IEICE TRANS. COMMUN., VOL.E88–B, NO.1 JANUARY 2005

PAPER Special Section on Multi-carrier Signal Processing Techniques for Next Generation Mobile Communications

# Diversity-Coding-Orthogonality Trade-off for Coded MC-CDMA with High Level Modulation

**Deepshikha GARG**[†a)] ***and*** **Fumiyuki ADACHI**[†], ***Members***

**SUMMARY** In MC-CDMA, the data rate can be increased by reducing the spreading factor *SF* or by allowing multicode transmission. In this paper, we examine by computer simulations which gives a better bit error rate (BER) performance—lower *SF* or multicode operation—when high level modulation is used in addition to error control coding. For a coded system in a frequency selective channel, there is a trade-off among frequency diversity gain due to spreading, improved coding gain due to better frequency interleaving effect and orthogonality distortion. It is found that for QPSK, the performance of OFDM (MC-CDMA with $SF$=1) is almost the same as that of a fully spread MC-CDMA system. However, for 16QAM and 64QAM, the BER performance is better for lower *SF* unlike the uncoded system, wherein higher *SF* gives a better BER.
***key words:*** *MC-CDMA, high-level modulation, spreading factor, frequency diversity, channel coding, orthogonality*

## 1. Introduction

Recently, the combination of multicarrier (MC) modulation based on orthogonal frequency division multiplexing (OFDM) [1] and code division multiple access (CDMA), called MC-CDMA [1]–[4] or sometimes OFDM-CDMA [5], has gained a lot of attention because of its ability to allow high data rate transmission in a harsh mobile environment and has emerged as the most promising candidate for the next generation mobile communications systems [6]. In MC-CDMA, each user's data-modulated symbol to be transmitted is spread over a number of subcarriers using an orthogonal spreading code defined in the frequency-domain. Since the received signal suffers from frequency selective multipath fading, the orthogonality among different codes is partially lost, producing a large inter-code interference (ICI). There are several frequency-domain equalization (FDE) techniques to improve the bit error rate (BER) performance: zero-forcing (ZF), maximal ratio (MR) and minimum mean square error (MMSE) equalizations. It is shown [3]–[5], [7], [8] that the orthogonality property can be partially restored while achieving the frequency diversity effect with the MMSE-FDE technique and hence, a better BER performance can be achieved than using ZF and MR equalizations.

The next generation mobile communications systems will be characterized by very high speed data transmissions. For high speed data transmissions, high level modulation, like 16 quadrature amplitude modulation (QAM) and 64QAM becomes inevitable. The spreading factor *SF* can be lowered or multicode transmission, with multiple codes assigned to a user, can also be applied to increase the data rate. OFDM can be viewed as a special case of MC-CDMA with $SF$=1. With higher *SF*, the orthogonality distortion is severer in a frequency selective channel. But at the same time, the frequency diversity effect is also higher. Various studies have been done on the trade-off analysis among coding gain, frequency diversity and orthogonality distortion [9]–[11]. However, these papers have not treated the effect of modulation scheme in the presence of channel coding. In [9], the BER performance of multicode MC-CDMA and OFDM is evaluated for binary phase shift keying (BPSK) data modulation and concluded that multicode MC-CDMA provides a better BER for the same data rate and bandwidth. In [10], the trade-off between channel coding and spreading in MC-CDMA is evaluated when convolutional coding and quaternary phase shift keying (QPSK) data modulation is used. It is concluded [10] that for a full load system, MC-CDMA provides a better BER performance for high code rates. However, in [9] and [10], higher level modulation has not been considered. In [11], the trade-off between frequency diversity and ICI due to non-orthogonality is analyzed for higher level modulation, but channel coding has not been considered.

In all types of data transmission systems, some form of error control is needed to improve the transmission performance. Turbo coding [12]–[16] has been found to provide strong error correction capabilities. With higher level modulation, the Euclidean distance between the symbols are smaller and the orthogonality distortion is a major problem. However, to the best of authors' knowledge, it is not known as to whether it is better to reduce the spreading factor or use a high spreading factor and allow multicode transmission when high level modulation is applied in addition to channel coding.

The purpose of this paper is to find out the BER performance dependence of MC-CDMA on *SF* for a system with channel coding and see how the trend is different from when channel coding is not applied. We evaluate by computer simulations, the BER performance of MC-CDMA with high level modulation and discuss in detail the trade-off among frequency diversity gain due to spreading, additional coding gain due to better frequency interleaving effect and orthogonality distortion. For a fair comparison, we keep the transmission rate fixed as that attainable with an OFDM system.

Manuscript received May 28, 2004.
Manuscript revised August 9, 2004.
[†]The authors are with the Department of Electrical and Communication Engineering, Tohoku University, Sendai-shi, 980-8579 Japan.
a) E-mail: deep@mobile.ecei.tohoku.ac.jp

Turbo decoding principle is based on iterative algorithm that requires soft decision values as input. The soft value generation for higher level modulation like 16QAM and 64QAM is dealt in [15], [16]. In [15], a decision threshold is used to generate the soft value, whereas in [16], log-likelihood ratio (LLR) approximation is presented for the generation of soft values. In this paper, we present the soft decision value generation for MC-CDMA with MMSE-FDE using the LLR approximation.

The remainder of the paper is organized as follows. Section 2 presents the transmission system model and shows how the soft decision values are generated for turbo decoding. The computer simulation results are presented and discussed in Sect. 3. Section 4 concludes the paper.

## 2. Transmission System Model

The transmission system model is shown in Fig. 1. The information sequence of length $K$ is turbo coded with a coding rate $R$ and modulated as QPSK, 16QAM or 64QAM symbol sequence. Let $d(n)$ be the $n$th modulated symbol, with symbol length $T$, in the sequence. We consider MC-CDMA having $N_c$ orthogonal subcarriers. The data is spread using the frequency-domain orthogonal spreading code with spreading factor $SF$. Throughout the paper, $T_c$-spaced discrete-time representation of the MC-CDMA signal is used, where $T_c$ is the sampling interval. Without loss of generality, we consider the time interval of one signaling period, i.e., $0 \le t < N$ with $N = N_c + N_g$, where $N_c$ and $N_g$ are respectively the window size of fast Fourier transform (FFT) (or the number of subcarriers) and the guard interval (GI).

The modulated symbol sequence is serial-to-parallel (S/P) converted into $C$ streams $\{d_c(n);\ c = 0 \sim C-1\}$. For each parallel stream, the signal is again S/P converted to $N_c/SF$ streams; each symbol is then copied $SF$ times and spread by multiplying with an orthogonal code $\{c_{oc,c}(k),\ c = 0 \sim C-1,\ k = 0 \sim SF-1\}$ with spreading factor $SF$. The $C$ different streams are then added ($C$ is the code multiplexing order) and further multiplied by a long scramble sequence $\{c_{scr}(k)\}$. The low-pass equivalent of the code multiplexed signal to be transmitted on the $k$th subcarrier can be written as

$$x(k) = \sqrt{\frac{2P}{SF}} \sum_{c=0}^{C-1} c_{oc,c}(k \bmod SF) c_{scr}(k) d_c\left(\left\lfloor \frac{k}{SF} \right\rfloor\right), \tag{1}$$

where $P$ is the transmit power per code and $\lfloor a \rfloor$ denotes the largest integer smaller than or equal to $a$. $N_c$-point inverse FFT (IFFT) is applied to the sequence $\{x(k);\ k = 0 \sim N_c-1\}$ to generate the MC-CDMA signal $\{s(t);\ t = 0 \sim N_c-1\}$ in time-domain:

$$s(t) = \sum_{k=0}^{N_c-1} x(k) \exp\left(j2\pi k \frac{t}{N_c}\right), \tag{2}$$

where $t$ represents the sample position within the signaling interval $0 \le t < N_c$. After insertion of the $N_g$-sample GI, the resultant MC-CDMA signal $\{s(t); t = -N_g \sim N_c-1\}$ is transmitted over a propagation channel.

A sample-spaced time delay model for the propagation channel is assumed. $M$-branch antenna diversity reception is considered. Assuming $L$ independent propagation paths with distinct time delays $\{\tau_l;\ l = 0 \sim L-1\}$, the impulse response $\xi_m(\tau)$ of the multipath channel experienced by the $m$th antenna, $m = 0 \sim M-1$, may be expressed as

$$\xi_m(\tau) = \sum_{l=0}^{L-1} \xi_{m,l} \delta(\tau - \tau_l) \tag{3}$$

with $\sum_{l=0}^{L-1} E[|\xi_{m,l}|^2] = 1$, where $\delta(t)$ is the delta function and E[.] denotes ensemble average operation. Time dependency of the channel has been dropped for simplicity.

At the receiver, ideal sampling timing is assumed. The MC-CDMA signal received on the $m$th antenna is sampled to obtain $\{r_m(t); t = -N_g \sim N_c-1\}$, which is expressed as

$$r_m(t) = \sum_{l=0}^{L-1} \xi_{m,l} s(t - \tau_l) + \eta_m(t) \tag{4}$$

for $t = -N_g \sim N_c-1$, where $\eta_m(t)$ represents the additive white Gaussian noise (AWGN) process with the single sided power spectrum density $N_0$. The $N_g$-sample GI is removed and the $N_c$-point FFT is applied to decompose the received MC-CDMA signal into the $N_c$ subcarrier components $\{R_m(k); k = 0 \sim N_c-1\}$:

$$R_m(k) = \frac{1}{N_c} \sum_{t=0}^{N_c-1} r_m(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \tag{5}$$

for $m$=0 $\sim M-1$. If the channel gain at the $k$th subcarrier on the $m$th antenna is $H_m(k)$, the $k$th subcarrier component

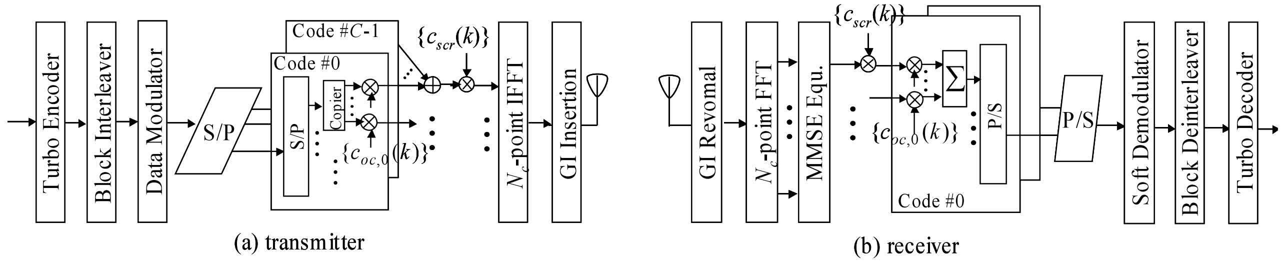



**Fig. 1** Transmission system model.

$R_m(k)$ received on the $m$th antenna can be written as

$$R_m(k) = H_m(k)x(k) + \Pi_m(k), \quad (6)$$

where $\{H_m(k); k = 0 \sim N_c - 1\}$ and $\{\Pi_m(k); k = 0 \sim N_c - 1\}$ are respectively the Fourier transforms of the channel impulse response $\xi_m(\tau)$ and the AWGN process $\eta_m(t)$. They are given by

$$\begin{cases} H_m(k) = \sum_{l=0}^{L-1} \xi_{m,l} \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right) \\ \Pi_m(k) = \frac{1}{N_c} \sum_{t=0}^{N_c-1} \eta_m(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \end{cases}. \quad (7)$$

$\{H_m(k); m = 0 \sim M - 1\}$ on the $k$th subcarrier are the independent and identically distributed (iid) complex random variables with zero mean and unit variance.

MMSE-FDE and antenna diversity combining is performed together by multiplying the received signal in Eq. (6) by the equalization weight $w_m(k)$ [8]

$$w_m(k) = \frac{H_m^*(k)}{\sum_{m=0}^{M-1} |H_m(k)|^2 + \left[\frac{C}{SF}\frac{E_s}{N_0}\right]^{-1}}, \quad (8)$$

where $E_s$ is the symbol energy with $E_s{=}PT_cN_c$. The $k$th subcarrier component obtained after MMSE-FDE is

$$\tilde{R}(k) = \tilde{H}(k)x(k) + \tilde{\Pi}(k) \quad (9)$$

for $k = 0 \sim N_c - 1$, where

$$\begin{cases} \tilde{H}(k) = \sum_{m=0}^{M-1} H_m(k)w_m(k) \\ \tilde{\Pi}(k) = \sum_{m=0}^{M-1} \Pi_m(k)w_m(k) \end{cases}. \quad (10)$$

Frequency-domain despreading is applied to obtain

$$\begin{aligned} \hat{d}_c(n) &= \frac{1}{SF} \sum_{k=nSF}^{(n+1)SF-1} \tilde{R}(k)\{c_{oc,c}(k \bmod SF)c_{scr}(k)\}^* \\ &= \sqrt{\frac{2P}{SF}} \left(\frac{1}{SF} \sum_{k=nSF}^{(n+1)SF-1} \tilde{H}(k)\right) d_c(n) \\ &\quad + \mu_{ICI}(n) + \mu_{noise}(n) \end{aligned} \quad (11)$$

for $n = 0 \sim N_c/SF$ and $c = 0 \sim C - 1$. The first term represents the desired signal component and the second and third terms are the ICI and noise due to AWGN, respectively. $\mu_{ICI}(n)$ and $\mu_{noise}(n)$ are given by

$$\begin{cases} \mu_{ICI}(n) = \frac{1}{SF}\sqrt{\frac{2P}{SF}} \\ \quad \times \sum_{k=nSF}^{(n+1)SF-1} \tilde{H}(k)\{c_{oc,c}(k \bmod SF)c_{scr}(k)\}^* \\ \quad \times \sum_{c'=0, \neq c}^{C-1} x_{c'}(k)\{c_{oc,c'}(k \bmod SF)c_{scr}(k)\} \\ \mu_{noise}(n) = \frac{1}{SF} \sum_{k=nSF}^{(n+1)SF-1} \{c_{oc,c}(k \bmod SF)c_{scr}(k)\}^* \tilde{\Pi}(k) \end{cases}. \quad (12)$$

It can be understood from Eq. (11) that $\hat{H}(k) = \frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1} \tilde{H}(k)$ is the equivalent channel gain for each symbol and the frequency diversity gain is a function of $SF$. After parallel-to-serial (P/S) conversion, the sequence $\{\hat{d}_c(n)\}$ is soft demodulated using the log-likelihood ratio (LLR) approximation [16] given by

$$L(b) = \frac{\left|\hat{d}_c(n) - \hat{H}(n)\hat{s}_0\right|^2}{2\sigma^2} - \frac{\left|\hat{d}_c(n) - \hat{H}(n)\hat{s}_1\right|^2}{2\sigma^2} \quad (13)$$

for the $b$th bit in the $n$th symbol; $b = 0 \sim 1$ or 3 or 5 for QPSK, 16QAM and 64QAM, respectively. Here, $\hat{s}_0$ (or $\hat{s}_1$) is the candidate symbol, with 0 (or 1) in the $b$th bit position, for which the Euclidean distance from $\hat{d}_c(n)$ is minimum. $2\sigma^2$ is the Gaussian approximated ICI plus noise variance given by [see Appendix for derivation]

$$\begin{aligned} 2\sigma^2 = \frac{2N_0}{T_cN_cSF}\Bigg[&\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\sum_{m=0}^{M-1}|w_m(k)|^2 \\ &+ \left(\frac{C-1}{SF}\frac{E_s}{N_0}\right)\Bigg\{\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}|\tilde{H}(k)|^2 \\ &- \left|\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\tilde{H}(k)\right|^2\Bigg\}\Bigg]. \end{aligned} \quad (14)$$

These LLR values are computed for $c = 0 \sim C-1$ and for all the bits in the symbol. Turbo decoding is performed using these LLR values as soft input.

## 3. Simulation Results

Table 1 summarizes the computer simulation conditions. We assume MC-CDMA using $N_c$=256 subcarriers, GI of $N_g$=32, and ideal coherent QPSK, 16QAM and 64QAM data-modulation. A frequency-selective Rayleigh fading channel having $L$=16-path exponential power delay profile with decay factor $\alpha$ and $\tau_l = l$ is assumed. Unless otherwise stated, the decay factor $\alpha$ is taken to be 0 dB. The normalized maximum Doppler frequency $f_DT$=0.001, which corresponds to a mobile velocity of about 100 km/hr when the carrier frequency is 5 GHz and the transmission rate is 100 M symbols/sec; $T$ is the MC-CDMA signaling period including GI. Uncorrelated, time-varying Rayleigh faded

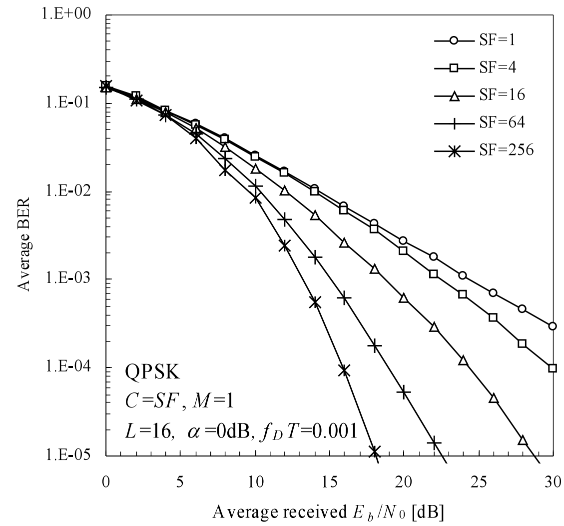
**Table 1** Simulation conditions.

| | | |
|---|---|---|
| Turbo coding | $R$=1/2~9/10<br>(13, 15) RSC encoder<br>Log-MAP decoding with 8 iterations | |
| Channel interleaver | Block interleaver | |
| Data modulation | Coherent QPSK, 16QAM, 64QAM | |
| MC-CDMA | No. of subcarriers | $N_c$=256 |
| | GI | $N_g$=32 |
| | Spreading factor | $SF$=1~256 |
| Channel model | Rayleigh fading ($L$=16)<br>$\tau_l = l$<br>$f_DT$=0.001 | |

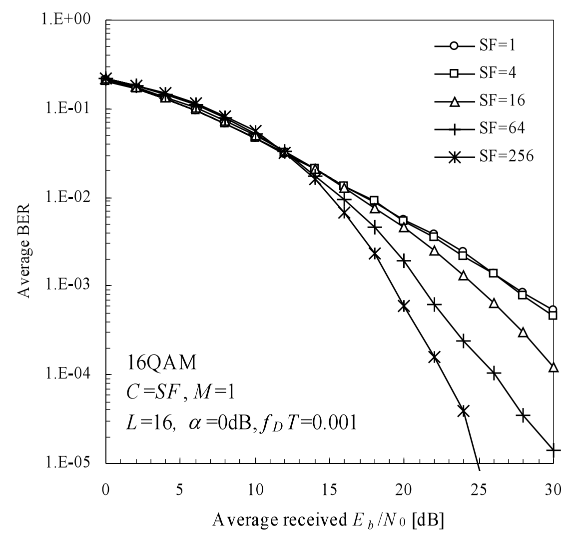
paths are generated using Dent's model [17]. We assume the full load condition with $SF$=$C$ to maintain the data rate fixed as that of an OFDM system (MC-CDMA with $SF$=1). A rate 1/3 turbo encoder with a constraint length 4 and (13, 15) RSC component encoders is assumed. The parity sequences are punctured to obtain other coding rates $R$. Unless otherwise stated, $R = 1/2$. Log-MAP decoding with 8 iterations is carried out at the receiver. The data sequence length is taken to be $K$=1024 bits. The turbo coded and punctured sequence is interleaved before data-modulation. A block bit interleaver is used as channel interleaver. The estimations of the channel gain and the AWGN power spectrum density are assumed to be ideal.

We show how the average BER changes with the change in $SF$ when $C$=$SF$ for different modulation levels. We perform the analysis first for the uncoded case and then for the coded case when $R = 1/2$-turbo coding is used. With higher $SF$, the frequency diversity gain is higher, but at the same time the orthogonality distortion is severer. So, we try to find the $SF$ which gives the lowest BER for the same data rate and bandwidth under the same propagation conditions.

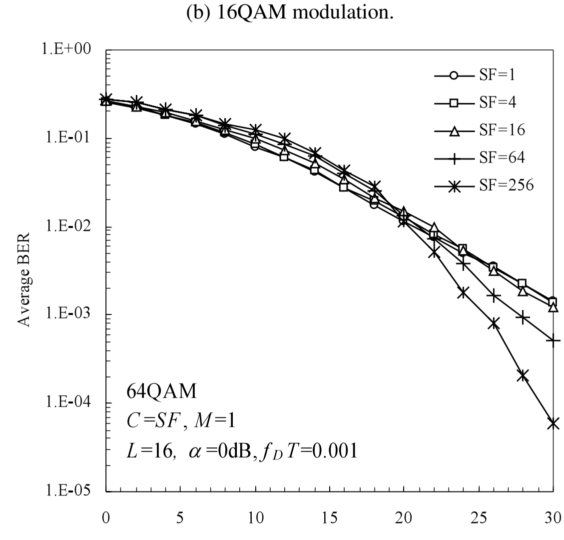
Figures 2(a)–(c) plot the uncoded average BER as a function of the average received signal energy per information bit-to-the noise power spectral density ratio $E_b/N_0$ with $SF$ as a parameter for QPSK, 16QAM and 64QAM, respectively, when $M$=1 (no antenna diversity reception). The power penalty due to GI insertion has been included in the $E_b/N_0$. From Fig. 2(a), we see that for QPSK modulation, the BER improves with the increase in $SF$ and is the lowest for $SF$=256. As said earlier, the orthogonality distortion is severer for higher $SF$ in a frequency selective channel, but MMSE-FDE is applied which restores orthogonality to a certain extent. For $SF$=1 (OFDM), there is no diversity gain and no orthogonality distortion. For $SF > 1$, there is a frequency diversity gain due to the spreading of each symbol over more subcarriers, but at the same time the orthogonality among the codes is partially lost. For 16QAM and 64QAM, however, the result is a little different. In the lower $E_b/N_0$ regions (noise dominant region), the average BER is better for lower $SF$. The Euclidean distance between the different symbols is shorter and even a small loss in or-




(a) QPSK modulation.




(b) 16QAM modulation.




(c) 64QAM modulation.

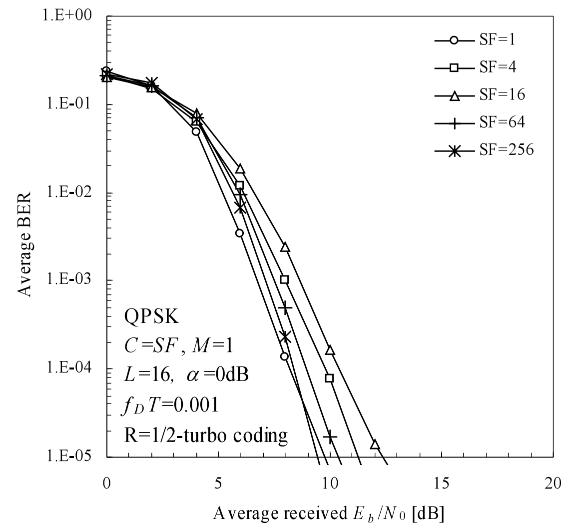
**Fig. 2** Uncoded BER performance for different modulation level with $SF$ as a parameter.

thogonality results in a decision error. In the larger $E_b/N_0$ regions, $E_b/N_0$ > 12 dB and 20 dB for 16QAM and 64QAM respectively, the BER is better for higher *SF*. Hence, it can be said that for the uncoded case, multicode transmission with $SF$=$N_c$ provides the best BER performance.

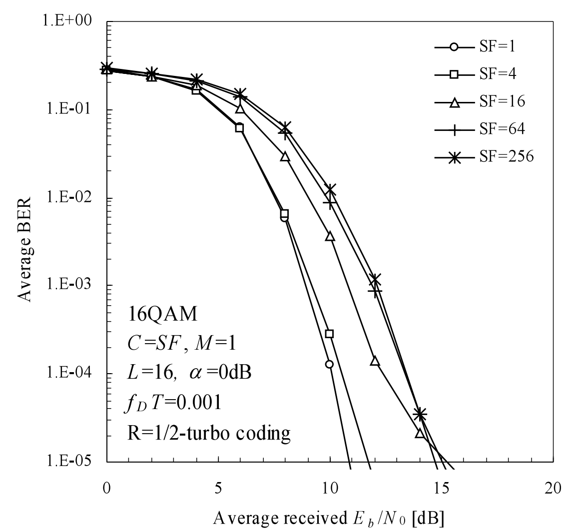
The scenario is completely different when channel coding is applied. The average BER with rate 1/2-turbo coding is plotted in Figs. 3(a)–(c) for QPSK, 16QAM and 64QAM, respectively, when *M*=1. It is seen that in sharp contrast to the uncoded case, the BER dependence on *SF* is very different. With channel coding, there is a trade-off among frequency diversity gain due to spreading, coding gain due to better frequency interleaving effect and orthogonality distortion. OFDM also avails from coding gain due to better frequency interleaving effect when turbo coding is applied; each subcarrier carries a different symbol and experiences different fading resulting in a better interleaving effect. For multicode MC-CDMA, the equivalent channel gain is the same for *C* symbols. Therefore, the interleaving becomes less effective, and orthogonality becomes worse as *SF* increases since *C*=*SF*. For QPSK, it is seen that the average BER for *SF*=1 (OFDM) is almost the same as that for $SF$=$N_c$ and *SF*=16 provides the worst BER performance. This is because for lower *SF* the coding gain is higher due to better frequency interleaving effect, and for higher *SF* orthogonality distortion is larger but the frequency diversity gain is higher. For 16QAM and 64QAM, the BER performance improves as *SF* decreases and the best performance is obtained by *SF*=1 (OFDM). It is found that coding gain due to better interleaving is more desirable than frequency diversity gain due to spreading. For 16QAM (64QAM), the average $E_b/N_0$ for a BER=$10^{-4}$ is 3 dB (6 dB) less for *SF*=1 compared to *SF*=256.

Figure 4 plots the required average received $E_b/N_0$ for a BER=$10^{-4}$ as a function of *SF* with the coding rate *R* as a parameter for QPSK, 16QAM and 64QAM when *M*=1. It can be observed from Fig. 4 that when *R*=1/2, the required average $E_b/N_0$ is the lowest for *SF*=1 (OFDM). As the coding rate increases, the coding gain decreases. Coding improves the performance for all *SF* at the cost of redundancy, however the improvement is the largest for OFDM with all modulation levels and the required average $E_b/N_0$ for OFDM decreases by about 16 dB even with a small amount of redundancy (*R*=9/10). From Fig. 4, it can be observed that for *R*=9/10, 5/6 and 3/4, the required $E_b/N_0$ is the highest for *SF* of around 16 or 32. This can be explained as follows. Higher *SF* benefits from frequency diversity due to spreading. However, higher coding gain due to better frequency interleaving and less orthogonality distortion accounts for the lack of frequency diversity gain for smaller *SF*. Therefore, as *SF* increases, the required $E_b/N_0$ increases but it starts to decrease beyond a certain *SF*. This is because the frequency diversity gain becomes stronger and offsets the adverse effect of orthogonality distortion.

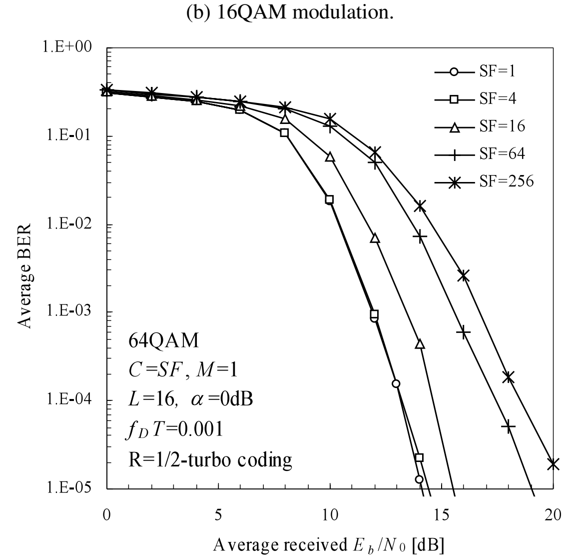
Figure 5 plots the coded average BER as a function of the decay factor $\alpha$ of the exponential delay profile of the channel for 64QAM and *M*=1 when *R*=1/2 and 5/6. The




(a) QPSK modulation.

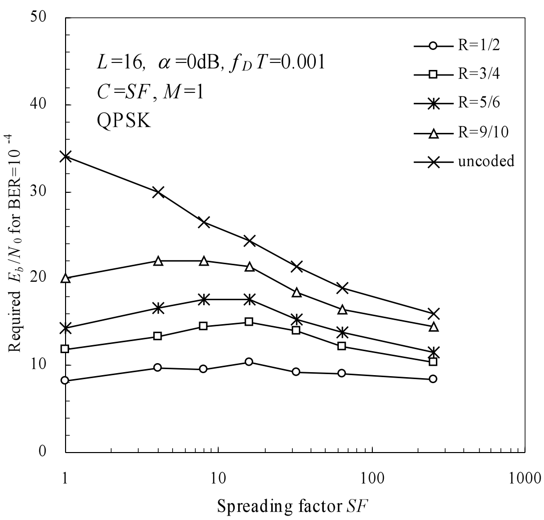



(b) 16QAM modulation.

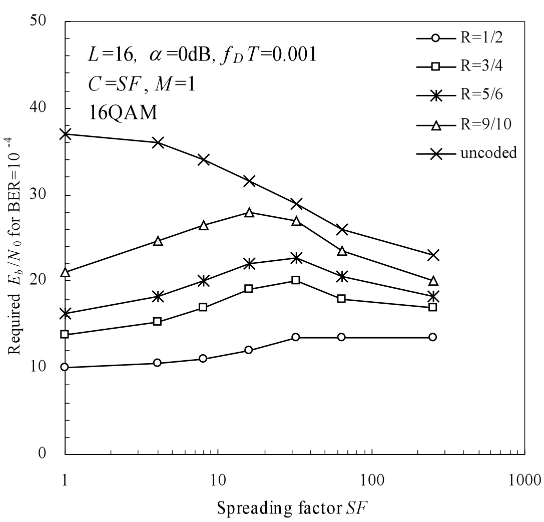



(c) 64QAM modulation.

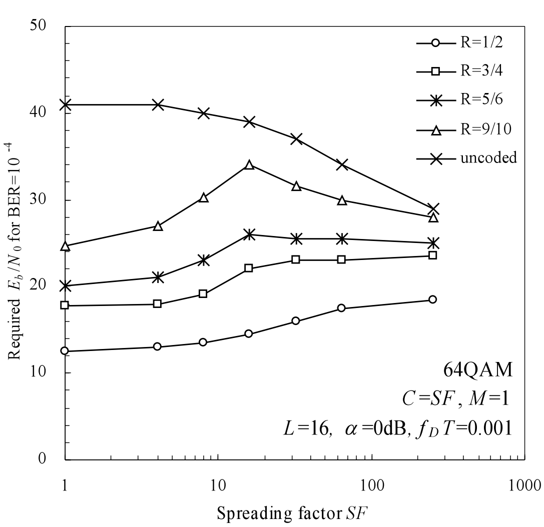
**Fig. 3** Coded BER performance for different modulation level with *SF* as a parameter.




(a) QPSK modulation.

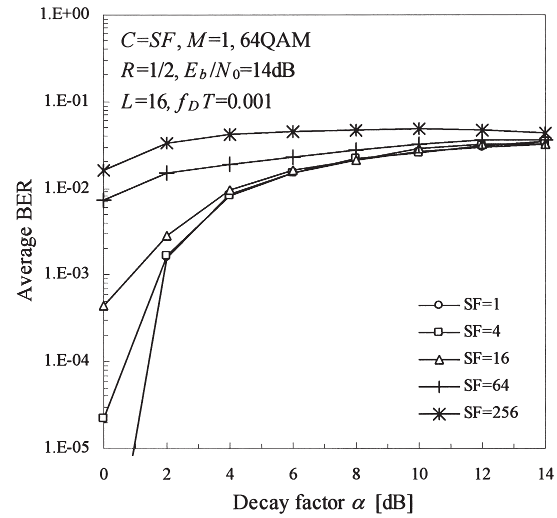



(b) 16QAM modulation.

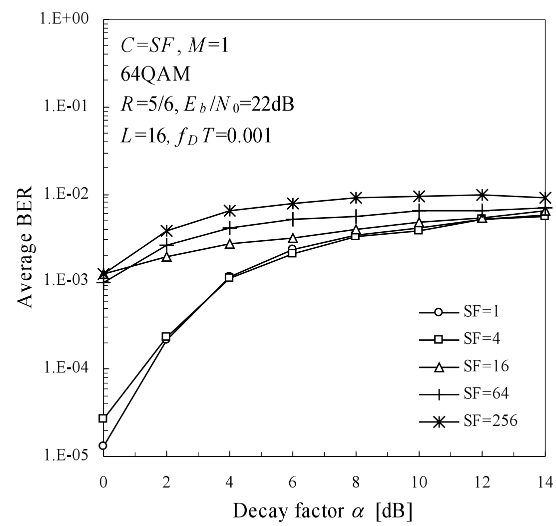



(c) 64QAM modulation.

**Fig. 4** Required average $E_b/N_0$ for different coding rate.




(a) $R$=1/2.




(b) $R$=5/6.

**Fig. 5** Coded BER for 64QAM as a function of the decay factor $\alpha$.

channel is highly frequency selective for $\alpha$=0 dB and approaches a single path channel as $\alpha \rightarrow \infty$. It can be observed from Fig. 5 that as the frequency-selectivity of the channel decreases (as $\alpha$ increases), the performance degrades for all *SF*, due to the decrease in the frequency diversity effect for large *SF* and reduced interleaving effect for low *SF*. For very high $\alpha$, the performance is almost independent of *SF*; however for lower $\alpha$, when $R$=1/2, smaller *SF* has low BER. Hence, it can be said that irrespective of the channel's frequency-selectivity, lower *SF* provides a better BER performance. For $R$=5/6, however, it is seen that for low $\alpha$, i.e., strong frequency selectivity, *SF*=16 has the worst performance. This is due to less frequency diversity compared to high *SF* and less coding gain compared to small *SF*. But as the frequency selectivity of the channel decreases, the higher the *SF*, the worse is the BER performance. This is because of the reduced frequency diversity associated with higher *SF*.

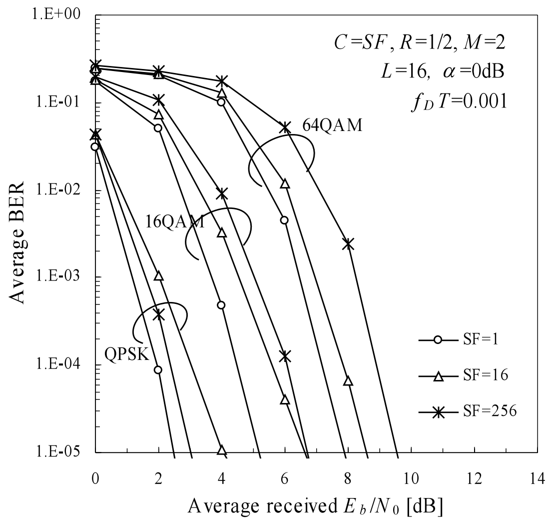



**Fig. 6** Coded BER performance with antenna diversity reception ($M$=2).

The average BER with 2-antenna diversity reception ($M$=2) and rate 1/2-turbo coding is plotted in Fig. 6. It is seen that even with antenna diversity reception, for QPSK, $SF$=1 (OFDM) has the best performance followed by that of $SF$=256. $SF$=16 is seen to have the worst BER performance. This is due to the diversity-coding-orthogonality trade-off. As for the single antenna case ($M$=1), the frequency diversity gain is the highest for $SF$=256 and the coding gain is the highest for $SF$=1. However, for 16QAM and 64QAM, the lower the $SF$, the better is the BER performance. The coding gain associated with better frequency interleaving is more desirable than higher frequency diversity gain for larger $SF$. Since the Euclidean distance between the signal points is shorter, even a small loss in orthogonality results in decision errors. Therefore $SF$=256 gives the worst performance.

## 4. Conclusion

The BER performance of MC-CDMA with high level modulation was evaluated by computer simulations. It was found that without channel coding, BER is better for higher $SF$ due to higher frequency diversity gain for all modulation levels. However with channel coding, there is a trade-off among frequency diversity gain due to spreading, additional coding gain due to better frequency interleaving effect and orthogonality distortion. When QPSK modulation is used, OFDM ($SF$=1) provides almost the same BER performance as MC-CDMA using $SF$=256. However, for 16QAM and 64QAM, OFDM provides the best BER performance, since for higher $SF$, the performance is sensitive to the orthogonality distortion and the frequency diversity gain is offset by the orthogonality distortion. This is true for different coding rate irrespective of the channel frequency-selectivity.

### References


[1] R. Van Nee and R. Prasad, OFDM for Wireless Multimedia Communications, Artech House, 2000.
[2] N. Lee, J.P. Linnarts, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," IEICE Trans. Commun., vol.E77-B, no.7, pp.900–904, July 1994.
[3] S. Hara and R. Prasad, "Overview of multicarrier CDMA," IEEE Commun. Mag., vol.35, no.12, pp.126–133, Dec. 1997.
[4] S. Hara and R. Prassad, Multicarrier Techniques for 4G Mobile Communications, Artech House, 2003.
[5] T. Sao and F. Adachi, "Comparative study of various frequency equalization techniques for downlink of a wireless OFDM-CDMA system," IEICE Trans. Commun., vol.E86-B, no.1, pp.352–364, Jan. 2003.
[6] H. Atarashi, S. Abeta, and M. Sawahashi, "Variable spreading factor-orthogonal frequency and code division multiplexing (VSF-OFCDM) for broadband packet wireless access," IEICE Trans. Commun., vol.E86-B, no.1, pp.291–299, Jan. 2003.
[7] A. Chouly, A. Brajal, and S. Jourdan, "Orthogonal multicarrier techniques applied to direct sequence spread spectrum CDMA system," Proc. IEEE Globecom'93, pp.1723–1728, Nov. 1993.
[8] F. Adachi and T. Sao, "Joint antenna diversity and frequency-domain equalization for multi-rate MC-CDMA," IEICE Trans. Commun., vol.E86-B, no.11, pp.3217–3224, Nov. 2003.
[9] R. Kimura and F. Adachi, "Comparison of OFDM and multicode MC-CDMA in a frequency selective fading channel," Electron. Lett., vol.39, no.3, pp.317–318, Feb. 2003.
[10] S. Kaiser, "Trade-off between channel coding and spreading in multi-carrier CDMA systems," Proc. IEEE ISSSTA'96, pp.1366–1370, Munich, Germany, Sept. 1996.
[11] T.Q.S. Quek, N. Maeda, H. Atarashi, and M. Sawahashi, "Analysis on tradeoff between frequency diversity and inter-code interference considering fading correlation in forward link for VSF-OFCDM wireless access," IEICE Technical Report, RCS2003-143, Oct. 2003.
[12] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near optimum error correcting coding and decoding: Turbo codes," IEEE Trans. Commun., vol.44, no., pp.1261–1271, 1996.
[13] J.P. Woodard and L. Hanzo, "Comparative study of turbo decoding techniques: An overview," IEEE Trans. Veh. Technol., vol.49, no.6, pp.2208–2233, Nov. 2000.
[14] D. Divsalar and F. Pollara, "Turbo codes for PCS applications," Proc. IEEE ICC'95, Seattle, Washington, June 1995.
[15] R. Pyndiah, A. Picart, and A. Glavieux, "Performance of block turbo coded 16-QAM and 64-QAM modulations," Proc. IEEE GLOBECOM95, pp.1039–1044, Singapore, Nov. 1995.
[16] A. Stefanov and T. Duman, "Turbo coded modulation for wireless communications with antenna diversity," Proc. IEEE VTC99-Fall, pp.1565–1569, Netherlands, Sept. 1999.
[17] P. Dent, G.E. Bottomley, and T. Croft, "Jakes fading model revisited," Electron. Lett., vol.29, no.13, pp.1162–1163, June 1993.
[18] W. Davenport and W. Root, An introduction to the theory of random signals and noise, McGraw Hill, 1958.


## Appendix: Derivation of $\sigma^2_{ICI}$ and $\sigma^2_{noise}$ for MC-CDMA

For simplicity we express $c_{oc,c}(k \bmod SF)c_{scr}(k)$ as $c_c(k)$. The ICI in Eq. (12) can be rewritten as

$$\mu_{ICI}(n) = \frac{1}{SF}\sqrt{\frac{2P}{SF}}\sum_{k=nSF}^{(n+1)SF-1} c_c^*(k) \tilde{H}(k)\sum_{\substack{c'=0\\ \neq c}}^{C-1} x_{c'}(k)c_{c'}(k). \quad \text{(A· 1)}$$

If we define

$$\bar{H}(n) = \frac{1}{SF} \sum_{k=nSF}^{(n+1)SF-1} \tilde{H}(k), \tag{A·2}$$

then Eq. (A· 1) can be further rewritten as

$$\mu_{ICI}(n) = \frac{1}{SF} \sqrt{\frac{2P}{SF}} \sum_{k=nSF}^{(n+1)SF-1} \sum_{\substack{c'=0 \\ \neq c}}^{C-1} \left\{ \tilde{H}(k) - \bar{H}(n) \right\} x_{c'}(k) c_{c'}(k) c_c^*(k) \tag{A·3}$$

since

$$\tilde{H}(k) = \left\{ \tilde{H}(k) - \bar{H}(n) \right\} + \bar{H}(n) \quad \text{and}$$
$$\sum_{k=nSF}^{(n+1)SF} c_c^*(k) c_{c'}(k) = 0 \quad \text{if} \quad c \neq c'. \tag{A·4}$$

The variance of $\mu_{ICI}(n)$ is given by

$$2\sigma_{ICI}^2 = E\left[ |\mu_{ICI}(n)|^2 \right] = \frac{1}{SF} \frac{2P}{SF^2} \sum_{k=nSF}^{(n+1)SF-1} \sum_{k'=nSF}^{(n+1)SF-1} \left| \tilde{H}(k) - \bar{H}(n) \right|^2 \times^* \sum_{\substack{c'=0 \\ \neq c}}^{C-1} c_{c'}(k) c_c^*(k) c_{c'}^*(k') c_c(k') \tag{A·5}$$

since $E\left[ x_c(k) x_{c'}^*(k) \right] = 0$ for $c \neq c'$. Further manipulations give

$$\begin{aligned} 2\sigma_{ICI}^2 = {} & \frac{2P}{SF^3} \sum_{k=nSF}^{(n+1)SF-1} \left| \tilde{H}(k) - \bar{H}(n) \right|^2 \times \sum_{\substack{c'=0 \\ \neq c}}^{C-1} |c_{c'}(k)|^2 \left| c_c(k) \right|^2 \\ & + \frac{2P}{SF^3} \sum_{k=nSF}^{(n+1)SF-1} \sum_{\substack{k'=nSF \\ \neq k}}^{(n+1)SF-1} \left| \tilde{H}(k) - \bar{H}(n) \right|^2 \times \sum_{\substack{c'=0 \\ \neq c}}^{C-1} |c_{c'}(k)|^2 \left| c_c(k) \right|^2. \end{aligned} \tag{A·6}$$

Here we make an assumption that the second term tends to 0 for large $C$ and $SF$, according to the law of large numbers [18]. Since we are assuming $|c_c(k)| = 1$ for spreading and scramble sequences, we obtain

$$\begin{aligned} \sigma_{ICI}^2 & \approx \frac{P}{SF^2} \frac{C-1}{SF} \sum_{k=nSF}^{(n+1)SF-1} \left| \tilde{H}(k) - \bar{H}(n) \right|^2 \\ & = \frac{P}{SF^2} \frac{C-1}{SF} \left( \sum_{k=nSF}^{(n+1)SF-1} \left| \tilde{H}(k) \right|^2 - \frac{1}{SF} \left| \sum_{k=nSF}^{(n+1)SF-1} \tilde{H}(k) \right|^2 \right). \end{aligned} \tag{A·7}$$

Next, we obtain $\sigma_{noise}^2$. We have

$$\begin{aligned} 2\sigma_{noise}^2 & = E\left[ |\mu_{noise}(n)|^2 \right] \\ & = \frac{1}{SF^2} \sum_{k=nSF}^{(n+1)SF-1} \sum_{m=0}^{M-1} |\Pi_m(k) w_m(k)|^2. \end{aligned} \tag{A·8}$$

Since $\{\Pi_m(k);\, m = 0 \sim M-1 \text{ and } k = nSF \sim (n+1)SF-1\}$ are zero-mean and i.i.d. complex Gaussian variables having a variance of $2N_0/(T_c N_c)$, we have

$$\sigma_{noise}^2 = \frac{1}{SF^2} \frac{N_0}{T_c N_c} \left( \sum_{k=0}^{N_c-1} \sum_{m=0}^{M-1} |w_m(k)|^2 \right). \tag{A·9}$$

**Deepshikha Garg** received her B.E. degree in Electrical and Communications engineering from Kathmandu University, Nepal in 1998. Currently she is in the Ph.D. program in the department of electrical and communications engineering at Tohoku University, Sendai, Japan. Her research interests include error control schemes and accessing techniques for wireless communications. She was the recipient of the 2002 active research award in radio communication systems from IEICE in 2002.

**Fumiyuki Adachi** received his B.S. and Dr. Eng. degrees in electrical engineering from Tohoku University, Sendai, Japan, in 1973 and 1984, respectively. In April 1973, he joined the Electrical Communications Laboratories of Nippon Telegraph & Telephone Corporation (now NTT) and conducted various types of research related to digital cellular mobile communications. From July 1992 to December 1999, he was with NTT Mobile Communications Network, Inc. (now NTT DoCoMo, Inc.), where he led a research group on wideband/broadband CDMA wireless access for IMT-2000 and beyond. Since January 2000, he has been with Tohoku University, Sendai, Japan, where he is a Professor of Electrical and Communication Engineering at Graduate School of Engineering. His research interests are in CDMA and TDMA wireless access techniques, CDMA spreading code design, Rake receiver, transmit/receive antenna diversity, adaptive antenna array, bandwidth-efficient digital modulation, and channel coding, with particular application to broadband wireless communications systems. From October 1984 to September 1985, he was a United Kingdom SERC Visiting Research Fellow in the Department of Electrical Engineering and Electronics at Liverpool University. From April 1997 to March 2000, he was a visiting Professor at Nara Institute of Science and Technology, Japan. He was a recipient of IEICE Achievement Award 2002 and was a co-recipient of the IEICE Transactions best paper of the year award 1996 and again 1998. He is an IEEE Fellow and was a co-recipient of the IEEE Vehicular Technology Transactions best paper of the year award 1980 and again 1990 and also a recipient of Avant Garde award 2000.