

LETTER

A Study on Spatial MMSE Despreading for OSTSTD in a Fast Fading Channel

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SUMMARY Spatial despreading weight based on minimum mean square error (MMSE) criterion is derived for orthogonal space-time spreading transmit diversity (OSTSTD) in a fast fading channel, taking into account the inter-antenna interference (IAI) and the inter-code interference (ICI) caused by orthogonality distortion of time-domain spreading codes. Average bit error rate (BER) performance is theoretically analyzed and confirmed by computer simulation to show that the diversity gain can be obtained even in a fast fading.

key words: OSTSTD, spatial MMSE despreading, fast fading

1. Introduction

In mobile radio, the transmitted signal is affected by multipath fading and thus, degrading the transmission performance [1]. Transmit antenna diversity is an effective technique for reducing the adverse effect of multipath fading [2], [3]. Recently, we have proposed orthogonal space-time spreading transmit diversity (OSTSTD) [4]. OSTSTD uses two transmit antennas and 2-dimensional (space and time) orthogonal spreading combined with delay transmission. In [4], the spatial minimum mean square error (MMSE) weight under the assumption of slow fading is used and thus, the bit error rate (BER) performance significantly degrades in fast fading. This is because inter-antenna interference (IAI) and inter-code interference (ICI) is produced by the orthogonality distortion of time-domain spreading codes due to the fast fading. In this letter, spatial MMSE despreading weight is derived taking into account the IAI and ICI to improve the BER performance in fast fading. The average BER performance is theoretically analyzed and confirmed by computer simulation.

The rest of the letter is organized as follows. Section 2 briefly introduces OSTSTD transmission system model. In Sect. 3, spatial MMSE despreading weight that can suppress the IAI and ICI is derived. The conditional BER for the given fading channel gains is derived in Sect. 4. Section 5 evaluates the achievable BER performance in a Rayleigh fading channel. Section 6 offers the conclusion.

2. OSTSTD Transmission System Model

We consider the OSTSTD with two transmit antennas and

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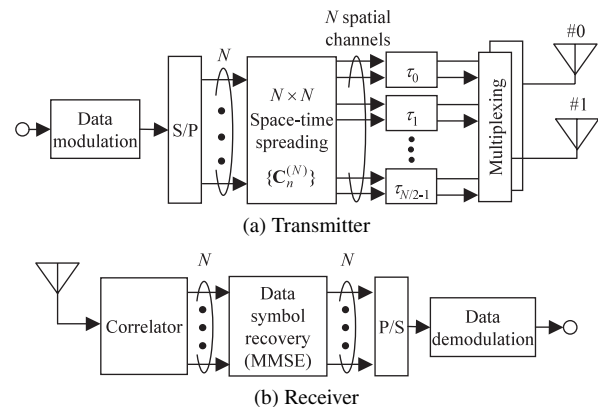


Fig. 1 Transmission system model of OSTSTD.

one receive antenna as illustrated in Fig. 1. But, the application to the OSTSTD with multiple receive antennas [4] is easily extended. In OSTSTD, N data-modulated symbols $\{d_n(t); n = 0 - (N - 1)\}$ during a period of NT_s are transmitted in parallel, where $1/T_s$ is the transmit symbol rate. In the following, symbol (T_s)-spaced discrete time representation of the space-time spread signal is used.

At a transmitter, the data-modulated symbol sequence is transformed into N parallel data symbol sequences with symbol rate $1/(NT_s)$. Each symbol sequence is spread using 2-dimensional orthogonal space-time spreading code $C_n^{(N)}$, $n = 0 - (N - 1)$, with chip rate $1/T_s$ (since the chip rate of the space-time spreading code is $1/T_s$ and equals the transmit data symbol rate, there is no bandwidth expansion in OSTSTD [4]). The element $c_n^{(N)}(m, t) = \pm 1$ of $C_n^{(N)}$ at the m -th row and t -th column is expressed as [4], $c_n^{(N)}(m, t) = h^{(N)}(m, t)h^{(N)}(n, m)$, where $h^{(N)}(m, t) = \pm 1$ is the element of the $N \times N$ -Hadamard matrix $H^{(N)}$ at the m -th row and t -th column. $h^{(N)}(m, t)$ (or $h^{(N)}(n, m)$) is used for orthogonal temporal (or spatial)-spreading of the transmitted data. The resultant N space-time spread signals $s(m, t)$, $m = 0 - (N - 1)$, are multiplexed to produce N spatial channels. Then, the space-time spread signals $s(m, t)$'s, with $m=2i$ and $2i+1$ to be transmitted, are added the same time delay $\tau_i = iDN$ for delay transmission with D denoting the time delay separation and i being an integer [4], where $s(m, t)$ is given by

$$s(m, t) = \sqrt{2P/N^2} \sum_{n=0}^{N-1} d_n(\lfloor t/N \rfloor) c_n^{(N)}(m, t \bmod N) \quad (1)$$

for $m = 0 - (N - 1)$, where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x and P is the total transmit

power. Finally, $N/2$ space-time spread signals with $m=2i$, $i = 0-(N/2 - 1)$, are combined, and those with $m=2i+1$, $i = 0-(N/2 - 1)$, are also combined to produce two spread signal sequences, $\bar{s}_0(t)$ and $\bar{s}_1(t)$, which are to be transmitted from two transmit antennas. $\bar{s}_{m_i}(t)$ is given by

$$\bar{s}_{m_i}(t) = \sum_{i=0}^{N/2-1} s(2i + m_i, t - \tau_i) \quad \text{for } m_i = 0 \text{ and } 1. \quad (2)$$

The space-time spread signals transmitted over frequency non-selective fading channels are received by one receive antenna. The received signal at time t can be represented as

$$r(t) = \sum_{m_i=0}^1 \xi_{m_i}(t) \bar{s}_{m_i}(t) + \eta(t), \quad (3)$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) with zero mean and a variance of $2N_0/T_s$ with N_0 being the AWGN power spectrum density. The complex-valued propagation channel gain between the m_i -th transmit antenna and the receive antenna is represented by $\xi_{m_i}(t)$ with the ensemble average of $|\xi_{m_i}(t)|^2$ being unity for all m_i ($m_i = 0-1$).

Space-time despreading of N transmitted data symbols consists of two steps [4]. The first step is the time-domain despreading to recover the N spatial channels $\{\tilde{r}(m, k); m = 0-(N-1)\}$ by taking the correlation between the received signal $r(t)$ and N time-domain spreading codes $\{h^{(N)}(m, t); m = 0-(N-1)\}$. From Eq. (3), we have

$$\tilde{r}(m, k) = \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} r(t' + \tau_{\lfloor m/2 \rfloor}) h^{(N)}(m, t' \bmod N) \quad (4)$$

for $m = 0-(N-1)$, where $k = \lfloor t/N \rfloor$.

The second step is the spatial MMSE despreading. N transmitted data symbols are spread over the N spatial channels using the spatial-domain spreading codes $\{h^{(N)}(n, m); n = 0-(N-1)\}$. The second step yields the decision variable $\{\tilde{d}_n(k); n = 0-(N-1)\}$ for the recovery of the n -th transmitted data symbol. $\tilde{d}_n(k)$ is given as

$$\tilde{d}_n(k) = \frac{1}{N} \sum_{m=0}^{N-1} \hat{r}(m, k) h^{(N)}(n, m), \quad (5)$$

where

$$\hat{r}(m, k) = w(m, k) \tilde{r}(m, k) \quad (6)$$

with $w(m, k)$, $m = 0-(N-1)$, denoting the spatial MMSE despreading weight which will be derived in Sect. 3. Finally, the data demodulation is done using $\{\tilde{d}_n(k); n = 0-(N-1)\}$.

3. Spatial MMSE Despreading Weight

3.1 Interference Modeling

Time-domain despreading output $\tilde{r}(m, k)$ of Eq. (4) can be rewritten as

$$\tilde{r}(m, k) = D(m, k) + I_{ICI}(m, k) + I_{IAI}(m, k) + Z(m, k) \quad (7)$$

for $m = 0-(N-1)$. In Eq. (7), $D(m, k)$ and $Z(m, k)$ respectively denote the desired signal and the noise components and $I_{ICI}(m, k)$ and $I_{IAI}(m, k)$ respectively represent the ICI and IAI components from the same (or different) antenna. They are given in Appendix A. For a fast fading, $I_{ICI}(m, k)$ and $I_{IAI}(m, k)$ are produced by orthogonality distortion of time-domain spreading codes $\{h^{(N)}(m, t); m = 0-(N-1)\}$ and degrades the BER performance.

We treat the sum of $I_{ICI}(m, k)$, $I_{IAI}(m, k)$ and $Z(m, k)$ as the equivalent noise $\beta(m, k)$. The time-domain despreading output is rewritten as

$$\tilde{r}(m, k) = D(m, k) + \beta(m, k). \quad (8)$$

Assuming a random transmit data symbol sequence, $I_{ICI}(m, k)$, $I_{IAI}(m, k)$ and $Z(m, k)$ are independent random variables. Thus, the variance of $\beta(m, k)$ can be given by

$$2\sigma_{\beta}^2(m, k) = 2\sigma_{ICI}^2(m, k) + 2\sigma_{IAI}^2(m, k) + 2\sigma_Z^2(m, k), \quad (9)$$

where $2\sigma_{ICI}^2(m, k)$, $2\sigma_{IAI}^2(m, k)$ and $2\sigma_Z^2(m, k)$ represent the variances of $I_{ICI}(m, k)$, $I_{IAI}(m, k)$ and $Z(m, k)$, respectively, which are given in Appendix B.

3.2 Spatial MMSE Despreading Weight

We derive the weight that minimizes the mean square error $E[|e(m, k)|^2]$ between $\hat{r}(m, k)$ and the reference signal, where

$$e(m, k) = \sqrt{2P/N^2} \sum_{n=0}^{N-1} d_n(k) h^{(n)}(n, m) - \hat{r}(m, k) \quad (10)$$

for $m = 0-(N-1)$. In Eq. (10), $\sqrt{2P/N^2} \sum_{n=0}^{N-1} d_n(k) h^{(n)}(n, m)$ is taken as the reference signal, since it is the desired signal component after time-domain despreading (see Eq. (A-1)). From the orthogonal principle [5], the minimization of $E[|e(m, k)|^2]$ results in

$$E[e(m, k) \tilde{r}^*(m, k)] = 0 \quad \text{for } m = 0-(N-1), \quad (11)$$

where $*$ stands for the complex conjugate operation. Using Eqs. (6), (8), (10) and (11), the spatial MMSE despreading weight is derived as

$$w(m, k) = \frac{\tilde{\xi}_{m \bmod 2}^* (\lfloor m/2 \rfloor DN)}{|\tilde{\xi}_{m \bmod 2} (\lfloor m/2 \rfloor DN)|^2 + [(E_s/T_s)/N\sigma_{\beta}^2(m, k)]^{-1}} \quad (12)$$

for $m = 0-(N-1)$, where $E_s (=PT_s)$ is the average transmit symbol energy. In a slow fading, both $\sigma_{ICI}^2(m, k)$ and $\sigma_{IAI}^2(m, k)$ become zero. Hence, the derived weight of Eq. (12) reduces to the previously used weight in [4], given by

$$w(m, k) = \frac{\tilde{\xi}_{m \bmod 2}^* (\lfloor m/2 \rfloor DN)}{|\tilde{\xi}_{m \bmod 2} (\lfloor m/2 \rfloor DN)|^2 + (E_s/N_0)^{-1}}. \quad (13)$$

4. BER Analysis

The conditional BER is derived for a given set of the fading channel gains $\{\xi_{m_i}(t); m_i = 0, 1\}$. Substituting Eq. (8) into Eq. (6) and then using Eq. (5), The decision variable $\tilde{d}_n(k)$ can be expressed as

$$\tilde{d}_n(k) = \frac{\sqrt{2P}}{N^3} \sum_{m=0}^{N-1} \hat{\Phi}(m, k) d_n(k) + \mu_I + \mu_N, \quad (14)$$

where the first and third terms represent the desired signal and the equivalent noise components, respectively. The second term is the ICI due to the orthogonality distortion of *spatial-domain* spreading codes $\{h^{(N)}(n, m); n = 0 - (N-1)\}$ (note that μ_I is different from $I_{CI}(m, k)$ produced by the *time-domain* deprecoding). μ_I and μ_N are given by

$$\begin{cases} \mu_I = \frac{\sqrt{2P}}{N^3} \sum_{m=0}^{N-1} \hat{\Phi}(m, k) \sum_{\substack{n=0 \\ n \neq n'}}^{N-1} d_n(k) h^{(N)}(n, m) h^{(N)}(n', m) \\ \mu_N = \frac{1}{N} \sum_{m=0}^{N-1} \hat{\Pi}(m, k) h^{(N)}(n, m) \end{cases} \quad (15)$$

with

$$\begin{cases} \hat{\Phi}(m, k) = w(m, k) \sum_{t'=kN}^{(k+1)N-1} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ \hat{\Pi}(m, k) = w(m, k) \beta(m, k) \end{cases} \quad (16)$$

Approximating μ_I as a zero-mean complex-valued Gaussian process, the sum of μ_I and μ_N can be treated as a new zero-mean complex-valued Gaussian noise μ . After some manipulations, the variance of μ can be obtained as

$$\begin{aligned} 2\sigma_\mu^2 &= \left(\frac{2E_s}{T_s}\right) \frac{(N-1)}{N^4} \left(\frac{1}{N} \sum_{m=0}^{N-1} |\hat{\Phi}(m, k)|^2 - |\bar{\Phi}(k)|^2 \right) \\ &+ \frac{1}{N^2} \sum_{m=0}^{N-1} |w(m, k)|^2 (2\sigma_\beta^2(m, k)) \end{aligned} \quad (17)$$

for the given set of $\{\xi_{m_i}(t)\}$, where

$$\bar{\Phi}(k) = \frac{1}{N} \sum_{m=0}^{N-1} \hat{\Phi}(m, k). \quad (18)$$

The decision variable $\tilde{d}_n(k)$ is a complex-valued random variable with mean $\left(\sqrt{2P}/N^3\right) \sum_{m=0}^{N-1} \hat{\Phi}(m, k) d_n(k)$ and the variance $2\sigma_\mu^2$.

We assume quaternary phase shift keying (QPSK) data-modulation and all "1's" transmission without loss of generality. Since μ_I can be assumed to be circularly symmetric, the conditional BER for the given set of $\{\xi_{m_i}(t)\}$ can be given by

$$p_b(E_s/N_0, \{\xi_{m_i}(t)\}) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma(E_s/N_0, \{\xi_{m_i}(t)\})/4}\right), \quad (19)$$

where $\operatorname{erfc}[x] = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$ is the complementary error function and $\gamma(E_s/N_0, \{\xi_{m_i}(t)\})$ is the conditional SINR, which is given, using Eqs. (14), (15), (17), (A·5) and (A·6) by

$$\begin{aligned} &\gamma(E_s/N_0, \{\xi_{m_i}(t)\}) \\ &= \frac{\left(\frac{E_s}{N_0}\right) \frac{2}{N} \left| \frac{1}{N} \sum_{m=0}^{N-1} \hat{\Phi}(m, k) \right|^2}{\left(\frac{E_s}{N_0}\right) \frac{1}{N} \sum_{m=0}^{N-1} |w(m, k)|^2} \\ &\quad \times \left[\left(\frac{N}{2} - 1\right) \left(\frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} |\xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN)|^2 \right. \right. \\ &\quad \left. \left. - |\bar{\xi}_{m \bmod 2}(\lfloor m/2 \rfloor DN)|^2 \right) \right. \\ &\quad \left. + \left(\frac{N}{2}\right) \left(\frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} |\xi_{(m+1) \bmod 2}(t' + \lfloor m/2 \rfloor DN)|^2 \right) \right. \\ &\quad \left. - |\bar{\xi}_{(m+1) \bmod 2}(\lfloor m/2 \rfloor DN)|^2 \right) \\ &\quad + \left(\frac{E_s}{N_0}\right) \frac{N-1}{N^2} \left(\frac{1}{N} \sum_{m=0}^{N-1} |\hat{\Phi}(m, k)|^2 - |\bar{\Phi}(k)|^2 \right) + \sum_{m=0}^{N-1} |w(m, k)|^2 \end{aligned} \quad (20)$$

The theoretical average BER can be numerically evaluated by averaging Eq. (20) over $\{\xi_{m_i}(t); m_i = 0, 1\}$:

$$\begin{aligned} p_b(E_s/N_0) &= \operatorname{ave}_{\{\xi_{m_i}(t)\}} \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma(E_s/N_0, \{\xi_{m_i}(t)\})/4}\right). \end{aligned} \quad (21)$$

5. Numerical and Simulation Results

Table 1 summarizes the numerical and simulation conditions. Propagation channels between each transmit antenna and the receive antenna are assumed to be characterized by independent frequency-nonselctive Rayleigh fading processes.

Following the same procedure as in [6], the numerical result of the theoretical average BER performance with the spatial MMSE weight of Eq. (12) is obtained by Monte-Carlo numerical computation method. For comparison, the theoretical average BER using Eq. (13) instead of Eq. (12) is also obtained.

Figure 2 compares the theoretical average BER performances achievable with OSTSTD using spatial MMSE

Table 1 Numerical and simulation conditions.

Data modulation	QPSK
Orthogonal spreading codes	Walsh-Hadamard codes
Space-time spreading code length	$N=32$
Time delay separation	$D=20$
No. of transmit antennas	2
Propagation channel model	Frequency non-selective Rayleigh fading
No. of receive antenna	1

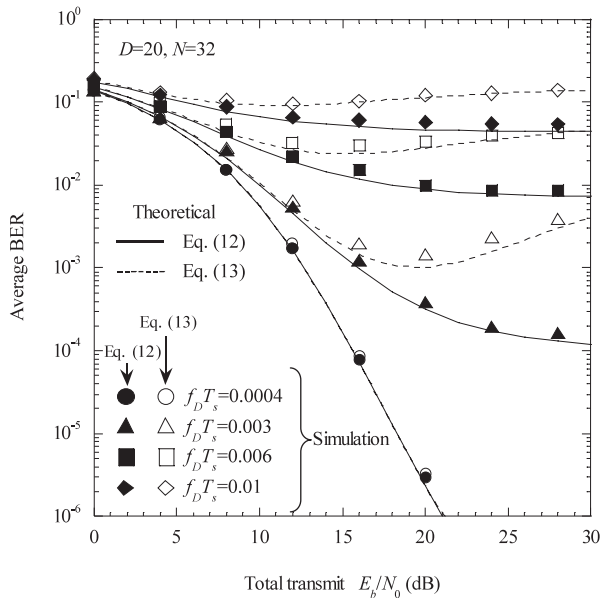


Fig. 2 BER performance.

despreading weight of Eqs. (12) and (13) as a function of the total transmit $E_b/N_0 (=0.5(E_s/N_0))$ with the normalized maximum Doppler frequency $f_D T_s$ as a parameter, where f_D denotes the maximum Doppler frequency. For comparison, results for the simulation of signal transmission using OSTSTD are also plotted. The theoretical and simulated results are in a good agreement. This confirms the propriety of the theoretical analysis.

It is seen from Fig. 2 that spatial MMSE despreading weight of Eq. (12) provides a better BER performance in a high E_b/N_0 region, where the predominant cause of the decision error is the ICI and IAI. On the other hand, the use of the spatial MMSE weight of Eq. (13), which is derived assuming slow fading (i.e., the orthogonality destruction of the time-domain spreading codes is not considered), degrades the BER performance and its BER floor increases as the total transmit E_b/N_0 increases beyond a certain value. A possible reason for this is explained as follows. The time-domain despreading output is the sum of the desired signal, ICI, IAI and noise components as shown in Eq. (7). The spatial MMSE weight needs to be derived by taking into account the ICI and IAI as well as the noise. However, the conventional MMSE weight of Eq. (13) is derived assuming that ICI and IAI components are negligible and thus, it is not optimal in the MMSE sense in a fast fading channel. As a consequence, the use of the conventional MMSE weight increases the residual ICI and IAI after the spatial despreading in a high E_s/N_0 region (where the ICI and IAI are a dominant cause of decision error rather than the AWGN) and thus, the BER floor increases.

It is seen from Fig. 2 that as the fading rate becomes faster, the BER performance of OSTSTD degrades due to the increase of the residual ICI and IAI powers and BER = 10^{-3} cannot be maintained when $f_D T_s > 0.003$ even if Eq. (12) is used as the spatial MMSE weight. Therefore, the proposed spatial MMSE despreading weight of Eq. (12)

is effective when $f_D T_s < 0.003$ (which corresponds to a vehicular speed of less than 100 km/h, assuming a carrier frequency of 2 GHz and a transmission symbol rate of 64 kps).

6. Conclusion

In this letter, spatial MMSE despreading weight for OSTSTD was derived, taking into account the interference due to the orthogonality distortion of time-domain spreading codes. The average BER performance was theoretically evaluated and confirmed by computer simulation. It was found that the derived spatial MMSE weight can suppress the interference and hence, the BER performance can be improved in a fast fading.

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Appendix A

$D(m, k)$, $I_{ICI}(m, k)$, $I_{IAI}(m, k)$ and $Z(m, k)$ are respectively represented as

$$\left\{ \begin{array}{l} D(m, k) = \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ \quad \cdot \left(\sqrt{\frac{2P}{N^2}} \sum_{n=0}^{N-1} d_n(k) h^{(N)}(n, m) \right) \\ I_{ICI}(m, k) = \frac{1}{N} \sum_{\substack{i=0 \\ \neq \lfloor m/2 \rfloor}}^{N/2-1} \rho_{ICI,i}(m, k) \sqrt{\frac{2P}{N^2}} \\ \quad \cdot \sum_{n=0}^{N-1} \left(d_n(k + (\lfloor m/2 \rfloor - i)D) \right. \\ \quad \left. \times h^{(N)}(n, 2i + m \bmod 2) \right) \\ I_{IAI}(m, k) = \frac{1}{N} \sum_{i=0}^{N/2-1} \rho_{IAI,i}(m, k) \sqrt{\frac{2P}{N^2}} \\ \quad \cdot \sum_{n=0}^{N-1} \left(d_n(k + (\lfloor m/2 \rfloor - i)D) \right. \\ \quad \left. \times h^{(N)}(n, 2i + (m+1) \bmod 2) \right) \\ Z(m, k) = \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} \eta(t' + \lfloor m/2 \rfloor DN) h^{(N)}(m, t' \bmod N) \end{array} \right. \quad (A.1)$$

where $\rho_{ICI,i}(m, k)$ and $\rho_{IAI,i}(m, k)$ are given by

$$\begin{cases} \rho_{ICI,i}(m, k) = \sum_{t'=kN}^{(k+1)N-1} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ \quad \times h^{(N)}(2i + m \bmod 2, t' \bmod N) h^{(N)}(m, t' \bmod N) \\ \rho_{IAI,i}(m, k) = \sum_{t'=kN}^{(k+1)N-1} \xi_{(m+1) \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ \quad \times h^{(N)}(2i + (m + 1) \bmod 2, t' \bmod N) \\ \quad \cdot h^{(N)}(m, t' \bmod N) \end{cases} \quad (\text{A} \cdot 2)$$

Appendix B

Using $(1/N) \sum_{t=0}^{N-1} h^{(N)}(m, t) h^{(N)}(m', t) = 0$ for $m \neq m'$, $I_{ICI}(m, k)$ and $I_{IAI}(m, k)$ of Eq. (A.1) can be respectively rewritten as

$$\begin{cases} I_{ICI}(m, k) = \frac{1}{N} \sum_{\substack{i=0 \\ \neq \lfloor m/2 \rfloor}}^{N/2-1} \sum_{t'=kN}^{(k+1)N-1} \begin{pmatrix} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ -\bar{\xi}_{m \bmod 2}(\lfloor m/2 \rfloor DN) \end{pmatrix} \\ \quad \times h^{(N)}(2i + m \bmod 2, t' \bmod N) h^{(N)}(m, t' \bmod N) \\ \quad \times \sqrt{\frac{2P}{N^2}} \sum_{n=0}^{N-1} \begin{pmatrix} d_n(k + (\lfloor m/2 \rfloor - i)D) \\ \times h^{(N)}(n, 2i + m \bmod 2) \end{pmatrix} \\ I_{IAI}(m, k) = \frac{1}{N} \sum_{i=0}^{N/2-1} \sum_{t'=kN}^{(k+1)N-1} \begin{pmatrix} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ -\bar{\xi}_{(m+1) \bmod 2}(\lfloor m/2 \rfloor DN) \end{pmatrix} \\ \quad \times h^{(N)}(2i + (m + 1) \bmod 2, t' \bmod N) h^{(N)}(m, t' \bmod N) \\ \quad \times \sqrt{\frac{2P}{N^2}} \sum_{n=0}^{N-1} \begin{pmatrix} d_n(k + (\lfloor m/2 \rfloor - i)D) \\ \times h^{(N)}(n, 2i + (m + 1) \bmod 2) \end{pmatrix} \end{cases} \quad (\text{A} \cdot 3)$$

where $\bar{\xi}_{m \bmod 2}(\lfloor m/2 \rfloor DN)$ and $\bar{\xi}_{(m+1) \bmod 2}(\lfloor m/2 \rfloor DN)$ are defined as

$$\begin{cases} \bar{\xi}_{m \bmod 2}(\lfloor m/2 \rfloor DN) \\ = \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} \xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN) \\ \bar{\xi}_{(m+1) \bmod 2}(\lfloor m/2 \rfloor DN) \\ = \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} \xi_{(m+1) \bmod 2}(t' + \lfloor m/2 \rfloor DN) \end{cases} \quad (\text{A} \cdot 4)$$

Using $E[d_n(k) d_n^{*}(k')] = 1(0)$ if $k = k'$ and $n = n'$ (elsewhere) and the law of large numbers [7], the variances of $I_{ICI}(m, k)$ and $I_{IAI}(m, k)$ can be approximately obtained as

$$\begin{cases} 2\sigma_{ICI}^2(m, k) = E[|I_{ICI}(m, k)|^2] \approx \left(\frac{2E_s}{T_s}\right) \left(\frac{1}{N^2}\right) \left(\frac{N}{2} - 1\right) \\ \quad \cdot \left(\begin{matrix} \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} |\xi_{m \bmod 2}(t' + \lfloor m/2 \rfloor DN)|^2 \\ - |\bar{\xi}_{m \bmod 2}(\lfloor m/2 \rfloor DN)|^2 \end{matrix} \right) \\ 2\sigma_{IAI}^2(m, k) = E[|I_{IAI}(m, k)|^2] \approx \left(\frac{2E_s}{T_s}\right) \left(\frac{1}{N^2}\right) \left(\frac{N}{2}\right) \\ \quad \cdot \left(\begin{matrix} \frac{1}{N} \sum_{t'=kN}^{(k+1)N-1} |\xi_{(m+1) \bmod 2}(t' + \lfloor m/2 \rfloor DN)|^2 \\ - |\bar{\xi}_{(m+1) \bmod 2}(\lfloor m/2 \rfloor DN)|^2 \end{matrix} \right) \end{cases} \quad (\text{A} \cdot 5)$$

Since $\eta(t)$ is zero-mean independent complex-valued Gaussian variable having a variance of $2N_0/T_s$, the variance of $Z(m, k)$ can be obtained as

$$2\sigma_Z^2(m, k) = E[|Z(m, k)|^2] = \frac{1}{N} \left(2 \frac{N_0}{T_s}\right). \quad (\text{A} \cdot 6)$$