

## PAPER

# Bit Error Rate Analysis of OFDM/TDM with Frequency-Domain Equalization

Haris GACANIN<sup>†a)</sup>, Shinsuke TAKAOKA<sup>†</sup>, *Student Members*, and Fumiya ADACHI<sup>†</sup>, *Member*

**SUMMARY** For alleviating the high peak-to-average power ratio (PAPR) problem of orthogonal frequency division multiplexing (OFDM), the OFDM combined with time division multiplexing (TDM) using frequency-domain equalization (FDE) was proposed. In this paper, the theoretical bit error rate (BER) analysis of the OFDM/TDM in a frequency-selective fading channel is presented. The conditional BER expression is derived, based on a Gaussian approximation of the inter-symbol interference (ISI) arising from channel frequency-selectivity, for the given set of channel gains. Various FDE techniques as in multi-carrier code division multiple access (MC-CDMA), i.e., zero forcing (ZF), maximum ratio combining (MRC) and minimum mean square error (MMSE) criteria are considered. The average BER performance is evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression.

**key words:** OFDM, time division multiplexing, frequency-domain equalization, frequency-selective fading

## 1. Introduction

The next generation wireless communications systems require high-speed data transmissions, e.g., 100 Mbps or higher [1]. However, in a wireless channel, the presence of many propagation paths with different time delays causes frequency-selective fading [2], which produces inter-symbol interference (ISI) and degrades the transmission performance of the single carrier (SC) systems [3]. Recently, orthogonal frequency division multiplexing (OFDM) has been attracting much attention because of its robustness against frequency-selective fading [4]. In OFDM, high-speed data is transmitted using a number of orthogonal subcarriers, where each modulated subcarrier bandwidth is narrow enough to experience frequency-nonselective fading. However, one of the main problems is its high peak-to-average-power ratio (PAPR).

For overcoming the PAPR problem of OFDM, recently we proposed [5], [6] to use OFDM combined with time division multiplexing (OFDM/TDM). Our work is an extension and modification of the work presented in [7]. The objective of [7] is different from ours and is to increase the transmission data rate for the given bandwidth. However, our objective is to reduce the number of subcarriers while keeping the data rate the same as the conventional OFDM. In our OFDM/TDM design, a sequence of  $K$  OFDM signals

with  $N_c/K$  subcarriers is transmitted during a block interval of the conventional OFDM with  $N_c$  subcarriers;  $K$  is an important design parameter. In [7], only frequency-domain equalization (FDE) based on zero-forcing (ZF) criterion is considered. To combat the frequency-selective fading, various FDE techniques can be applied as in multicarrier code division multiple access (MC-CDMA) [8], [9], SC transmission [10] and quite recently in direct sequence CDMA (DS-SS) [11], [12]. It was found [5], [6] by computer simulation that the OFDM/TDM with FDE based on minimum mean square error criterion (MMSE-FDE) provides a much better bit error rate (BER) performance compared to the conventional OFDM owing to frequency diversity effect resulting from FDE. It is also interesting to note that the OFDM/TDM with MMSE-FDE bridges the conventional OFDM and SC transmissions; OFDM/TDM becomes SC when  $K = N_c$  and becomes conventional OFDM when  $K=1$ .

So far, we have presented only the computer simulation results to show the BER performance of OFDM/TDM with FDE. This paper is intended to give a theoretical foundation to OFDM/TDM. Channel coding is a very powerful technique to improve the transmission performance in fading channels. However, in this paper, we consider only uncoded case for the theoretical analysis. The remainder of this paper is organized as follows. Section 2 presents the OFDM/TDM transmission system model with FDE based on maximal-ratio combining (MRC), ZF and MMSE. An expression for the conditional BER in a frequency-selective Rayleigh fading channel is derived for the given set of channel gains and the theoretical average BER is evaluated by Monte-Carlo numerical computation method using the derived BER expression in Sect. 3. In Sect. 4, the average BER performance is compared with the computer simulation results to confirm the theoretical analysis. Section 5 gives some conclusions.

## 2. Transmission System Model

The OFDM/TDM transmission system model is illustrated in Fig. 1. Throughout this paper,  $T_c$ -spaced discrete time representation is used, where  $T_c$  represents the fast Fourier transform (FFT) sampling period.

### 2.1 OFDM/TDM Signal Generation

Without loss of generality, the transmission of  $N_c$  data symbols is considered, where  $N_c$  is the inverse FFT (IFFT) block size. A sequence of  $N_c$  data-modulated symbols  $\{d(i)\}$

Manuscript received March 16, 2005.

Manuscript revised August 8, 2005.

<sup>†</sup>The authors are with the Department of Electrical and Communications Engineering, Tohoku University, Sendai-shi, 980-8579 Japan.

a) E-mail: haris@mobile.ecei.tohoku.ac.jp

DOI: 10.1093/ietcom/e89-b.2.509

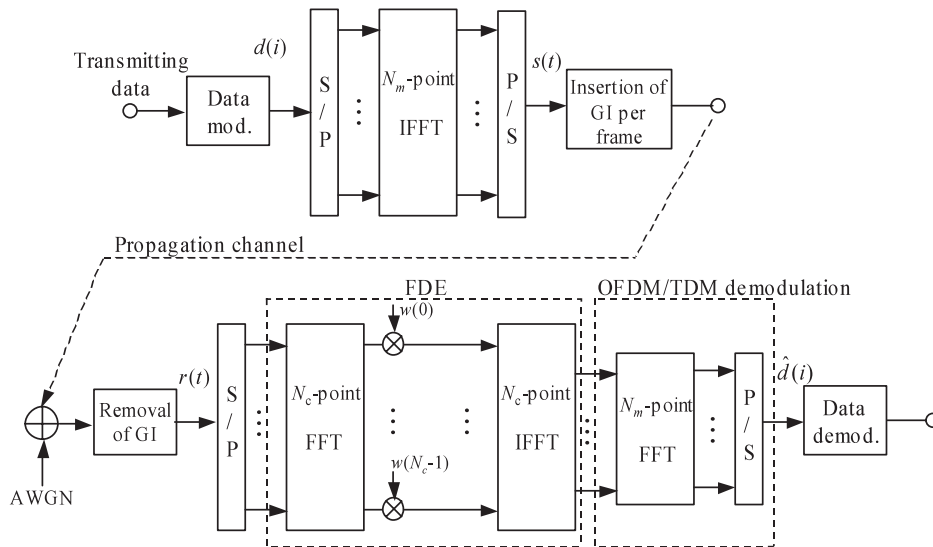


Fig. 1 OFDM/TDM transceiver.

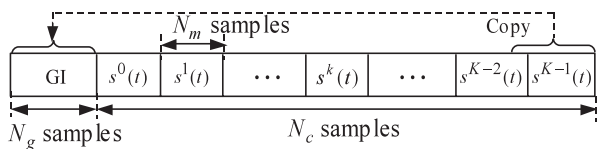


Fig. 2 OFDM/TDM frame structure.

$i = 0 \sim N_c - 1$  with  $E[|d(i)|^2] = 1$  is transmitted during one OFDM/TDM frame (equal to the IFFT block size of the conventional OFDM), where  $N_c$  and  $E[\cdot]$  are the number of subcarriers in the conventional OFDM and the ensemble average operation, respectively. The data-modulated symbol sequence  $\{d(i)\}$  is divided into  $K$  blocks of  $N_m = N_c/K$  symbols each. The  $k$ -th block symbol sequence is denoted by  $\{d^k(i); i = 0 \sim N_m - 1\}$ , where  $d^k(i) = d(kN_m + i)$  for  $k = 0 \sim K - 1$ .  $N_m$ -point IFFT is applied to generate a sequence of  $K$  OFDM signals with  $N_m$  subcarriers, as illustrated in Fig. 2. The transmission data rate of OFDM/TDM is kept the same as that of conventional OFDM.

The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t) = \sum_{k=0}^{K-1} s^k(t - kN_m)u(t - kN_m) \quad (1)$$

for  $t = 0 \sim N_c - 1$ , where  $u(t) = 1(0)$  for  $t = 0 \sim N_m - 1$  (elsewhere) and  $s^k(t)$  is the  $k$ -th OFDM signal with  $N_m$  subcarriers, given by

$$s^k(t) = \sqrt{\frac{2E_s}{T_c} \frac{1}{N_m}} \sum_{i=0}^{N_m-1} d^k(i) \exp\left(j2\pi t \frac{i}{N_m}\right). \quad (2)$$

for  $t = 0 \sim N_m - 1$ , where  $E_s$  represents the data-modulated symbol energy. Before transmission, the last  $N_g$  samples in the OFDM/TDM frame are copied as a cyclic prefix (CP) and inserted into the guard interval (GI) at the beginning of the frame (see Fig. 2).

## 2.2 Channel Model

A  $T_c$ -spaced time-delay model of the propagation channel is assumed. Assuming that the channel has  $L$  independent propagation paths with distinct time delays  $\{\tau_l; l = 0 \sim L - 1\}$ , the discrete-time impulse response  $h(t)$  of the channel is expressed as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l), \quad (3)$$

where  $h_l$  is the  $l$ th path gain with  $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ . Assuming block fading so that the path gains remain constant over one OFDM/TDM frame, time dependency of the channel has been dropped for simplicity. It is assumed that the maximum time delay of the channel is shorter than the GI.

## 2.3 Received Signal Representation

The received signal can be expressed as

$$r(t) = \sum_{l=0}^{L-1} h_l s(t - \tau_l) + \eta(t) \quad (4)$$

for  $t = -N_g \sim N_c - 1$ , where  $\eta(t)$  is the additive white Gaussian noise (AWGN) process with zero mean and variance  $2N_0/T_c$  with  $N_0$  being the single-sided power spectrum density. After removing the GI from the received signal, the received OFDM/TDM signal  $\{r(t); t = 0 \sim N_c - 1\}$  is decomposed into  $N_c$  frequency components  $\{R(n); n = 0 \sim N_c - 1\}$  by applying  $N_c$ -point FFT:

$$\begin{aligned} R(n) &= \sum_{t=0}^{N_c-1} r(t) \exp\left(-j2\pi n \frac{t}{N_c}\right) \\ &= S(n)H(n) + \Pi(n), \end{aligned} \quad (5)$$

where  $S(n)$ ,  $H(n)$  and  $\Pi(n)$  are the transmitted OFDM/TDM signal component, the channel gain and the noise component at the  $n$ th frequency, respectively, given by

$$\begin{cases} S(n) = \sum_{t=0}^{N_c-1} s(t) \exp\left(-j2\pi n \frac{t}{N_c}\right) \\ H(n) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi n \frac{\tau_l}{N_c}\right) \\ \Pi(n) = \sum_{t=0}^{N_c-1} \eta(t) \exp\left(-j2\pi n \frac{t}{N_c}\right). \end{cases} \quad (6)$$

## 2.4 Frequency-Domain Equalization

One-tap FDE is applied to obtain

$$\hat{R}(n) = w(n)R(n) = S(n)\hat{H}(n) + \hat{\Pi}(n), \quad (7)$$

where

$$\begin{cases} \hat{H}(n) = w(n)H(n) \\ \hat{\Pi}(n) = w(n)\Pi(n) \end{cases}, \quad (8)$$

$w(n)$  is the equalization weight for the  $n$ th frequency, and  $\hat{\Pi}(n)$  is the noise component after equalization. We consider ZF-, MRC- and MMSE-FDE. Their equalization weights are given by [8]–[10]

$$w(n) = \begin{cases} \frac{H^*(n)}{|H(n)|^2} & \text{for ZF} \\ H^*(n) & \text{for MRC} \\ \frac{H^*(n)}{|H(n)|^2 + \left(\frac{E_s}{N_0}\right)^{-1}} & \text{for MMSE} \end{cases}. \quad (9)$$

Comparing Eqs. (5) and (7),  $\hat{H}(n)$  is called the equivalent channel gain. The ZF-FDE perfectly restores the channel frequency-nonselctivity, but produces the noise enhancement. On the other hand, the MRC-FDE produces no noise enhancement, but enhances the frequency-selectivity of the channel and increases the ISI; hence, it does not necessarily maximize the signal-to-interference plus noise power ratio (SINR), and degrades the BER performance. The MMSE-FDE minimizes the SINR by giving up perfect restoration of the channel frequency-nonselctivity.

## 2.5 Recovery of OFDM/TDM Signal

By applying  $N_c$ -point IFFT to  $\{\hat{R}(n); n = 0 \sim N_c - 1\}$ , we obtain the time-domain OFDM/TDM signal  $\hat{r}(t)$ , which can be expressed as

$$\begin{aligned} \hat{r}(t) &= \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{R}(n) \exp\left(j2\pi t \frac{n}{N_c}\right) \\ &= \sum_{k=0}^{K-1} \hat{r}^k(t - kN_m) u(t - kN_m) \end{aligned} \quad (10)$$

for  $t=0 \sim N_c - 1$ , where  $\hat{r}^k(t)$  corresponds to the  $k$ -th OFDM signal with  $N_m$  subcarriers. Thereby, the decision variable for the  $i$ th data symbol of the  $k$ th OFDM signal can be obtained by applying  $N_m$ -point FFT as

$$\hat{d}^k(i) = \frac{1}{N_m} \sum_{t=kN_m}^{(k+1)N_m-1} \hat{r}(t) \exp\left(-j2\pi i \frac{t - kN_m}{N_m}\right) \quad (11)$$

for  $i=0 \sim N_m - 1$  and  $k = 0 \sim K - 1$ . Substituting Eq. (10) into Eq. (11), we obtain

$$\hat{d}^k(i) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{R}(n) \Psi(n; i, k), \quad (12)$$

where

$$\begin{aligned} \Psi(n; i, k) &= \frac{1}{N_m} \sum_{t=kN_m}^{(k+1)N_m-1} \exp\left[-j2\pi t \frac{iK - n}{N_c}\right] \\ &= \frac{\sin\left(\pi N_m \frac{iK - n}{N_c}\right)}{N_m \sin\left(\pi \frac{iK - n}{N_c}\right)} \exp\left[-j\pi\{(2k + 1)N_m - 1\} \frac{iK - n}{N_c}\right]. \end{aligned} \quad (13)$$

Some properties of  $\Psi(n; i, k)$  are given below. If  $K=1$ , then  $N_m = N_c$  and  $k = 0$  and hence,

$$\Psi(n; i, k = 0) = \frac{1}{N_c} \sum_{t=0}^{N_c-1} \exp\left(-j2\pi t \frac{i-n}{N_c}\right) = \delta(i-n). \quad (14)$$

The sum of the squared  $\Psi(n; i, k)$  is given by

$$\begin{aligned} &\sum_{n=0}^{N_c-1} |\Psi(n; i, k)|^2 \\ &= \frac{1}{N_m^2} \sum_{t=kN_m}^{(k+1)N_m-1} \sum_{t'=kN_m}^{(k+1)N_m-1} \exp\left[-j2\pi(t-t') \frac{iK}{N_c}\right] N_c \delta(t-t') \\ &= \frac{N_c}{N_m}. \end{aligned} \quad (15)$$

## 2.6 Complexity Comparison

Let  $N_c$  be the number of subcarriers of conventional OFDM. Remember that  $N$ -point IFFT requires  $N \log_2 N$  complex multiplications if  $N$  is a power of 2 [13]. The number of complex multiplications required at the transmitter and receiver is given in Table 1 for conventional OFDM, OFDM/TDM and SC transmissions.

The conventional OFDM transmitter requires one  $N_c$ -point IFFT while OFDM/TDM requires  $KN_m$ -point IFFTs. Hence, the number of complex multiplications is  $N_c \log_2 N_c$  for conventional OFDM transmitter and  $N_c \log_2(N_c/K)$  for OFDM/TDM transmitter. For FDE,  $N_c$ -point FFT is required for conventional OFDM while OFDM/TDM and SC require one more  $N_c$ -point IFFT (see Fig. 1). Since one-tap FDE is used, additional  $N_c$  complex multiplications are necessary for weight multiplication. Therefore, the total number of complex multiplications required for FDE

**Table 1** Required complex multiplications.

		OFDM ( $K=1$ )	OFDM/TDM ( $K=2\sim 128$ )	SC ( $K=256$ )
Transmitter		$N_c \log_2 N_c$	$N_c \log_2(N_c/K)$	-
Receiver	FDE	$N_c \log_2 N_c$ $+N_c$	$2N_c \log_2 N_c$ $+N_c$	$2N_c \log_2 N_c$ $+N_c$
	Dem.	-	$N_c \log_2(N_c/K)$	-
Total		$2N_c \log_2 N_c$ $+N_c$	$2N_c \log_2(N_c/K)$ $+2N_c \log_2 N_c$ $+N_c$	$2N_c \log_2 N_c$ $+N_c$

is  $N_c \log_2 N_c + N_c$  for the conventional OFDM while it is  $2N_c \log_2 N_c + N_c$  for OFDM/TDM and SC. For OFDM/TDM signal demodulation,  $KN_m$ -point FFTs are required (see Fig. 1). Hence, the number of complex multiplications required for OFDM/TDM demodulation is  $N_c \log_2(N_c/K)$ .

It can be understood from Table 1 that the transmitter complexity is less than the receiver complexity for OFDM/TDM while they are almost the same for the conventional OFDM. The total (transmitter/receiver) complexity is larger for OFDM/TDM than for conventional OFDM. However, OFDM/TDM can achieve much better BER performance at the cost of complexity (this is shown in Sect. 4).

### 3. BER Analysis

In this section, we first theoretically derive the conditional BER based on the Gaussian approximation of the ISI and then, evaluate the theoretical average BER performance by Monte-Carlo numerical computation method. In the following analysis, as stated in Sect. 2, we assume block fading (i.e., the path gains remain constant over one OFDM/TDM frame) and the maximum time delay of the channel does not exceeds the GI.

#### 3.1 OFDM/TDM Demodulated Output

Substituting  $S(n)$  given by Eqs. (6) and (7) into Eq. (10), we obtain

$$\begin{aligned} \hat{r}(t) = & s(t) \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right) \\ & + \sum_{\substack{t'=0 \\ \neq t}}^{N_c-1} s(t') \left\{ \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \exp \left( -j2\pi n \frac{t' - t}{N_c} \right) \right\} \\ & + \hat{\eta}(t), \end{aligned} \quad (16)$$

where the first term represents the desired signal component, the second term is the ISI component and the third term is the noise component. By applying  $N_c$ -point FFT to Eq. (16), we obtain  $\hat{R}(n)$  as

$$\begin{aligned} \hat{R}(n) = & S(n) \left( \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right) \\ & + S(n) \left[ \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right] + \hat{\Pi}(n). \end{aligned} \quad (17)$$

Substituting Eq. (17) into Eq. (12), the decision variable for  $d^k(i)$  becomes

$$\begin{aligned} \hat{d}^k(i) = & \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} S(n) \Psi(n; i, k) \right) \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right) \\ & + \frac{1}{N_c} \sum_{n=0}^{N_c-1} S(n) \left[ \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right] \Psi(n; i, k) \\ & + \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{\Pi}(n) \Psi(n; i, k). \end{aligned} \quad (18)$$

Since

$$\begin{aligned} & \frac{1}{N_c} \sum_{n=0}^{N_c-1} S(n) \Psi(n; i, k) \\ & = \frac{1}{N_m} \sqrt{\frac{2E_s}{T_c}} \frac{1}{N_m} \sum_{t=0}^{N_m-1} \sum_{i'=0}^{N_m-1} d^k(i') \exp \left( j2\pi t \frac{i' - i}{N_m} \right) \\ & = \sqrt{\frac{2E_s}{T_c}} \frac{1}{N_m} d^k(i), \end{aligned} \quad (19)$$

Eq. (18) can be rewritten as

$$\begin{aligned} \hat{d}^k(i) = & \sqrt{\frac{2E_s}{T_c}} \frac{1}{N_m} d^k(i) \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right) \\ & + \frac{1}{N_c} \sum_{n=0}^{N_c-1} S(n) \left[ \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right] \Psi(n; i, k) \\ & + \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{\Pi}(n) \Psi(n; i, k), \end{aligned} \quad (20)$$

where the first term is the desired signal component, the second term is the ISI component and the third term is the noise component.

#### 3.2 Expression for Conditional BER

It can be understood from Eq. (20) that decision variable  $\hat{d}^k(i)$  for  $d^k(i)$  is a random variable with mean  $\sqrt{\frac{2E_s}{T_c}} \frac{1}{N_m} d^k(i) \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right)$ . Assuming that the ISI can be approximated as a zero-mean complex-valued Gaussian variable, the sum of the ISI and noise due to the AWGN can be treated as a new zero-mean complex-valued Gaussian noise with variance:

$$2\sigma^2 = 2\sigma_{ISI}^2 + 2\sigma_{AWGN}^2, \quad (21)$$

where [see Appendix]

$$\begin{cases} 2\sigma_{ISI}^2 = \frac{2E_s}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} \left| \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right|^2 \\ \quad |\Psi(n; i, k)|^2 \\ 2\sigma_{AWGN}^2 = \frac{2N_0}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} |w(n)|^2 |\Psi(n; i, k)|^2 \end{cases} \quad (22)$$

for the given set of  $\{H(n)$  and  $w(n); n = 0 \sim N_c - 1\}$ . Therefore, we have

$$2\sigma^2 = \frac{2N_0}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} |w(n)|^2 + \frac{E_s}{N_0} \left| \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right|^2 |\Psi(n; i, k)|^2. \quad (23)$$

We assume all “1” transmission (i.e.,  $d^k(i) = (1 + j)/\sqrt{2}$ ) without loss of generality and quaternary phase shift keying (QPSK) data-modulation. Since the ISI can be assumed to be circularly symmetric, the conditional BER for the given set of  $\{H(n); n = 0 \sim N_c - 1\}$  (or equivalently, the given set of path gains and time delays  $\{h_l$  and  $\tau_l; l = 0 \sim L - 1\}$ ) can be expressed as [3]

$$p_b \left( \frac{E_s}{N_0}, \{H(n)\}; i, k \right) = \frac{1}{2} \text{Prob} [\text{Re}[\hat{d}^k(i)] < 0 | \{H(n)\}] + \frac{1}{2} \text{Prob} [\text{Im}[\hat{d}^k(i)] < 0 | \{H(n)\}] = \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{1}{4} \gamma \left( \frac{E_s}{N_0}, \{H(n)\}; i, k \right)} \right], \quad (24)$$

where  $\text{erfc}[x] = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$  is the complementary error function and  $\gamma(E_s/N_0, \{H(n)\})$  is the conditional SINR. The signal component after demodulation is given by the first term of Eq. (20). The ISI plus noise power is given by  $\sigma^2$ . Hence, the conditional SINR is given by

$$\gamma \left( \frac{E_s}{N_0}, \{H(n)\}; i, k \right) = \frac{\left| \sqrt{\frac{2E_s}{T_c}} \frac{1}{N_m} d^k(i) \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right) \right|^2}{\sigma^2} = \frac{2 \left( \frac{E_s}{N_0} \right) \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right|^2}{\left( \frac{N_m}{N_c} \right) \sum_{n=0}^{N_c-1} \left\{ |w(n)|^2 + \left( \frac{E_s}{N_0} \right) \left| \hat{H}(n) - \left( \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right) \right|^2 \right\} |\Psi(n; i, k)|^2}. \quad (25)$$

The theoretical average BER can be numerically evaluated by averaging Eq. (24) over  $\{H(n); n = 0 \sim N_c - 1\}$ :

$$P_b \left( \frac{E_s}{N_0}; i, k \right) = \int \cdots \int \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{1}{4} \gamma \left( \frac{E_s}{N_0}, \{H(n)\}; i, k \right)} \right] \cdot p(\{H(n)\}) \prod_n dH(n), \quad (26)$$

where  $p(\{H(k)\})$  is the joint probability density function (pdf) of  $\{H(k); n = 0 \sim N_c - 1\}$ .

Since  $N_c = KN_m$ ,  $|\Psi(n; i, k)|^2$  is not a function of  $k$  (see Eq. (15)). Remembering that the statistical property of  $H(n)$  is equally likely for all  $n$ ,  $\gamma$  is not a function of either  $i$  or  $k$ . Hence, the average BER is the same for all  $i$  and  $k$ . Therefore, the average BER can be evaluated by

$$P_b \left( \frac{E_s}{N_0} \right) = \int \cdots \int \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{1}{4} \gamma \left( \frac{E_s}{N_0}, \{H(n)\} \right)} \right] \cdot p(\{H(n)\}) \prod_n dH(n), \quad (27)$$

where

$$\gamma \left( \frac{E_s}{N_0}, \{H(n)\} \right) = \frac{2 \left( \frac{E_s}{N_0} \right) \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right|^2}{\left( \frac{N_m}{N_c} \right) \sum_{n=0}^{N_c-1} \left\{ |w(n)|^2 + \left( \frac{E_s}{N_0} \right) \left| \hat{H}(n) - \left( \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right) \right|^2 \right\} |\Psi(n)|^2} \quad (28)$$

with

$$|\Psi(n)|^2 = \frac{\left| \sin \left( \pi N_m \frac{n}{N_c} \right) \right|^2}{\left| N_m \sin \left( \pi \frac{n}{N_c} \right) \right|^2} \leq 1. \quad (29)$$

### 3.2.1 Special Case of Frequency-Nonselective Fading

When the channel is frequency-nonselective,  $\hat{H}(n) = \hat{H} = wH$  for all  $n$ , and ISI disappears. From Eq. (28), we have

$$\gamma \left( \frac{E_s}{N_0}, H \right) = \frac{2 \left( \frac{E_s}{N_0} \right) |wH|^2}{|w|^2 \left( \frac{N_m}{N_c} \sum_{n=0}^{N_c-1} |\Psi(n)|^2 \right)} = 2 \left( \frac{E_s}{N_0} \right) |H|^2. \quad (30)$$

In this case, the exact average BER can be obtained. Since  $H$  is zero-mean complex Gaussian distributed with unity variance, we have [3]

$$P_b \left( \frac{E_s}{N_0} \right) = \int \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{1}{2} \frac{E_s}{N_0} |H|^2} \right] p(H) dH = \frac{1}{2} \left[ 1 - \sqrt{\frac{\frac{E_s}{N_0}}{\frac{E_s}{N_0} + 2}} \right] \quad (31)$$

for QPSK data modulation.

### 3.2.2 Special Case of $K = 1$ (Conventional OFDM)

When  $K=1$ ,  $N_m = N_c$  and

$$\Psi(n; i, k) = \delta(i - n). \quad (32)$$

Then, Eq. (20) becomes

$$\hat{d}(i) = \sqrt{\frac{2E_s}{T_c} \frac{1}{N_c}} d(i) \hat{H}(i) + \frac{1}{N_c} \hat{\Pi}(i), \quad (33)$$

which means that no ISI is produced. The conditional SNR is given by

$$\gamma\left(\frac{E_s}{N_0}; i\right) = \frac{2E_s}{N_0} \frac{|\hat{H}(i)|^2}{|w(i)|^2} = \frac{2E_s}{N_0} |H(i)|^2. \quad (34)$$

Hence, the average BER of OFDM is the same as for the frequency-nonselective fading case.

### 3.2.3 Special Case of $K = N_c$ (SC Transmission)

When  $K = N_c$ , then  $N_m=1$  and Eq. (29) becomes

$$|\Psi(n)|^2 = 1. \quad (35)$$

Hence, Eq. (28) becomes

$$\begin{aligned} & \gamma\left(\frac{E_s}{N_0}, \{H(n)\}\right) \\ &= \frac{2\left(\frac{E_s}{N_0}\right) \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right|^2}{\frac{1}{N_c} \sum_{n=0}^{N_c-1} |w(n)|^2 + \left(\frac{E_s}{N_0}\right) \left\{ \frac{1}{N_c} \sum_{n=0}^{N_c-1} |\hat{H}(n)|^2 - \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right|^2 \right\}}. \end{aligned} \quad (36)$$

### 3.3 Lower Bounded BER

MRC equalization, which is the channel-matched filter in the frequency-domain, provides the maximum SNR. Therefore, substituting  $w(n) = H^*(n)$  into Eq. (28) and neglecting the ISI gives the maximum achievable SNR, which is given by

$$\gamma\left(\frac{E_s}{N_0}, \{H(n)\}\right) = \frac{2\left(\frac{E_s}{N_0}\right) \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} |H(n)|^2 \right|^2}{\left(\frac{N_m}{N_c}\right) \sum_{n=0}^{N_c-1} |H(n)|^2 |\Psi(n)|^2}. \quad (37)$$

Using Eq. (37), the lower bounded BER can be evaluated.

## 4. Numerical and Simulation Results

The conditions for numerical evaluation of the theoretical average BER and computer simulation are given in Table 2. We assume an OFDM/TDM frame size of  $N_c=256$  samples, GI length of  $N_g=32$  samples and ideal coherent QPSK data modulation/demodulation. As the propagation channel, we assume an  $L = 16$ -path block Rayleigh fading channel having the exponential power delay profile  $\Omega(\tau)$ , given by

$$\Omega(\tau) = \left( \frac{1 - \alpha^{-1}}{1 - \alpha^{-L}} \right) \sum_{l=0}^{L-1} \alpha^{-l} \delta(\tau - \tau_l), \quad (38)$$

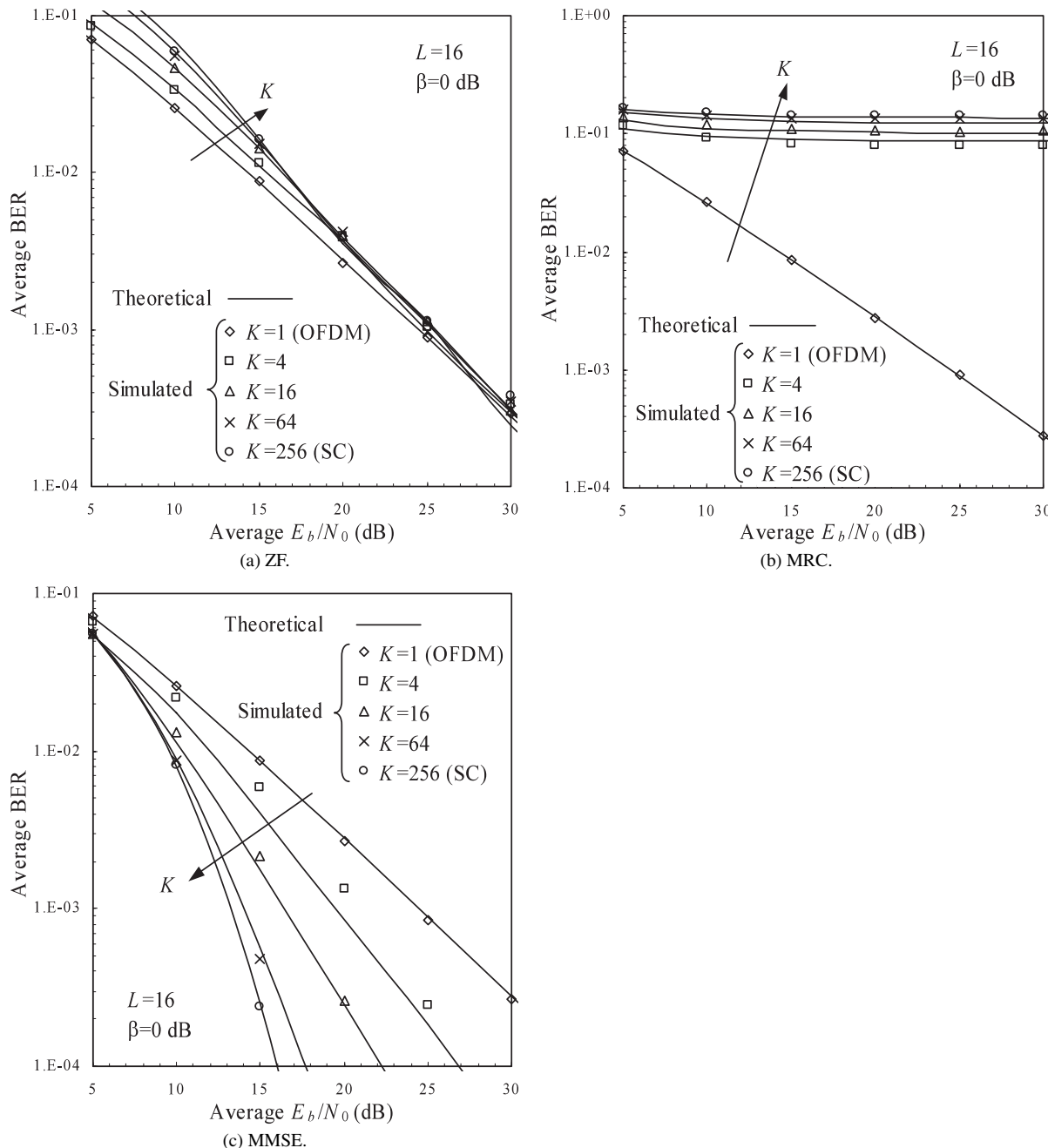
where  $\alpha$  represents the decay factor and  $\{h_l; l = 0 \sim L-1\}$  are zero-mean independent complex Gaussian variables. It is assumed that the time delay of the  $l$ th path is  $\tau_l = l$  samples (i.e., the maximum delay difference is less than the GI length since  $L \leq N_g$ ).

The evaluation of the theoretical average BER is done by Monte-Carlo numerical computation method as follows. A set of path gains  $\{h_l; l = 0 \sim L-1\}$  is generated for obtaining  $\{H(k); k = 0 \sim N_c-1\}$  using Eq. (6) and then  $\{w(k); k = 0 \sim N_c-1\}$  is computed using Eq. (9). The conditional BER for the given average received  $E_s/N_0$  is computed using Eq. (24). This is repeated a sufficient number of times to obtain the theoretical average BER of Eq. (27). Also presented below are the computer simulation results for the OFDM/TDM signal transmission to confirm the validity of the theoretical analysis.

The theoretical average BER performance is plotted with  $K$  as a parameter in Fig. 3 for MMSE, MRC and ZF equalizations as a function of the average received bit energy-to-the AWGN power spectrum density ratio  $E_b/N_0$ , which is given by  $E_b/N_0 = 0.5(E_s/N_0)(1 + N_g/N_c)$ . It is seen that as  $K$  increases, the MMSE equalization consistently improves the BER. The best performance is obtained when  $K = N_c$ , which is the single carrier transmission system. This is because, as  $K$  increases, the transmitted symbol energy is distributed over a wider bandwidth and this is exploited in the MMSE-FDE to achieve the frequency diversity effect. Moreover, the BER performance with ZF equalization is almost insensitive to  $K$  since no ISI is produced,

**Table 2** Simulation conditions.

Transmitter	Data modulation	QPSK
	No. of IFFT points	$N_m=256/K$
	No. of slots	$K=1\sim 256$
	Frame length	$N_c=256$
	GI	$N_g=32$
Channel	$L=16$ -path frequency-selective block Rayleigh fading	
Receiver	No. of FFT points	$N_c=256$ $N_m=256/K$
	FDE	ZF, MRC, MMSE
	Channel estimation	Ideal



**Fig. 3** Average BER performances with MMSE, MRC, and ZF equalizations: (a) ZF, (b) MRC, and (c) MMSE.

but the BER performance is worse than with MMSE equalization because of the noise enhancement. On the other hand, with MRC equalization, the noise enhancement can be suppressed, but a large ISI is produced due to enhanced frequency-selectivity. Hence, the BER floor appears when  $K > 1$ .

The performance improvement of OFDM/TDM is attributed to the frequency diversity gain achieved by the MMSE-FDE. The frequency diversity gain depends on the channel frequency-selectivity. A good measure of the channel frequency selectivity is the delay spread, defined as [3]

$$\tau_{rms} = \sqrt{\int_{-\infty}^{\infty} \Omega(\tau)(\tau - \bar{\tau})^2 d\tau} \tag{39}$$

with  $\bar{\tau} = \int_{-\infty}^{\infty} \tau \Omega(\tau) d\tau$  representing the mean time delay. The delay spread is a function of  $\alpha$  for the given  $L$ . As decay factor  $\alpha$  increases, the channel becomes less frequency-selective and when  $\alpha \rightarrow \infty$  dB it approaches a frequency-nonselective channel (single-path channel). The dependency of the achievable BER performance on  $\alpha$  is shown in Fig. 4 for  $K=32$  with MMSE-FDE. We have also measured the BER performance with  $L$  as a parameter for the uniform

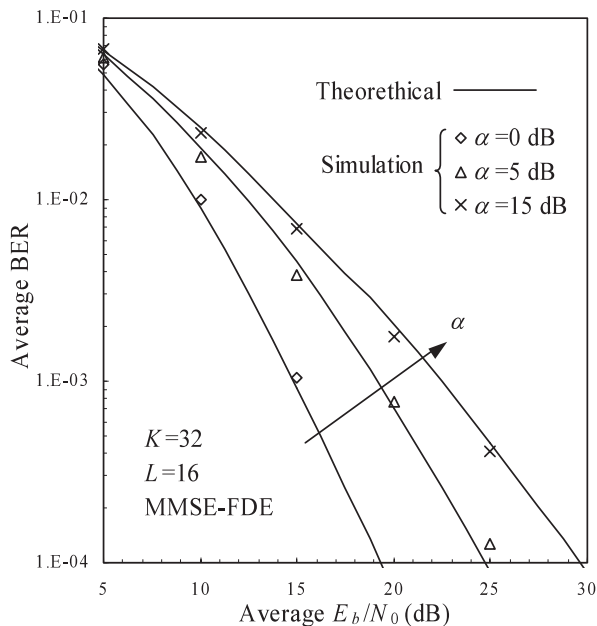


Fig. 4 Dependency of BER performance with MMSE equalization on channel frequency-selectivity:  $K = 32$ .

power delay profile case ( $\alpha = 1$  (or 0 dB)) and confirmed that if the delay spread is the same, the same BER performance is obtained as the  $L = 16$ -path exponential power delay profile case. Therefore, we only plot the BER performance dependency on  $\alpha$  in Fig. 4 for the case of  $L = 16$ -path exponential power delay profile. As was expected, as  $\alpha$  becomes larger, the performance consistently degrades due to less frequency diversity effect resulting from a weaker channel frequency-selectivity.

The computer-simulated average BERs are plotted in Figs. 3 and 4 to compare with theoretical ones. A fairly good agreement with theoretical and computer simulated results is seen. This confirms the validity of our BER analysis based on the Gaussian approximation of the ISI.

## 5. Conclusions

In this paper, theoretical foundation was developed for OFDM/TDM signal transmissions in a frequency-selective fading channel. Unlike the conventional OFDM, a sequence of OFDM signals with reduced number of subcarriers is time-multiplexed in a frame with a cyclic prefix at its beginning. At the receiver, FDE is applied to reduce the ISI resulting from the channel frequency-selectivity.

In the theoretical analysis, FDE schemes based on MRC, ZF and MMSE criteria were considered. The theoretical expression for conditional BER for the given set of path gains was derived based on the Gaussian approximation of the ISI. The numerical evaluation of the theoretical average BER performance was presented to show that the MMSE equalization provides the best BER performance among the three equalization schemes and the OFDM/TDM with MMSE-FDE can achieve a better BER performance

than the conventional OFDM. The BER performance of OFDM/TDM is bounded between the conventional OFDM (as upper bound) and the SC (as lower bound). This performance improvement is due to the frequency diversity effect achieved by the MMSE-FDE. The theoretical results were compared with the computer simulation results and a fairly good agreement between the two results was observed.

In this paper, we assumed an ideal linear power amplifier. Distortion and power efficiency loss due to high PAPR was not considered in this paper and it is left as a future work. When channel coding (e.g., turbo coding) is applied, there is a trade-off relationship between the frequency diversity gain and coding gain through  $K$ . Theoretical analysis of this trade-off is not easy and it is left as another future work.

## References

- [1] F. Adachi, "Wireless past and future-evolving mobile communications systems," *IEICE Trans. Fundamentals*, vol.E84-A, no.1, pp.55–60, Jan. 2001.
- [2] W.C. Jakes, Jr., ed., *Microwave mobile communications*, Wiley, New York, 1974.
- [3] J.G. Proakis, *Digital communications*, 2nd ed., McGraw-Hill, 1995.
- [4] R.D.J. van Nee, R. Prasad, and R. van Nee, *OFDM for wireless multimedia communications*, Artech House, 2000.
- [5] H. Gacanin, S. Takaoka, and F. Adachi, "Generalized OFDM with frequency-domain equalization," *IEICE Technical Report*, RCS2003-306, March 2004.
- [6] H. Gacanin, S. Takaoka, and F. Adachi, "Introduction of the OFDM/TDM for improving robustness against the severe frequency-selective fading channel," *IEICE Technical Report*, RCS2004-63, May 2004.
- [7] C.V. Sinn, J. Götze, and M. Haardt, "Common architectures for TD-CDMA and OFDM based mobile radio systems without the necessity of a cyclic prefix," *MS-SS Workshop, DLR, Oberpfaffenhofen*, pp.65–76, Germany, Sept. 2001.
- [8] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, vol.35, pp.126–144, Dec. 1997.
- [9] M. Helard, R. Le Gouable, J.-F. Helard, and J.-Y. Baudais, "Multicarrier CDMA techniques for future wideband wireless access," *Ann. Telecommun.*, vol.56, pp.260–274, 2001.
- [10] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol.40, pp.58–66, April 2002.
- [11] F. Adachi, T. Sao, and T. Itagaki, "Performance of multicode DS-CDMA using frequency domain equalization in a frequency selective fading channel," *Electron. Lett.*, vol.39, no.2, pp.239–241, Jan. 2003.
- [12] F. Adachi and T. Itagaki, "Frequency-domain rake combining for antenna diversity reception of DS-CDMA signals," *IEICE Trans. Commun.*, vol.E86-B, no.9, pp.2781–2784, Sept. 2003.
- [13] J.G. Proakis and D. Manolakis, *Digital signal processing*, 3rd ed., Prentice Hall, 1996.

## Appendix

The variance of ISI is given by

$$2\sigma_{ISI}^2 = E \left[ \left| \frac{1}{N_c} \sum_{n=0}^{N_c-1} S(n) \left\{ \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right\} \Psi(n; i, k) \right|^2 \right]$$



$$\begin{aligned}
&= \frac{1}{N_c^2} \sum_{n=0}^{N_c-1} \sum_{n'=0}^{N_c-1} E [S(n)S^*(n')] \\
&\times \left\{ \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right\} \left\{ \hat{H}(n') - \frac{1}{N_c} \sum_{m'=0}^{N_c-1} \hat{H}(m') \right\}^* \\
&\times \Psi(n; i, k) \Psi^*(n'; i, k). \quad (\text{A.1})
\end{aligned}$$

Since

$$\begin{aligned}
&E [S(n)S^*(n')] \\
&= \sum_{t=0}^{N_c-1} \sum_{t'=0}^{N_c-1} E [s(t)s^*(t')] \exp \left( -j2\pi \frac{tn - t'n'}{N_c} \right) \quad (\text{A.2})
\end{aligned}$$

and assuming  $kN_m \leq t, t' \leq (k+1)N_m - 1$ , we have

$$\begin{aligned}
&E [S(n)S^*(n')] \\
&= \frac{2E_s}{T_c} \frac{1}{N_m} \sum_{t=0}^{N_c-1} \sum_{t'=0}^{N_c-1} \sum_{i=0}^{N_m-1} \sum_{i'=0}^{N_m-1} E [d^k(i)d^{k*}(i')] \\
&\times \exp \left( j2\pi \frac{ti - t'i' - tn + t'n'}{N_c} \right). \quad (\text{A.3})
\end{aligned}$$

Different data symbols are independent and thus,  $E[d^k(i)d^{k*}(i')] = \delta(i - i')$ , resulting in

$$\begin{aligned}
&E [S(n)S^*(n')] \\
&= \sum_{t=0}^{N_c-1} \frac{2E_s}{T_c} \frac{1}{N_m} N_m \exp \left( -j2\pi t \frac{n - n'}{N_c} \right) \\
&= \frac{2E_s}{T_c} N_c \delta(n - n'). \quad (\text{A.4})
\end{aligned}$$

Therefore, we have

$$2\sigma_{ISI}^2 = \frac{2E_s}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} \left| \hat{H}(n) - \frac{1}{N_c} \sum_{n=0}^{N_c-1} \hat{H}(n) \right|^2 |\Psi(n; i, k)|^2, \quad (\text{A.5})$$

where

$$|\Psi(n; i, k)|^2 = \left| \frac{\sin \left( \pi N_m \frac{n - Ki}{N_c} \right)}{N_m \sin \left( \pi \frac{n - Ki}{N_c} \right)} \right|^2 \leq 1. \quad (\text{A.6})$$

The AWGN noise variance is given by

$$\begin{aligned}
2\sigma_{AWGN}^2 &= \frac{1}{N_c^2} \sum_{n=0}^{N_c-1} \sum_{n'=0}^{N_c-1} E [\Pi(n)\Pi^*(n')] \\
&\cdot w(n)w^*(n') \Psi(n; i, k) \Psi^*(n'; i, k). \quad (\text{A.7})
\end{aligned}$$

Since

$$E [\Pi(n)\Pi^*(n')] = \frac{2N_0}{T_c} N_c \delta(n - n'), \quad (\text{A.8})$$

we have

$$2\sigma_{AWGN}^2 = \frac{2N_0}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} |w(n)|^2 |\Psi(n; i, k)|^2. \quad (\text{A.9})$$



**Haris Gacanin** received his B.Sc., and M.E., degrees in electrical and communication engineering from Faculty of Electrical Engineering, University of Sarajevo, Bosnia and Herzegovina, in 2000 and Graduate School of Electrical Engineering, Tohoku University, Japan, in 2005, respectively. Currently, he is working toward his Ph.D. degree at Department of Electrical and Communications Engineering, Tohoku University. His research interests include wireless broadband communication systems,

OFDM, hybrid combination of OFDM and TDM, channel estimation and equalization, PAPR reduction, turbo coding, HARQ, and transmit/receive antenna diversity techniques. He is a recipient of the IEICE Communications Society Young Researcher Award of the year 2005 and the VTC 2005-Fall Student paper Award from the IEEE Japan chapter of the IEEE Vehicular Technology Society for the year 2005.



**Shinsuke Takaoka** received his B.S. and M.S. degrees in communications engineering from Tohoku University, Sendai, Japan, in 2001 and 2003, respectively. Currently, he is a graduate student at the Department of Electrical and Communications Engineering, Tohoku University. His research interests include digital signal transmission techniques, especially for mobile communication systems.



**Fumiyuki Adachi** received his B.S. and Dr. Eng. Degrees in electrical engineering from Tohoku University, Sendai, Japan, in 1973 and 1984, respectively. In April 1973, he joined the Electrical Communications Laboratories of Nippon Telegraph & Telephone Corporation (now NTT) and conducted various types of research related to digital cellular mobile communications. From July 1992 to December 1999, he was with NTT Mobile Communications Network, Inc. (now NTT DoCoMo, Inc.), where he

led a research group on wideband/broadband CDMA wireless access for IMT-2000 and beyond. Since January 2000, he has been with Tohoku University, Sendai, Japan, where he is a Professor of Electrical and Communication Engineering at Graduate School of Engineering. His research interests are in CDMA and TDMA wireless access techniques, CDMA spreading code design, Rake receiver, transmit/receive antenna diversity, adaptive antenna array, bandwidth-efficient digital modulation, and channel coding, with particular application to broadband wireless communication systems. From October 1984 to September 1985, he was a United Kingdom SERC Visiting Research Fellow in the Department of Electrical Engineering and Electronics at Liverpool University. He was a co-recipient of the IEICE Transactions best paper of the year award 1996 and again 1998. He is an IEEE Fellow and was a co-recipient of the IEEE Vehicular Technology Transactions best paper of the year award 1980 and again 1990 and also a recipient of Avant Garde award 2000.