PAPER

Impact of Shadowing Correlation on Spectrum Efficiency of a Power Controlled Cellular System

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SUMMARY Independent shadowing losses are often assumed for computing the frequency reuse distance of cellular mobile communication systems. However, shadowing losses may be partially correlated since the obstacles surrounding a mobile station block similarly the desired signal and interfering signals. We investigate, by computer simulation, how the shadowing correlation impacts the frequency reuse distance of a power controlled cellular system. It is pointed out that the shadowing correlation impacts the frequency reuse distance differently for the uplink and downlink.
key words: *cellular system, frequency reuse distance, transmit power control, outage, shadowing correlation*

1. Introduction

In cellular mobile communications systems, the same carrier frequency is reused in spatially separated cells [1]. The frequency reuse distance is an important design parameter. For a shorter reuse distance, the limited frequency band can be more efficiently utilized. When determining the frequency reuse distance, often it is assumed that the shadowing losses of desired and interfering signals vary independently. However, shadowing losses seen at a mobile station may be partially correlated since the obstacles surrounding a mobile station with low antenna height block similarly the desired signal and interfering signals [2].

Fast transmit power control (TPC) is a well known technique to increase the capacity of cellular systems [3]–[5]. The objective of this paper is to investigate how the shadowing correlation impacts the frequency reuse distance of a power controlled cellular system using frequency division multiple access (FDMA). Fast TPC based on signal-to-noise power ratio (SNR) [5] is assumed. The remainder of this paper is organized as follows. Section 2 introduces the interference and shadowing correlation models. The procedure to determine the frequency reuse distance is presented in Sect. 3. Section 4 discusses the computer simulation results. Some conclusions are drawn in Sect. 5.

2. Models of Interference and Shadowing Correlation

2.1 Interference Model

Only the six nearest co-channel cells are considered since they give predominant interference to the desired cell [5], [6]. Figure 1 illustrates the geographical relationship between the cell of interest (j=0) and the six co-channel cells (j=1–6) with a cell radius R and a distance D between adjacent cells. Hexagonal cell layout is assumed. A base station is located at the center of each cell.

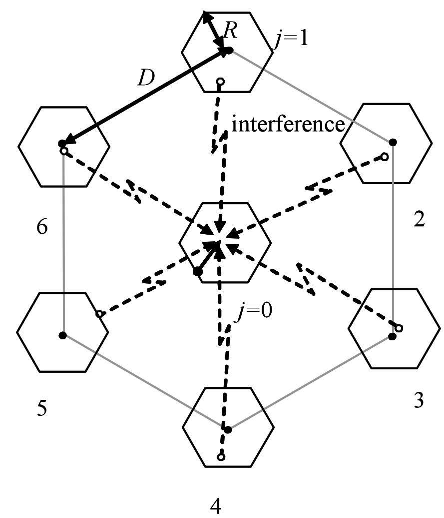

Fig. 1 Interference model (uplink).

2.2 Shadowing Correlation Model

The mobile radio propagation channel is modeled by the distance dependent path loss, the log-normally distributed shadowing loss and the multipath Rayleigh fading [1]. In this paper, frequency non-selective fading is assumed as in [5]. M-antenna diversity using maximal ratio combining (MRC) with ideal coherent detection is assumed at both base stations and mobile stations.

(a) Uplink

The instantaneous power $P_{j'\to j}(m)$ of the signal transmitted from the j'-th cell mobile station (j'=0–6) and received at the m-th antenna of the j-th cell base station can be represented as [5]

$$P_{j'\to j}(m) = A \cdot P_{T,j'} r_{j'\to j}^{-\alpha} 10^{-\eta_{j'\to j}/10} \left|\xi_{j'\to j}(m)\right|^2, \qquad (1)$$

where A is a constant, $P_{T,j'}$ is the transmit power of the j'-th cell mobile station, α is the path loss exponent, $r_{j'\to j}$ and $\eta_{j'\to j}$ are the distance and shadowing loss between the j'-th cell mobile station and the j-th cell base station, respectively. $\eta_{j'\to j}$ is a zero-mean Gaussian variable with a

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standard deviation σ. $\xi_{j'\to j}(m)$ is the complex channel gain between the j'-th cell mobile station and the m-th antenna of the j-th cell base station and is a zero-mean complex Gaussian variable with unity variance. Figure 2(a) illustrates the shadowing correlation relationship on the uplink. Shadowing correlation is produced due to the fact that the different channels are affected by the same obstacles. In this paper, we assume that the antenna height of a base station is sufficiently high so that there are no obstacles surrounding the base station and hence, shadowing is only due to the obstacles surrounding a mobile station with low antenna height. Therefore, we assume that the shadowing losses $\eta_{j'\to 0}$ and $\eta_{j'\to j'}$ are correlated and have the same correlation ρ for all j'. Likewise, we assume that the shadowing losses $\eta_{j'\to 0}$ and $\eta_{0\to 0}$ are uncorrelated.

(b) Downlink
The instantaneous power $P_{j'\to j}(m)$ of the signal transmitted from the j'-th cell base station and received at the m-th antenna of the j-th cell mobile station is represented as [5]

$$P_{j'\to j}(m) = A \cdot P_{T,j'} r_{j'\to j}^{-\alpha} 10^{-\eta_{j'\to j}/10} \left|\xi_{j'\to j}(m)\right|^2, \qquad (2)$$

where $P_{T,j'}$ is the transmit power of the j'-th cell base station, $r_{j'\to j}$ and $\eta_{j'\to j}$ are the distance and shadowing loss between the j'-th cell base station and the j-th cell mobile station, respectively. $\xi_{j'\to j}(m)$ is the complex channel gain between the j'-th cell base station and the m-th antenna of the j-th cell mobile station and is a zero-mean complex Gaussian variable with unity variance. Figure 2(b) illustrates the shadowing correlation relationship on the downlink. Similar to the uplink case, we assume that the shadowing losses $\eta_{j'\to 0}$ and $\eta_{0\to 0}$ are correlated and have the same correlation ρ for all j' and that the shadowing losses $\eta_{j'\to 0}$ and $\eta_{j'\to j}$ are uncorrelated.

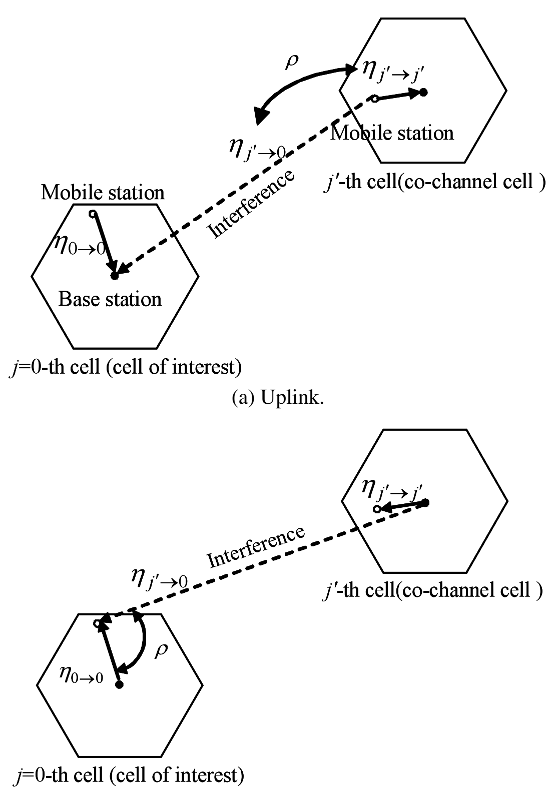


Fig. 2 Shadowing correlation relationship.

3. Determining Frequency Reuse Distance

3.1 Frequency Reuse Distance and Spectrum Efficiency

Communication quality, often measured as the bit error rate (BER), is a function of the received signal-to-interference plus noise power ratio (SINR). The outage probability is defined as the probability that the received SINR falls below the required SINR [6]. As the distance between the co-channel base stations reduces, the interference power from other cells increases and the outage probability also increases. The minimum distance between the co-channel base stations for the given allowable outage probability is denoted by D.

When the total number of available channels is G and the cluster size is F, the number C of channels per cell is given by $C = G/F$. The spectrum efficiency μ is defined as μ=C/G=$1/F$. Assuming the hexagonal cell layout, F is related to the cell radius R and distance D by [1]

$$F = \frac{1}{3}\left(\frac{D}{R}\right)^2, \qquad (3)$$

where $F = i^2 + j^2 + ij$, with i and j as positive integers, can take only limited integer numbers. The maximum reuse distance D/R for the given value of F is shown in Table 1.

3.2 Required SINR

SNR-based slow and fast TPC are assumed as in [5]. In the fast TPC case, the transmit power is controlled so that the instantaneous received SNR is kept constant. On the other hand, in the case of slow TPC, the transmit power is controlled so that the average received SNR is kept constant but the instantaneous received SNR variations due to fading remain intact. We use the target SNR value $(S/N)_{target}$ given by [5]

$$\left(\frac{S}{N}\right)_{target} = \chi \cdot \gamma_{req}, \qquad (4)$$

where N is the noise power and χ represents the allowable interference rise factor defined as the interference plus background noise-to-background noise power ratio and γ_{req} is the required SINR for achieving the required BER.

The required SINR is derived as follows. In this paper,

Table 1 Maximum reuse distance D/R.

F	1	3	4	7	9	12	13	16
D/R	1.73	3.00	3.46	4.58	5.20	6.00	6.24	6.93

Table 2 Required SINRs for average BERs of $P_b = 10^{-2}$ and 10^{-3}.

Required average BER		P_b=10^{-2}	10^{-3}
Slow TPC and no TPC	M=1	Λ_0=16.9 dB	27.0 dB
	2	11.5	17.1
	3	9.9	14.3
	4	9.2	13.1
Fast TPC		λ_0=7.3 dB	9.8 dB

Table 3 Simulation parameters.

Modulation	Coherent QPSK
TPC	SNR based ideal TPC
User distribution	Uniform
Path loss exponent	α=3.5
Antenna diversity	M=2-branch MRC
Allowable outage probability	Q_0=0.1

the sum of the background noise and the interference from 6 co-channel cells is approximated as a Gaussian variable [5]. Assuming coherent quadrature phase shift keying (QPSK) data modulation, the average BER $P_{b,\text{fast TPC}}$ for fast TPC is given by [7]

$$P_{b,\text{fast TPC}}(\lambda) = \frac{1}{2}\text{erfc}\sqrt{\lambda/2}, \tag{5}$$

where λ is the instantaneous received SINR and $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt$. In the case of slow TPC, the instantaneous power variations due to fading remain intact as in the case of no TPC. The average BER $P_{b,\text{slow TPC}}$ for M-antenna diversity reception using MRC is given by [7]

$$P_{b,\text{slow TPC}}(\Lambda, M) = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}}\sum_{k=0}^{M-1}\binom{2k}{k}\left(\frac{1-\mu^2}{4-2\mu^2}\right)^k\right], \tag{6}$$

where

$$\begin{cases} \mu = \sqrt{\dfrac{\Lambda/M}{1+\Lambda/M}} \\ \dbinom{2k}{k} = \dfrac{(2k)!}{k!\,k!} \end{cases}, \tag{7}$$

with Λ being the average received SINR.

The required SINR λ_0 with fast TPC and the required average SINR Λ_0 with slow and no TPC are found by using Eqs. (5) and (6), respectively, and are shown in Table 2 for the required BERs of $P_b = 10^{-2}$ and 10^{-3}.

4. Computer Simulation

Table 3 shows the simulation parameters. It is assumed that the shadowing loss and the location of each mobile station remain the same during communication. Mobile stations are assumed to be uniformly distributed over the entire area comprising of 7 cells.

4.1 Simulation Procedure

Figure 3 shows the flow chart for computer simulation. Before starting communication, the base station with which the communication is to be established is selected for each mobile station. Average received signal powers from the base stations surrounding the mobile station are measured and the base station providing the largest received signal power is selected. If the selected base station is one of the 7 co-channel cells, this mobile station is used in the simulation. This process is repeated until each of the 7 cells accommodates one user. Then the average received SINR is measured.

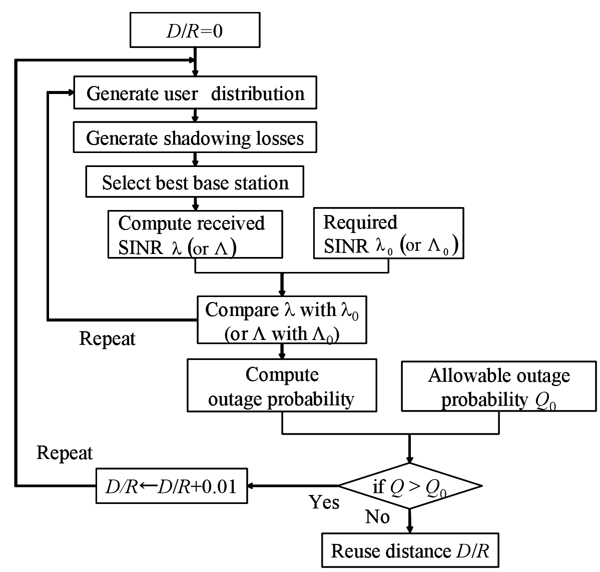


Fig. 3 Flow chart for computer simulation.

The average received SINR for the uplink is computed as follows. In the case of slow and no TPC, the average received SINR Λ is given by

$$\Lambda = \frac{\overline{S}}{N+\overline{I}} = \frac{(\bar{P}_{T,0}/N)r_{0\to0}^{-\alpha}10^{-\eta_{0\to0}/10}}{1+\sum_{j=1}^{6}(\bar{P}_{T,j}/N)r_{j\to0}^{-\alpha}10^{-\eta_{j\to0}/10}}, \tag{8}$$

where $\bar{S}$ and $\bar{I}$ are the average desired signal power and the average interfering signal power, respectively, and $\bar{P}_{T,j}$ is the average transmit power given by

$$\frac{P_{T,j}}{N} = \begin{cases} \chi R^{\alpha}\dfrac{\Lambda_0(M)}{M} & \text{without TPC} \\ \dfrac{\chi\Lambda_0(M)}{Mr_{j\to j}^{-\alpha}10^{-\eta_{j\to j}/10}} & \text{with slow TPC.} \end{cases} \tag{9}$$

In the case of fast TPC, the instantaneous received SINR λ is given by

$$\lambda = \frac{S}{N+\bar{I}} = \frac{(P_{T,0}/N) r_{0\to 0}^{-\alpha} 10^{-\eta_{0\to 0}/10} \sum_{m=0}^{M-1} |\xi_{0\to 0}(m)|^2}{1 + \sum_{j=1}^{6} (\bar{P}_{T,j}/N) r_{j\to 0}^{-\alpha} 10^{-\eta_{j\to 0}/10}}, \tag{10}$$

where $P_{T,0}$ and $\bar{P}_{T,j}$ are given by

$$\begin{cases} P_{T,0}/N = \dfrac{\chi\lambda_0}{r_{0\to 0}^{-\alpha} 10^{-\eta_{0\to 0}/10} \sum_{m=0}^{M-1} |\xi_{0\to 0}(m)|^2} \\ \overline{P}_{T,j}/N = \dfrac{\chi\lambda_0}{(M-1) r_{j\to j}^{-\alpha} 10^{-\eta_{j\to j}/10}}. \end{cases} \tag{11}$$

Similarly, the average SINR Λ and the instantaneous SINR λ for the downlink can be obtained.

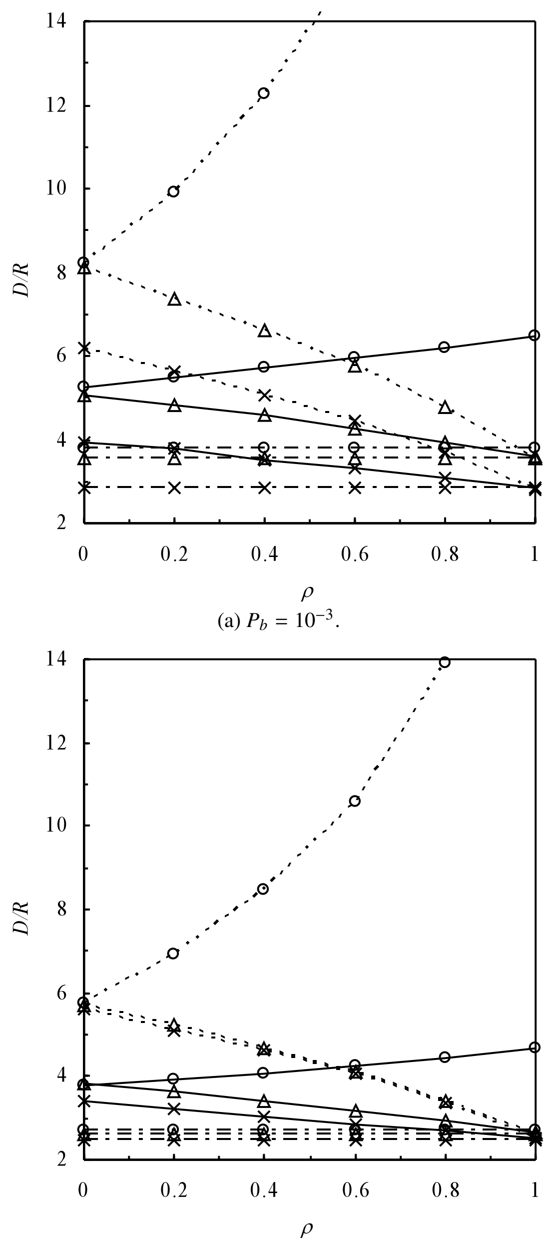
The received SINR is compared with the required SINR to decide that the communication quality is in outage or not. This trial is repeated 1,000,000 times to compute the outage probability Q = Prob $[\Lambda < \Lambda_0]$ or Prob $[\lambda < \lambda_0]$. If $Q > Q_0$, D/R is increased by 0.01 until $Q = Q_0$ to find the value of D/R for the given Q_0=0.1.

4.2 Uplink

Figure 4 plots the normalized uplink frequency reuse distance D/R as a function of shadowing correlation ρ with the shadowing standard deviation σ as a parameter when χ=10 dB, Q_0=0.1 and M=2. For comparison, the normalized reuse distance D/R without shadowing loss (σ=0 dB) is also plotted. As σ increases, the probability of large interference power increases and hence, D/R becomes larger, for the given value of shadowing correlation ρ. It is interesting to note that as ρ becomes larger, D/R becomes smaller when TPC is applied, while D/R becomes larger without TPC. The reason for this is given below.

(1) Without TPC: as ρ increases, the site diversity effect reduces when selecting the best base station (based on the minimum propagation loss, i.e., maximum average received signal power) and hence, the probability of having a large shadowing loss increases. This leads to a decreased received signal power from the 0-th cell mobile station. As a consequence, the probability of SINR being below the required value increases, resulting in an increased D/R.

(2) With TPC: the transmit powers from the 0-th cell and the j'-th cell mobile stations are controlled by their base stations, respectively, so that the received signal powers at their base stations are kept at the TPC target. As ρ increases, both $\eta_{j'\to j'}$ and $\eta_{j'\to 0}$ tend to vary similarly and hence, the variations in the interference power from the j'-th cell mobile station can be reduced. Therefore, the probability of the received SINR becoming below the required value reduces, resulting in a reduced D/R.



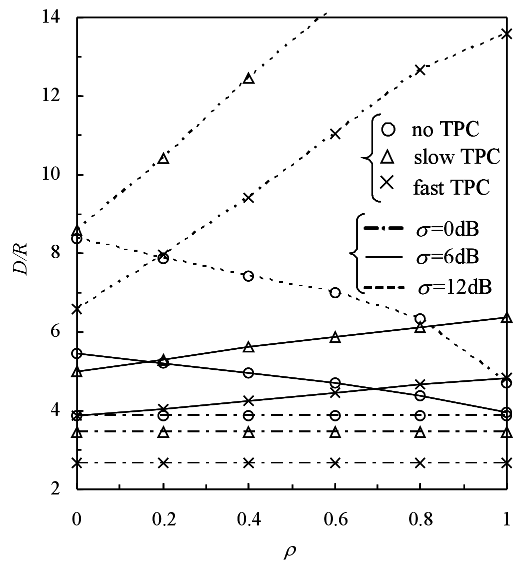
(a) $P_b = 10^{-3}$.

(b) $P_b = 10^{-2}$.

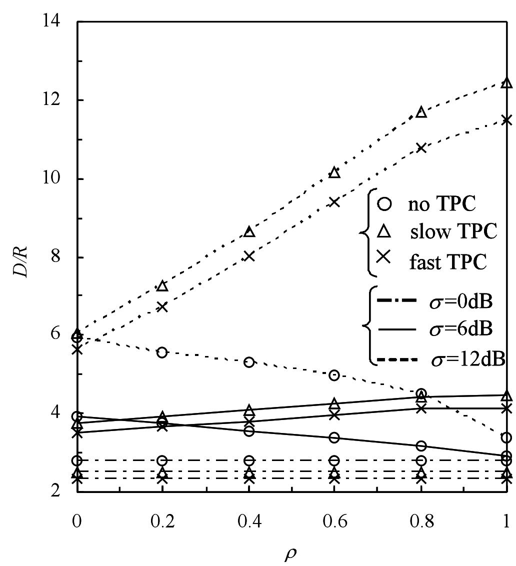
Fig. 4 Normalized frequency reuse distance as a function of shadowing correlation ρ for uplink (χ=10 dB, Q_0=0.1 and M=2).

4.3 Downlink

Figure 5 plots the normalized downlink frequency reuse distance as a function of shadowing correlation ρ with the shadowing standard deviation σ as a parameter when χ=10 dB, Q_0=0.1 and M=2. Similar to the uplink case, as σ increases, D/R becomes larger for the given value of ρ. Interestingly, opposite result from the uplink case is obtained for the ef-



(a) $P_b = 10^{-3}$.



(b) $P_b = 10^{-2}$.

Fig. 5 Normalized frequency reuse distance as a function of shadowing correlation ρ for downlink (χ=10 dB, Q_0=0.1 and M=2).

fect of ρ; as ρ becomes larger, D/R becomes larger (smaller) with TPC (without TPC). The reason for this is as follows:

(1) Without TPC: as ρ increases, both $\eta_{j'\to 0}$ and $\eta_{0\to 0}$ tend to vary similarly. Hence, both the desired and interference signal powers tend to vary similarly, reducing the variations in the SINR. Therefore, the probability of the received SINR being below the required value reduces, resulting in a reduced D/R.
(2) With TPC: the transmit powers from the 0-th cell and the j'-th cell base stations are controlled so that the received signal powers at their mobile stations are kept at the TPC target. However, variations due to shadowing in the interference power from the j'-th cell base station remain intact since $\eta_{j'\to j'}$ and $\eta_{j'\to 0}$ are independent. Note that as ρ increases, the site diversity effect reduces when selecting the best base station and hence the probability of having large shadowing losses increases. This leads to an increased transmit power from the j'-th cell base station when TPC is used. Hence, the 0-th cell mobile station experiences larger interference power from the j'-th cell base station. As a consequence, as ρ increases the probability of SINR being below the required value increases, resulting in an increased D/R.

5. Conclusions

This paper investigated, by computer simulation, the impact of the shadowing correlation on the frequency reuse distance in a power controlled FDMA cellular system. It was found that the impact of the shadowing correlation on the frequency reuse distance D/R is different for uplink and downlink:

(1) In the case of uplink, as the shadowing correlation increases, the D/R becomes smaller, when TPC is used, while it becomes larger when TPC is not used.
(2) In the case of downlink, the opposite result is obtained. As the shadowing correlation increases, the D/R becomes smaller when TPC is not used while it becomes larger when TPC is used.

In this paper, the shadowing correlation is assumed to be independent of the geographical locations of the mobile stations. The evaluation with more practical shadowing correlation model is left as an interesting future study. Extension to CDMA cellular systems in frequency-selective fading channels is also an interesting future study.

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