LETTER

On Received Signal Power Distribution of Wideband Signals in a Frequency-Selective Rayleigh Fading Channel

Fumiyuki ADACHI[†a)], ***Regular Member and*** **Akihito KATO**[†*], ***Student Member***

SUMMARY A mathematical expression for the received signal power in a severe frequency-selective fading channel is derived. Using the derived expression, the signal power distributions are obtained by Monte-Carlo simulation and compared with the Nakagami m-power distribution. It is found that the power distribution matches well with the Nakagami m-power distribution when the multipath channel has a uniform power delay profile.

key words: *signal power distribution, frequency-selective channel, Nakagami m-fading, mobile communication*

1. Introduction

A wideband mobile radio propagation channel consists of many distinct paths with different time delays. Wideband signals transmitted over such a channel undergo severe frequency-selective fading. Recently, multi-carrier code division multiple access (MC-CDMA) is under intensive study as a promising wireless access technique for high-speed data transmissions [1]. In MC-CDMA, the received signal power is the sum of powers of SF subcarriers, where SF represents the spreading factor. In direct sequence CDMA (DS-CDMA), rake combining is used to improve the transmission performance [2]. If rake combining is ideal, all powers of the received multipath signal components can be collected for data demodulation [3]. Hence, knowing the statistical properties of the received signal powers is of practical importance as it helps to assess the transmission performance, and also mathematically interesting. Objective of this paper is to find the approximate distributions of the received signal power in a severe frequency-selective fading channel.

2. Analysis

Let $h_T(\tau)$ and $h_C(t,\tau)$ be respectively the transmit filter impulse response and the propagation channel impulse response. The overall impulse response of the transmit filter and propagation channel is given by $h(t,\tau) = h_T(\tau)\otimes h_C(t,\tau)$, where $\otimes$ denotes the convolution operation. Note that $h_C(t,\tau)$ is time-variant while $h_T(\tau)$ is time-invariant. An alternative expression for the transmission channel is the overall transfer function taking into account the transmit filter and propagation channel, which is given by $H(t,f) = H_T(f)H_C(t,f)$, where $H_T(f)$ and $H_C(t,\, f)$ are respectively the Fourier transforms of $h_T(\tau)$ and $h_C(t,\tau)$, respectively.

The received signal $r(t)$ can be expressed using the equivalent low-pass representation as

$$r(t) = s(t)\otimes h(t,\tau) = \int_{-\infty}^{\infty} s(t-\tau)h(t,\tau)d\tau, \quad (1)$$

where $s(t)$ is the signal waveform before transmit filtering. We are interested in the instantaneous received signal power $Q(t)$ at time t. Slow fading is assumed such that the overall channel impulse response $h(t,\tau)$ stays almost constant over the power measurement time interval $[t-T/2,\, t+T/2]$. We obtain

$$\begin{aligned} Q(t) &= \frac{1}{T}\int_{t-T/2}^{t+T/2}\frac{1}{2}\left|\int_{-\infty}^{\infty} s(z-\tau)h(z,\tau)d\tau\right|^2 dz \\ &\approx \frac{1}{2T}\int_{t-T/2}^{t+T/2}\left|\int_{-\infty}^{\infty} s(z-\tau)h(t,\tau)d\tau\right|^2 dz \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{2T}\int_{t-T/2}^{t+T/2} s(z-\tau)s^*(z-\tau')dz\right) \\ &\quad \times h(t,\tau)h^*(t,\tau')d\tau d\tau' \\ &= S\int_{-\infty}^{\infty}|h(t,\tau)|^2 d\tau, \end{aligned} \quad (2)$$

where S is the power spectrum density of $s(t)$ and $(.)^*$ denotes the complex conjugate operation. In the above equation, transmitted wideband signals are assumed to be white noise-like and their autocorrelation functions are represented by the delta function, i.e.,

$$\frac{1}{2T}\int_{t-T/2}^{t+T/2} s(z-\tau)s^*(z-\tau')dz = S\delta(\tau-\tau'). \quad (3)$$

The above white noise assumption of $s(t)$ is valid for DS-CDMA and MC-CDMA signals. In DS-CDMA, the pseudo-noise (PN) spreading chip sequence (or the product of PN scramble sequence and orthogonal spreading chip sequence) is used and hence, the resultant DS-CDMA signal can be approximated as a random process. In MC-CDMA (the spreading sequence

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[†]The authors are with the Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University, Sendai-shi, 980-8579 Japan.

[*]Presently, with Sony Ericsson Mobile Communications Japan, Inc.

a) E-mail: adachi@ecei.tohoku.ac.jp

defined in the frequency-domain is used unlike DS-CDMA), the MC-CDMA signal is the sum of many data-modulated orthogonal subcarriers and its power spectrum is uniform (white).

As a consequence, the instantaneous received signal power normalized by its average can be expressed as

$$x(t) = \frac{\int_{-\infty}^{\infty} |h(t,\tau)|^2 d\tau}{\int_{-\infty}^{\infty} \overline{|h(t,\tau)|^2} d\tau}, \tag{4}$$

where $\bar{x}$ represents the time average operation. Assuming a discrete time delay model of the propagation channel, the channel impulse response may be expressed as

$$h_C(t,\tau) = \sum_{l=0}^{\infty} \xi_l(t)\delta(\tau - \tau_l) \tag{5}$$

with $\sum_{l=0}^{\infty} \overline{|\xi_l(t)|^2} = 1$, where $\xi_l(t)$ and τ_l respectively represent the path gain and time delay of the lth propagation path. Using Eq. (5), $h(t,\tau)$ can be expressed as

$$h(t,\tau) = \sum_{l=0}^{\infty} \xi_l(t) h_T(\tau - \tau_l). \tag{6}$$

Substituting Eq. (6) and $\sum_{l=0}^{\infty} \overline{|\xi_l(t)|^2} = 1$ into Eq. (4), we obtain

$$x(t) = \frac{\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \xi_l(t)\xi_m^*(t) \int_{-\infty}^{\infty} h_T(\tau-\tau_l)h_T^*(\tau-\tau_m)d\tau}{\int_{-\infty}^{\infty} |h_T(\tau)|^2 d\tau}. \tag{7}$$

Since

$$\begin{cases} \int_{-\infty}^{\infty} h_T(\tau-\tau_l)h_T^*(\tau-\tau_m)d\tau \\ \quad = \int_{-\infty}^{\infty} |H_T(f)|^2 \exp(j2\pi f(\tau_m - \tau_l))df \\ \int_{-\infty}^{\infty} |H_T(f)|^2 df = \int_{-\infty}^{\infty} |h_T(\tau)|^2 d\tau \end{cases}, \tag{8}$$

we have

$$x(t) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \xi_l(t)\xi_m^*(t) q(\tau_m - \tau_l), \tag{9}$$

where

$$q(\tau) = \frac{\int_{-\infty}^{\infty} |H_T(f)|^2 \exp(j2\pi f\tau)df}{\int_{-\infty}^{\infty} |H_T(f)|^2 df}. \tag{10}$$

3. Discussions

The frequency-selective fading channel can be characterized by the power delay profile $\Omega(\tau)$ which is defined as $\Omega(\tau) = \overline{|h_C(t,\tau)|^2} \Big/ \int_{-\infty}^{\infty} \overline{|h_C(t,\tau)|^2} d\tau$. We assume a propagation channel consisting of L Rayleigh-faded discrete paths with uniform power delay profile. In this case, $\Omega(\tau) = \sum_{l=0}^{L-1} \overline{|\xi_l(t)|^2}\delta(\tau-\tau_l)$ with $\sum_{l=0}^{L-1} \overline{|\xi_l(t)|^2} = 1$. Path gains $\{\xi_l(t);\ l = 0 \sim L-1\}$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian processes with the same variance of $1/L$ since $\overline{|\xi_l(t)|^2}$ is the same for all l. Also assumed is a transmit filter with rectangular-spectrum pulse shaping of bandwidth B, i.e., $H_T(f) = 1$ for $|f| \le B/2$ and 0 otherwise. In this case, we have $q(\tau) = \sin(\pi B\tau)/(\pi B\tau)$. Two extreme cases are considered: strong frequency-selective case where $\max|\tau_m - \tau_l| \gg 1/B$ and weak frequency-selective case where $\max|\tau_m - \tau_l| \ll 1/B$ for all m and l but $m \neq l$.

3.1 Strong Frequency-Selective Case

When $\max|\tau_m - \tau_l| \gg 1/B$, $q(\tau) \approx 1(0)$ for $\tau = 0$ (otherwise) and thus, the overall impulse response to the transmit pulse do not overlap at all. Substitution of $q(\tau_m - \tau_l) = 1\ (0)$ for $\tau_m = \tau_l$ (otherwise) into Eq. (9) gives

$$x(t) = \sum_{l=0}^{L-1} |\xi_l(t)|^2 \tag{11}$$

and the received signal power follows the chi-square distribution with $2L$ degrees of freedom [4]:

$$p(x) = \frac{L^L x^{L-1}}{(L-1)!}\exp(-Lx), \tag{12}$$

which is a special case of the power distribution of the Nakagami-m faded signal [5]:

$$p(x) = \frac{m^m x^{m-1}}{\Gamma(m)}\exp(-mx), \tag{13}$$

where $\Gamma(m)$ denotes the gamma function and m is

$$m = \frac{(\overline{x})^2}{\overline{x^2} - (\overline{x})^2} \ge \frac{1}{2}. \tag{14}$$

Eq. (13) with $m = L$ is identical to Eq. (12). The Nakagami m-distribution represents the distribution of the signal amplitude $R = \sqrt{x}$. In this paper, the power distribution of Eq. (13) is called the Nakagami m-power distribution for convenience. The Nakagami m-power distribution covers a wide range of distributions including exponential and the chi-square distribution.

3.2 Weak Frequency-Selective Case

When $\max|\tau_m - \tau_l| \ll 1/B$, $q(\tau) \approx 1$ over the range of τ of interest and thus, the overall impulse response to the transmit pulse totally overlap. Substitution of $q(\tau_m - \tau_l) = 1$ into Eq. (9) gives

$$x(t) = \left|\sum_{l=0}^{L-1} \xi_l(t)\right|^2. \tag{15}$$

Remember that the sum of complex Gaussian processes also becomes another complex Gaussian process. $\xi(t) = \sum_{l=0}^{L-1} \xi_l(t)$ becomes a zero-mean complex Gaussian process with unity variance. This means that the propagation channel becomes frequency non-selective as often encountered in narrowband mobile communications and the rms envelope of the received signal follows the well-known Rayleigh distribution. Hence, the received signal power follows the exponential distribution:

$$p(x) = \exp(-x), \tag{16}$$

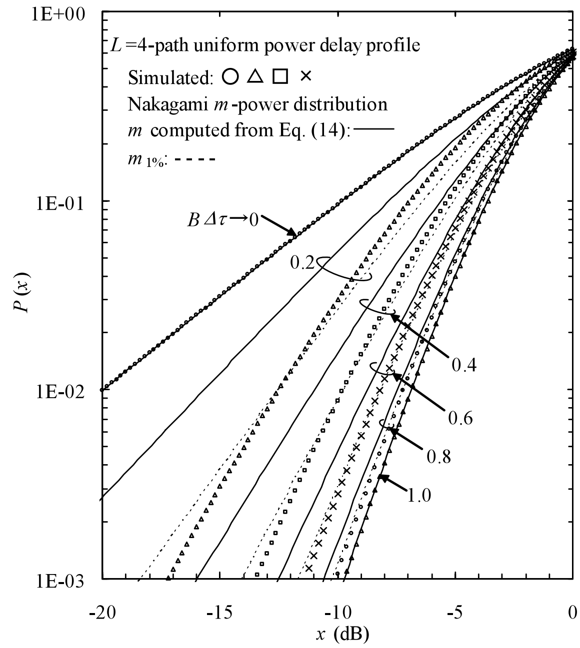
which is obtained by letting m=1 in Eq. (13).

3.3 General Case

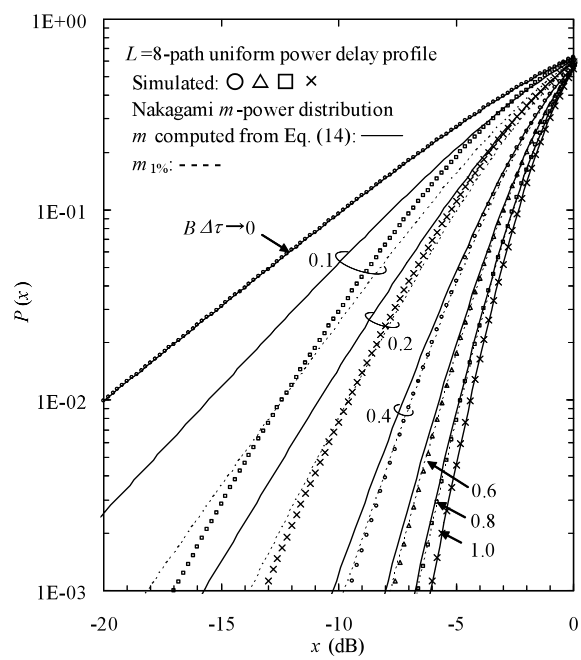
The power distribution may be between the Nakagami m-power distribution with m=1 and that with $m = L$. In the following, we discuss how the frequency-selectivity affects the received signal power statistics. The L-path uniform power delay profile with the time delays $\tau_l = l\Delta\tau$, $l = 0 \sim L-1$, is assumed. In this case, Eq. (9) becomes

$$x(t) = \sum_{l=0}^{L-1}\sum_{m=0}^{L-1} \xi_l(t)\xi_m^*(t)\left\{\frac{\sin(\pi(m-l)B\Delta\tau)}{\pi(m-l)B\Delta\tau}\right\}. \tag{17}$$

Assuming L=4 and 8, the cumulative distribution function $P(x) = \int_0^x p(x)dx$ is found by the Monte-Carlo simulation. In the Monte-Carlo simulation, time dependency of $\xi_l(t)$ and $x(t)$ is dropped. The set of $\{\xi_l\}$ is generated to compute x using Eq. (9). This is repeated sufficient number of times to obtain $P(x)$. The results are plotted in Fig. 1 for various values of $B\Delta\tau$ together with the Nakagami m-power distribution curves with the value of m determined using Eq. (14). The frequency-selectivity of the channel is represented by the parameter $B\Delta\tau$. It can be seen that the simulated power distribution matches well with the Nakagami m-power distribution for the strong frequency-selective case, i.e., $B\Delta\tau > 0.8$ (0.4) for L=4 (8). For the case of weak frequency-selectivity, i.e., $0 < B\Delta\tau < 0.8$ (0.4) for L=4 (8), the simulated power distribution deviates from the Nakagami m-power distribution having the value of m computed from Eq. (14). We found the value of m, denoted by $m_{1\%}$, that makes the distribution curve to fit the simulated one at a cumulative probability of 1%. The fitted Nakagami m-power distribution curves are plotted in Fig. 1 as dotted lines. It is found that if $B\Delta\tau < 0.8$ (0.4) for L=4 (8), $m_{1\%}$ can be used to better approximate the power distribution. The dependency of m on $B\Delta\tau$ is calculated using Eq. (14) and is plotted in Fig. 2 as a solid curve. Also



(a) $L = 4$



(b) $L = 8$

Fig. 1 Simulated cumulative power distributions and Nakagami-m power distributions for uniform power delay profile model.

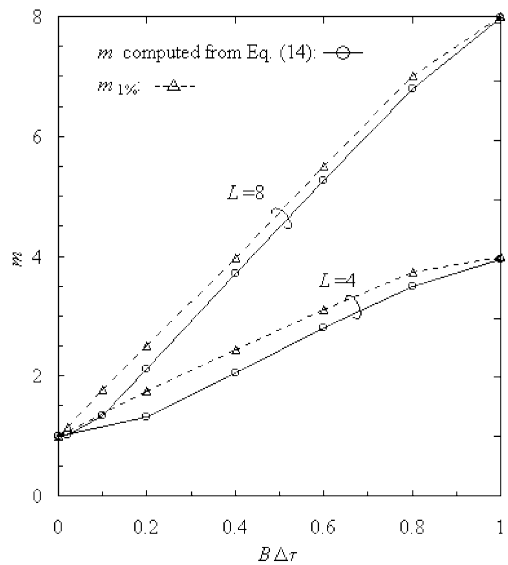


Fig. 2 Dependency of m on $B\Delta\tau$ for uniform power delay profile model.

plotted is the value of $m_{1\%}$.

4. Conclusion

It was found that the received signal power distribution in a frequency-selective Rayleigh channel having a uniform power delay profile well matches the Nakagami m-power distribution for the strong frequency-selective case. The value of m depends on the number of resolvable propagation paths and can be computed using Eq. (14). For the case of weak frequency-selectivity, however, the power distribution deviates from the Nakagami m-power distribution, but can still be well approximated by the Nakagami m-power distribution using $m_{1\%}$. Interesting future studies include finding the received signal power distribution when the multipath channel has a non-uniform power delay profile. Obviously, in this case, the power distribution is different from the Nakagami m-power distribution.

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