respectively, and they are given by (B1)(see Appendix B)

$$\begin{aligned} 2\sigma_{MUI}^2 &= \frac{4(K-1)}{SF} \left( \frac{E_s}{N_0} \right)^2 \\ &\cdot \left( \frac{1}{SF} \sum_{i=0}^{SF-1} \left( \sum_{m=0}^{M-1} w_m(q, i) H_m(i/T_s, qT) \right)^2 \right) \end{aligned} \quad (15a)$$

$$2\sigma_{impulse}^2 = \frac{4}{SF} \left( \frac{A/T_s}{\sqrt{2N_0/T_s}} \right)^2 \left( \frac{E_s}{N_0} \right) \cdot \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} |w_m(q, i)|^2, \quad (15b)$$

$$2\sigma_{AWGN}^2 = \frac{4}{SF} \frac{E_s}{N_0} \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} |w_m(q, i)|^2. \quad (15c)$$

The average BERs  $P_b(A)$ 's of MC-CDMA in the presence of impulsive interference can be computed by averaging Eq. (12) over  $\{\xi_{m,l}\}$ :

$$\begin{aligned} P_b(A) &= \int \cdots \int \Pr \\ &\cdot \left( \operatorname{Re}[\hat{d}_0(q)|d_0(q) = \frac{1+j}{\sqrt{2}} \text{ and } \{\xi_{m,l}\}] < 0 \right) \\ &\times \prod_{m,l} p(\xi_{m,l}) \prod_{m,l} d\xi_{m,l}. \end{aligned} \quad (16)$$

If the impulsive interference occurs during guard interval, it does not affect the decision at all. The overall BER  $P_b$  is given by

$$P_b = \frac{T_s}{T} P_{pulse} P_b(A) + \left( 1 - \frac{T_s}{T} P_{pulse} \right) P_b(A=0), \quad (17)$$

where  $P_b(A=0)$  is the average BER in the presence of no impulsive interference and  $P_{pulse}$  is the probability of the occurrence of the impulse per QPSK symbol.

#### 4. Numerical Results

An  $L=4$  path Rayleigh channel having uniform power delay profile is assumed for the multipath channel, i.e.,  $E[|\xi_{m,l}|^2] = 1/4$ . The time delay of the  $l$ th path is  $\tau_l = l \cdot \Delta\tau/T$  and  $\Delta\tau/T = 1.95 \times 10^{-2}$  is assumed here.  $M=4$  antenna diversity reception is considered. The multiple-integration required in Eq. (16) is performed based on the Monte Carlo simulation method. The impulsive interference amplitude-to-the rms noise amplitude ratio  $(A/T)/\sqrt{2N_0/T}$  is an important parameter. In the following numerical computation, we set the value of  $(A/T)/\sqrt{2N_0/T}$  to be 100 and the impulse occurrence rate  $P_{pulse}$  to be  $10^{-3}$  for an example. Assuming a transmission of 1M MC-CDMA symbol/s (i.e.,  $T=10^{-6}$ s), a receiver with the noise figure of 5 dB ( $N_F=3.16$ ), and a room temperature of 27 degrees ( $T_0 = 300$  Kelvin) and using  $N_0 = \kappa T_0 N_F$ , where  $\kappa$  is the Boltzman constant, the impulsive interference amplitude  $A/T$  becomes  $16.2 \mu\text{V}$  and the impulse occurring

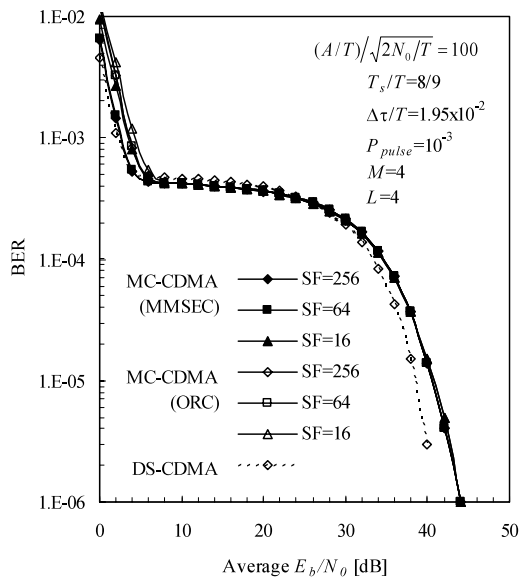


Fig. 3 Impact of  $SF$  on overall BER performance.

rate becomes 1000/s. Since the guard interval is not necessary for DS-CDMA, the signal bandwidth of DS-CDMA is  $T_s/T (< 1)$  times that of MC-CDMA.

4.1 Impacts of  $SF$  and  $T_s/T$

The spreading factor ( $SF$ ) impacts the BER performance of MC-CDMA as understood from Eqs. (13) and (14), even for the single user case ( $K=1$ ). On the other hand, the BER performance does not depend on  $SF$  in DS-CDMA for  $K=1$  [9]. The guard interval is introduced in MC-CDMA in order to keep the orthogonality among sub-carriers in a frequency-selective channel. Therefore,  $SF$  and  $T_s/T$  are important parameters that differentiate MC-CDMA performance from DS-CDMA. How they impact the BER performance of MC-CDMA is discussed below.

Figure 3 plots the overall BER performance curves with  $SF$  as a parameter for  $T_s/T = 8/9$  and  $K = 1$ . It is seen that  $SF$  has no impact on the BER floor value due to impulsive interference for both MC-CDMA and DS-CDMA. However, in the small  $E_b/N_0$  regions below BER floor region, where the AWGN is a predominant cause of errors, the BER value reduces with increasing  $SF$ . This is because, as  $SF$  increases, the frequency diversity effect becomes larger owing to frequency equalization used in MC-CDMA signal reception.

Figure 4 plots the overall BER performance curves with  $T_s/T$  as a parameter for  $SF = 256$  and  $K=1$ . The BER performance of DS-CDMA is almost identical to that of MC-CDMA with large  $T_s/T$ . The BER floor value caused by impulsive interference reduces as  $T_s/T$  decreases. This is because the probability that impulsive interference occurs in the effective symbol period  $T_s$  becomes smaller as  $T_s/T$  decreases for the given value of  $T$ . On the other hand, in the low (less than about 6 dB)

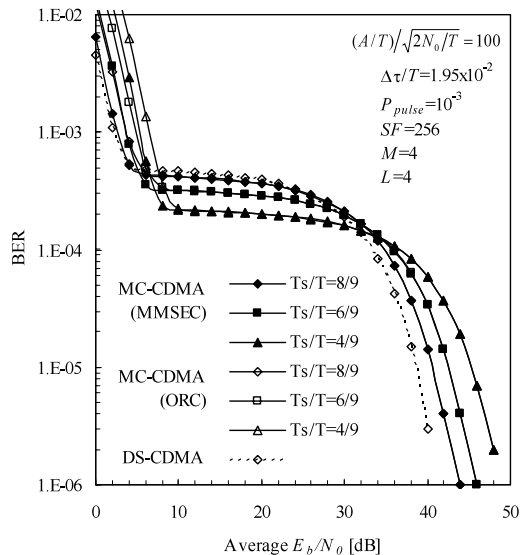


Fig. 4 Impact of  $T_s/T$  on overall BER performance.

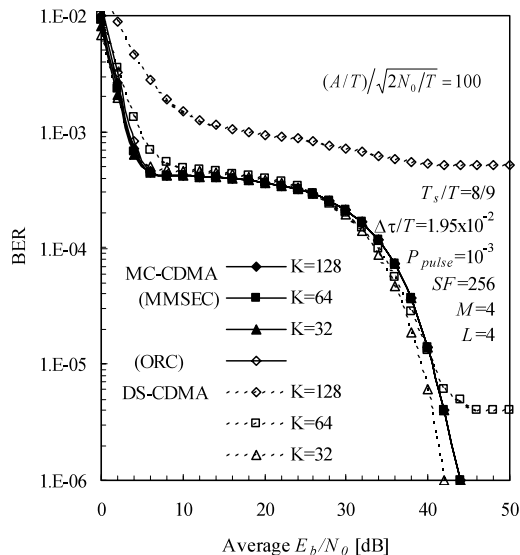


Fig. 5 Impact of number of users on overall BER performance.

and high (larger than about 34 dB)  $E_b/N_0$  regions, the BER value increases as  $T_s/T$  decreases. This is because, for the given value of  $E_b/N_0$ , the effective symbol energy  $E_s (=2E_bT_s/T)$  reduces as  $T_s/T$  decreases.

4.2 Impact of Number of Users

So far we have considered the single user case ( $K = 1$ ). Here, we will extend the evaluation to the multi-user case. In the multi-user case, two kinds of BER floors may be produced; one is the BER floor due to impulsive interference and the other is the BER floor due to MUI. Since MUI can be completely eliminated by using ORC and partially eliminated by using MMSEC (but, note that the MUI for MMSEC approaches zero as the average  $E_b/N_0$  increases), MC-CDMA does not

produce BER floor due to MUI, and hence, only the BER floor due to impulsive interference exists. On the other hand, DS-CDMA yields both BER floors due to impulsive interference and MUI. Figure 5 plots the BER performance with the number  $K$  of users as a parameter. As is expected, no BER floor due to MUI is seen for MC-CDMA, while it is seen for DS-CDMA in the large  $E_b/N_0$  regions; when  $K=128$ , the influence of impulsive interference is almost masked by MUI.

## 5. Conclusions

BER expressions for MC-CDMA in the presence of pure impulsive interference were derived and then, the BER performance was numerically evaluated based on Monte-Carlo simulation method. The BER performance of MC-CDMA with MMSEC or ORC was compared with that of DS-CDMA with coherent Rake combining. The results obtained in this paper can be summarized as follows:

- (a) MC-CDMA with MMSEC and that with ORC produce almost the same BER floor due to impulsive interference as DS-CDMA. The BER floor does not depend on the spreading factor for both MC-CDMA and DS-CDMA.
- (b) The BER floor value caused by impulsive interference reduces as  $T_s/T$  decreases in MC-CDMA.
- (c) For large number of users, e.g.,  $K=128$ , the influence of impulsive interference is almost masked by MUI for DS-CDMA, however the impulsive interference is still the major cause of errors for MC-CDMA.

The results obtained in this paper can be used to estimate the BER performance degradation in the presence of pure impulsive interference. The assumption made in this paper on the pure impulsive interference was that each incoming impulse always has the same area  $A$ . However, in a real environment, the value of  $A$  may be a random variable and impulses may arrive in groups. How these impact the BER performance is left for a future study. Also, an interesting extension of the analysis is to the case of Middleton's class A impulsive interference model.

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## Appendix A: Joint Weight For MRC and MMSEC or ORC of Eq. (10a)

The composite sub-carrier components of  $M$  received signals are coherently combined by MRC and then, MMSEC is performed (note that both MRC and MMSEC are not optimal combining in the presence of the impulsive interference). Assuming ideal channel estimation, the weight  $w_{m,MRC}(q,i)$  of MRC for the  $m$ th antenna is given by

$$w_{m,MRC}(q,i) = H_m^*(i/T_s, qT), \quad (\text{A.1})$$

where  $*$  denotes the complex conjugate operation. The  $i$ th sub-carrier component  $\tilde{u}(q,i)$  after MRC in the presence of no impulsive interference is given by

$$\begin{aligned} \tilde{u}(q,i) &= \sum_{m=0}^{M-1} \tilde{u}_m(q,i) w_{m,MRC}(q,i) \\ &= \sum_{m=0}^{M-1} \left\{ \sqrt{\frac{2S}{SF}} |H_m(i/T_s, qT)|^2 u(q,i) \right\} \end{aligned}$$

$$+ \tilde{n}_m(q, i) H_m^*(i/T_s, qT) \Big\}. \quad (\text{A}\cdot 2)$$

It is understood from Eq. (A·2) that  $\sqrt{2S/SF} \sum_{m=0}^{M-1} |H_m(i/T_s, aT)|^2$  can be viewed as a new channel gain and the noise variance becomes  $(2N_0/T_s) \sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2$ . Therefore, the MMSEC weight  $w_{MMSEC}(q, i)$  is given by [3], [4]

$$w_{MMSEC}(q, i) = \frac{1}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2 + \left(\frac{K}{SF} \frac{E_s}{N_0}\right)^{-1}}. \quad (\text{A}\cdot 3)$$

On the other hand, the ORC weight  $w_{ORC}(q, i)$  is derived by neglecting the background noise term of the MMSEC weight and is given by

$$w_{ORC}(q, i) = \frac{1}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2} \quad (\text{A}\cdot 4)$$

In the sequel, the joint weight  $w_m(q, i)$  is the product of the weights of MRC and MMSEC or ORC and is given by

$$w_m(q, i) = \begin{cases} \frac{H_m^*(i/T_s, qT)}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2 + \left(\frac{K}{SF} \frac{E_s}{N_0}\right)^{-1}} & \text{for MMSEC} \\ \frac{H_m^*(i/T_s, qT)}{\sum_{m=0}^{M-1} |H_m(i/T_s, qT)|^2} & \text{for ORC} \end{cases} \quad (\text{A}\cdot 5)$$

### Appendix B: Derivations of Eqs. (15)

$z$  is the sum of the MUI, the noise  $N_{impulse}$  due to impulsive interference, and the noise  $N_{AWGN}$  due to AWGN. The variance  $2\sigma^2$  of  $z$  is the sum of those of MUI, impulsive interference and the AWGN:

$$2\sigma^2 = 2\sigma_{MUI}^2 + 2\sigma_{impulse}^2 + 2\sigma_{AWGN}^2, \quad (\text{A}\cdot 6)$$

for MMSEC. However, in the ORC case, since MUI is completely removed, the variance  $2\sigma^2$  is given by

$$2\sigma^2 = 2\sigma_{impulse}^2 + 2\sigma_{AWGN}^2. \quad (\text{A}\cdot 7)$$

Below, we derive expressions of  $2\sigma_{MUI}^2$ ,  $2\sigma_{impulse}^2$  and  $2\sigma_{AWGN}^2$ .

First, we derive the expression for  $2\sigma_{MUI}^2$ . The MUI component, given in Eq. (10b), can be rewritten as

$$MUI = \frac{2}{SF} \frac{E_s}{N_0} \sum_{k=1}^{K-1} d_k(q) \cdot \left\{ Y + \bar{y} \sum_{i=0}^{SF-1} c_k(i) c_0(i) \right\}, \quad (\text{A}\cdot 8)$$

where

$$\begin{aligned} Y &= \sum_{i=0}^{SF-1} c_k(i) c_0(i) (y_i - \bar{y}) \\ y_i &= \sum_{m=0}^{M-1} w_m(q, i) H_m(i/T_s, qT) \\ \bar{y} &= \frac{1}{SF} \sum_{i=0}^{SF-1} y_i. \end{aligned} \quad (\text{A}\cdot 9)$$

Since the orthogonal spreading sequences  $\{c_k(i); k = 0 \dots K-1\}$  are used, the second term inside the brackets  $\{\cdot\}$  of Eq. (A·8) becomes zero. From the central limit theorem,  $Y$  can be approximated as a complex Gaussian variable for large  $SF$ . Since data sequence  $\{d_k(q)\}$  is a complex random process,  $MUI$  can be approximated as a zero-mean complex Gaussian variable, whose variance is given by

$$2\sigma_{MUI}^2 = \frac{4(K-1)}{SF} \left(\frac{E_s}{N_0}\right)^2 (\bar{y}^2 - \bar{y}^2), \quad (\text{A}\cdot 10)$$

where

$$\bar{y}^2 = \frac{1}{SF} \sum_{i=0}^{SF-1} y_i^2. \quad (\text{A}\cdot 11)$$

Next, the expressions for  $2\sigma_{impulse}^2$  is obtained. From Eq. (10c),  $N_{impulse}$  can be rewritten as

$$\begin{aligned} N_{impulse} &= 2X \left( \frac{A/T_s}{\sqrt{2N_0/T_s}} \right) \sqrt{\frac{1}{SF} \left(\frac{E_s}{N_0}\right)}, \end{aligned} \quad (\text{A}\cdot 12)$$

where

$$\begin{aligned} X &= \sum_{i=0}^{SF-1} c_i x_i \\ c_i &= c_0(i) c_{PN}(qSF + i) \\ x_i &= \sum_{m=0}^{M-1} \exp\left(j2\pi \left(m \frac{D}{\lambda} \cos \phi - \frac{t_n - qT}{T_s} i\right)\right) \\ &\quad \cdot w_m(q, i). \end{aligned} \quad (\text{A}\cdot 13)$$

Since  $c_{PN}(i)$  is the scramble sequence having a period of much longer than  $SF$ ,  $c_i$  can be treated as a random binary sequence. Therefore, from the central limit theorem,  $X$  can be approximated as a zero-mean complex Gaussian variable for large  $SF$ . As a consequence, the impulsive interference component  $N_{impulse}$

of Eq. (A·12) is well approximated as a zero-mean complex Gaussian variable with variance  $2\sigma_{impulse}^2$  given by

$$2\sigma_{impulse}^2 = \frac{4}{SF} \left( \frac{A/T_s}{\sqrt{2N_0/T_s}} \right)^2 \cdot \left( \frac{E_s}{N_0} \right) \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} |w_m(q, i)|^2. \quad (\text{A} \cdot 14)$$

Finally we obtain the expression for  $2\sigma_{AWGN}^2$ . Since  $\tilde{n}_m(q, i)$  is a zero mean complex Gaussian variable with the variance of  $2N_0/T_s$ ,  $N_{AWGN}$  of Eq. (10d) is also a zero mean complex Gaussian variable whose variance is given by

$$2\sigma_{AWGN}^2 = \frac{4}{SF} \frac{E_s}{N_0} \sum_{i=0}^{SF-1} \sum_{m=0}^{M-1} |w_m(q, i)|^2. \quad (\text{A} \cdot 15)$$



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