

PAPER

Decision Feedback Chip-Level Maximum Likelihood Detection for DS-CDMA in a Frequency-Selective Fading Channel

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SUMMARY In direct sequence code division multiple access (DS-CDMA), variable rate transmission can be realized by simply changing the spreading factor SF for the given chip rate. In a frequency-selective fading channel, the transmission performance can be improved by using rake combining. However, when a very low SF is used for achieving a high transmission rate, error floor is produced due to insufficient suppression of inter-chip interference (ICI). In this paper, decision feedback chip-level maximum likelihood detection (DF-CMLD) is proposed that can suppress the ICI. An upper-bound for the conditional bit error rate (BER) is theoretically derived for the given spreading sequence and path gains. The theoretical average BER performance is numerically evaluated by Monte-Carlo numerical computation using the derived conditional BER. The numerical computation results are confirmed by computer simulation of DS-CDMA signal transmission with DF-CMLD.

key words: DS-CDMA, frequency-selective fading channel, maximum likelihood detection, rake combining

1. Introduction

In the next generation mobile communications systems, high-speed and high-quality data transmission is required [1], [2]. Wideband mobile radio channels are characterized by a frequency-selective fading channel that consists of a number of propagation paths having different time delays. In the third generation (3G) mobile communications systems, the direct sequence code division multiple access (DS-CDMA) technique is adopted [3] to exploit the channel frequency-selectivity and then improve the bit error rate (BER) performance by the use of rake combining [4]–[6]. In DS-CDMA, variable rate transmission can be realized by simply changing the spreading factor SF for the given chip rate. However, when a very low SF is used for achieving a high transmission rate, error floor is produced due to insufficient suppression of inter-chip interference (ICI). In this paper, decision feedback chip-level maximum likelihood detection (DF-CMLD) is proposed to suppress the ICI and improve the BER performance. In DF-CMLD, the ICI from the past data symbols is removed by feeding back the past decisions and the MLD is performed. The objectives of this paper are two-fold; one is to show that the proposed DF-CMLD can suppress the ICI effectively even for a low spreading factor (e.g., $SF=4$) and hence it can offer a better BER performance than rake combining, and the other is to

give a theoretical foundation of DF-CMLD. We also show the BER performance comparison between DF-CMLD and rake combining.

The remainder of this paper is organized as follows. Section 2 proposes DF-CMLD. Section 3 presents the theoretical analysis of the conditional BER with DF-CMLD for the given spreading sequence and path gains. In Sect. 4, the theoretical average BER is numerically evaluated by Monte-Carlo numerical computation using the derived conditional BER to compare with rake combining. Section 5 concludes the paper.

2. DF-CMLD

The transmission system model with DF-CMLD and antenna diversity reception is illustrated in Fig. 1. First, the mathematical expressions for the transmit and receive signals are given and then DF-CMLD is described.

2.1 Transmit and Receive Signal Representation

At the transmitter, a binary transmit data sequence is transformed into a data-modulated symbol sequence $d(k)$ and then multiplied by a long random spreading sequence $\{c(n); n = \dots -1, 0, 1, \dots\}$ having a chip duration T_c . The transmitted spread signal $s(t)$ can be expressed using the equivalent lowpass representation as

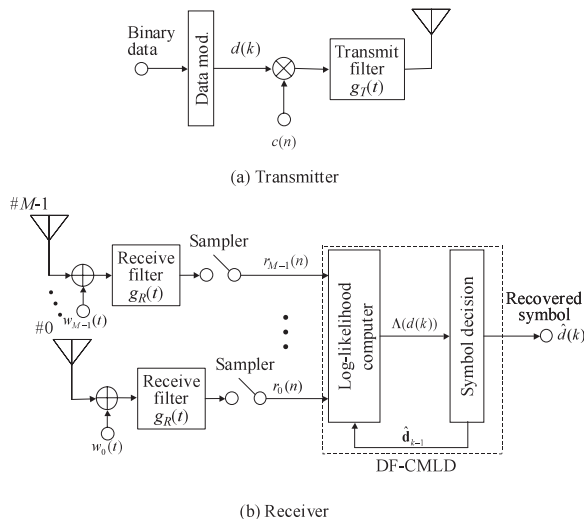


Fig. 1 Transmission system model.

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$$s(t) = \sqrt{2S} \sum_{n=-\infty}^{\infty} d[\lfloor n/SF \rfloor] c(n) g_T(t - nT_c), \quad (1)$$

where S is the transmit power, SF is the spreading factor and $\lfloor x \rfloor$ is the largest integer smaller than or equal to x . $g_T(t)$ is the transmit filter impulse response. The transmitted signal is received via a frequency-selective fading channel. M -branch antenna diversity reception is considered. Assuming that the channel has L independent propagation paths with T_c -spaced distinct time delays $\{\tau_l; l = 0 \sim L-1\}$, the discrete-time impulse response $h_m(t)$ of the channel seen at the m th antenna can be expressed as [7]

$$h_m(t) = \sum_{l=0}^{L-1} h_{m,l} \delta(t - \tau_l), \quad (2)$$

where $h_{m,l}$ is the l th path gain with $\sum_{l=0}^{L-1} E[|h_{m,l}|^2] = 1$. The maximum time delay among the multipaths is assumed to be less than one symbol duration $SF \cdot T_c$. We assume also a block fading where the path gains remain constant over one symbol interval of SF chips. Time dependency of the channel has been dropped for simplicity.

The received signal is passed through a receive filter with an impulse response $g_R(t)$ and sampled at time instant $t = nT_c$. This paper assumes that the overall (of the transmit/receive) filter response $g(t) = g_T(t) \otimes g_R(t)$, where \otimes denotes the convolution operation, satisfies the Nyquist condition, and that the receiver chip timing is perfectly locked at $t = nT_c$. Thus, the received signal sample $r_m(n)$ on the m th antenna is given by

$$r_m(n) = \sqrt{2S} \sum_{l=0}^{L-1} h_{m,l} d\left(\left\lfloor \frac{n-l}{SF} \right\rfloor\right) c(n-l) + w_m(n), \quad (3)$$

where $w_m(n)$ is the additive white Gaussian noise (AWGN) process with a zero mean and a variance of $2N_0/T_c$ with N_0 being the single-sided AWGN power spectrum density.

2.2 DF-CMLD

The k th symbol is to be detected. For the given transmitted symbol sequence $\mathbf{d} = \{d(0), d(1), \dots, d(k-1), d(k)\}$, the conditional joint probability density function (PDF) $p(\mathbf{r}|\mathbf{d})$ of the received signal sequence $\mathbf{r} = \{r_m(n); m = 0 \sim M-1$ and $n = 0 \sim (k+1)SF-1\}$ can be expressed as

$$p(\mathbf{r}|\mathbf{d}) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^{2(k+1)M \cdot SF} \times \exp \left[-\frac{1}{2\sigma^2} \sum_{m=0}^{M-1} \sum_{n=0}^{(k+1)SF-1} \left| r_m(n) - \sqrt{2S} \sum_{l=0}^{L-1} h_{m,l} d\left(\left\lfloor \frac{n-l}{SF} \right\rfloor\right) c(n-l) \right|^2 \right]. \quad (4)$$

Using Bayes theorem [5], the *a posteriori* probability $p(\mathbf{d}|\mathbf{r})$ is given by

$$p(\mathbf{d}|\mathbf{r}) = \frac{p(\mathbf{d})}{p(\mathbf{r})} p(\mathbf{r}|\mathbf{d}), \quad (5)$$

where $p(\mathbf{d})$ is the probability that the transmitted sequence is \mathbf{d} and $p(\mathbf{r})$ is the joint PDF of $\mathbf{r} = \{r_0(0), r_0(1), \dots, r_0((k+1)SF-1), \dots, r_{M-1}(0), r_{M-1}(1), \dots, r_{M-1}((k+1)SF-1)\}$. Therefore, assuming that all the symbol sequences are equally probable, maximum likelihood sequence estimation (MLSE) is to find the sequence \mathbf{d} that maximizes $p(\mathbf{r}|\mathbf{d})$ or equivalently maximizes

$$\log\{p(\mathbf{r}|\mathbf{d})\} = 2(k+1)M \cdot SF \log\left(\frac{1}{\sqrt{2\pi\sigma}}\right) - \frac{1}{2\sigma^2} \times \sum_{m=0}^{M-1} \sum_{n=0}^{(k+1)SF-1} \left| r_m(n) - \sqrt{2S} \sum_{l=0}^{L-1} h_{m,l} d\left(\left\lfloor \frac{n-l}{SF} \right\rfloor\right) c(n-l) \right|^2. \quad (6)$$

However, the computational complexity of MLSE grows exponentially with the sequence length.

To reduce the complexity, the past decisions $\hat{\mathbf{d}}_{k-1} = \{\hat{d}(0), \hat{d}(1), \dots, \hat{d}(k-1)\}$ are feedback for performing the symbol-by-symbol decision using Eq. (6). Since the 1st term and the coefficient $1/(2\sigma^2)$ of the 2nd term in Eq. (6) do not contribute to the symbol decision, decision on the k th symbol can be based on the log-likelihood $\Lambda(d(k))$ given by

$$\Lambda(d(k)) = \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{n=kSF}^{(k+1)SF-1} \left| r_m(n) - \sqrt{2S} d(k) \sum_{l=0}^{n-kSF} h_{m,l} c(n-l) - \sqrt{2S} \sum_{l=n-kSF+1}^{L-1} h_{m,l} \hat{d}\left(\left\lfloor \frac{n-l}{SF} \right\rfloor\right) c(n-l) \right|^2. \quad (7)$$

Decision on the k th symbol is performed as

$$\hat{d}(k) = \arg \min_{d(k)} \Lambda(d(k)). \quad (8)$$

The above symbol decision is called DF-CMLD in this paper.

3. BER Analysis

Quaternary phase shift keying (QPSK) data modulation is assumed. The signal constellation used in the theoretical

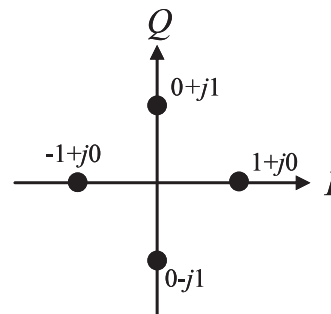


Fig. 2 Signal constellation.

analysis is illustrated in Fig. 2. Without loss of generality, we assume the transmission of all “1 + j0” symbols and that past decisions are all correct, i.e., $\hat{\mathbf{d}}_{k-1} = \mathbf{d}_{k-1}$. In the following, the upper-bound for conditional BER is derived.

The log-likelihood $\Lambda(d(k))$ is, from Eq. (7), given by

$$\Lambda(d(k)) = \begin{cases} \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{n=kSF}^{(k+1)SF-1} |w_m(n)|^2, & \text{for } d(k) = 1 + j0 \\ \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{n=kSF}^{(k+1)SF-1} \left| \sqrt{2S}(1-j) \sum_{l=0}^{L-1} h_{m,l} c(n-l) + w_m(n) \right|^2, & \text{for } d(k) = 0 + j1 \\ \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{n=kSF}^{(k+1)SF-1} \left| \sqrt{2S}(1+j) \sum_{l=0}^{L-1} h_{m,l} c(n-l) + w_m(n) \right|^2, & \text{for } d(k) = 0 - j1 \\ \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{n=kSF}^{(k+1)SF-1} \left| 2\sqrt{2S} \sum_{l=0}^{L-1} h_{m,l} c(n-l) + w_m(n) \right|^2, & \text{for } d(k) = -1 + j0 \end{cases} \quad (9)$$

Decision error occurs if either $\Lambda(j)$, $\Lambda(-j)$ or $\Lambda(-1)$ is smaller than $\Lambda(1)$. The probability of decision error is given by

$$P_s = \text{Prob}[\min\{\Lambda(j), \Lambda(-j), \Lambda(-1)\} < \Lambda(1)]. \quad (10)$$

Therefore, the decision error probability can be upper-bounded by

$$\begin{aligned} P_{s,\text{upper}} &= \text{Prob}[\Lambda(j) < \Lambda(1)] + \text{Prob}[\Lambda(-j) < \Lambda(1)] \\ &\quad + \text{Prob}[\Lambda(-1) < \Lambda(1)] \\ &\approx \text{Prob}[\Lambda(j) < \Lambda(1)] + \text{Prob}[\Lambda(-j) < \Lambda(1)] \\ &= 2 \text{Prob}[\Lambda(j) < \Lambda(1)]. \end{aligned} \quad (11)$$

Using Gray coding, since decision error to the nearest symbol produces 1 bit error, the conditional upper bound BER can be approximated as

$$P_{b,\text{upper}} \approx \frac{1}{2} P_{s,\text{upper}} \approx \text{Prob}[\Delta\Lambda < 0], \quad (12)$$

where $\Delta\Lambda = \Lambda(j) - \Lambda(1)$. $\Delta\Lambda$ is given by

$$\Delta\Lambda = 4S \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \text{Re}[\mu] + \text{Re}[w], \quad (13)$$

where μ is the ICI component and w is the noise component. μ and w are given by

$$\mu = \frac{8S}{SF} \sum_{m=0}^{M-1} \begin{bmatrix} h_{m,L-1} \sum_{n=kSF+L-1}^{(k+1)SF-1} c(n-(L-1)) \sum_{l=0}^{L-2} h_{m,l}^* c^*(n-l) \\ + h_{m,L-2} \sum_{n=kSF+L-2}^{(k+1)SF-1} c(n-(L-2)) \sum_{l=0}^{L-3} h_{m,l}^* c^*(n-l) \\ \cdots + h_{m,1} \sum_{n=kSF+1}^{(k+1)SF-1} h_{m,0}^* c(n-1) c^*(n) \end{bmatrix}$$

$$= \frac{8S}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{j=l+1}^{L-1} h_{m,l} h_{m,j}^* \sum_{n=kSF+j}^{(k+1)SF-1} c(n) c^*(n-j), \quad (14)$$

and

$$w = \frac{2\sqrt{2S}(1-j)}{SF} \sum_{m=0}^{M-1} \left[\sum_{l=0}^{L-1} h_{m,l}^* \sum_{n=kSF+l}^{(k+1)SF-1} w_m(n) c^*(n-l) \right]. \quad (15)$$

In the performance analysis of DS-SS-CDMA, the Gaussian approximation of ICI is used [7]–[9] and the sum of μ and w is treated as a new zero-mean complex-valued Gaussian noise. However, the accuracy of the Gaussian approximation of ICI degrades for a small SF . In this paper, we do not rely on the Gaussian approximation of ICI and derive an exact conditional BER expression for the given spreading sequence $\{c(n); n = kSF - 1 \sim (k+1)SF - 1\}$ and path gains $\{h_{m,l}; m = 0 \sim M-1 \text{ and } l = 0 \sim L-1\}$. For the given $\{h_{m,l}\}$ and $\{c(t)\}$, $\Delta\Lambda$ is a Gaussian variable with a mean of $4S \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \text{Re}[\mu]$ and a variance of σ^2 . σ^2 is given by

$$\begin{aligned} \sigma^2 &= \frac{1}{2} E[|w_{\text{MLD}}|^2] = \frac{8S}{SF^2} \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} h_{m,l}^* h_{m',l'} \\ &\quad \times \sum_{n=kSF+l}^{(k+1)SF-1} \sum_{n'=kSF+l'}^{(k+1)SF-1} E[w_m(n) w_{m'}^*(n')] c^*(n-l) c(n'-l') \\ &= \frac{N_0}{T_c} \frac{16S}{SF} \sum_{m=0}^{M-1} \left[\sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \frac{2}{SF} \right. \\ &\quad \left. \times \sum_{l=0}^{L-1} \sum_{j=l+1}^{L-1} \text{Re} \left[h_{m,l} h_{m,j}^* \times \sum_{n=kSF+j}^{(k+1)SF-1} c(n) c^*(n-j) \right] \right]. \end{aligned} \quad (16)$$

Therefore, the upper-bound BER can be given by

$$P_{b,\text{upper}} \left(\frac{E_b}{N_0}, \{h_{m,l}\} \right) = \frac{1}{2} \text{erfc}(\gamma), \quad (17)$$

where $\text{erfc}(x) = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$ and

$\gamma =$

$$\frac{4S \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \text{Re}[\mu]}{\sqrt{2 \frac{N_0}{T_c} \frac{16S}{SF} \sum_{m=0}^{M-1} \left[\sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \frac{2}{SF} \right. \left. \times \sum_{l=0}^{L-1} \sum_{j=l+1}^{L-1} \text{Re} \left[h_{m,l} h_{m,j}^* \times \sum_{n=kSF+j}^{(k+1)SF-1} c(n) c^*(n-j) \right] \right]}}$$

$$= \sqrt{\frac{E_b}{N_0}} \frac{\left(\sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \text{Re} \left[\frac{2}{SF} \times \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{j=l+1}^{L-1} h_{m,l} h_{m,j}^* \sum_{n=kSF+j}^{(k+1)SF-1} c(n)c^*(n-j) \right] \right)}{\sqrt{\sum_{m=0}^{M-1} \left[\sum_{l=0}^{L-1} \left(1 - \frac{l}{SF}\right) |h_{m,l}|^2 + \frac{2}{SF} \times \sum_{l=0}^{L-1} \sum_{j=l+1}^{L-1} \text{Re} \left[\sum_{n=kSF+j}^{(k+1)SF-1} h_{m,l} h_{m,j}^* c(n)c^*(n-j) \right] \right]}} \quad (18)$$

with $E_b/N_0 = 0.5SF \cdot S \cdot T_c(E_c/N_0)$ representing the average received signal energy per bit-to-AWGN power spectrum density ratio. The theoretical average BER can be numerically evaluated by averaging Eq. (17) over $\{c(n); n = kSF \sim (k+1)SF - 1\}$ and $\{h_{m,l}; m = 0 \sim M - 1 \text{ and } l = 0 \sim L - 1\}$:

$$P_{b,\text{upper}} \left(\frac{E_b}{N_0} \right) = \int \cdots \int P_{b,\text{upper}} \left(\frac{E_b}{N_0}, \{h_{m,l}\} \right) \times p(\{h_{m,l}\}) \prod_{m,l} dh_{m,l}, \quad (19)$$

where $p(\{h_{m,l}\})$ is the joint PDF of $\{h_{m,l}; m = 0 \sim M - 1 \text{ and } l = 0 \sim L - 1\}$.

4. Numerical and Simulation Results

The numerical and simulation conditions are shown in Table 1. We assume an ideal coherent QPSK data-modulation/demodulation and the use of a long binary pseudo-noise (PN) sequence having a period of 4095. As a propagation channel, a chip-spaced L -path frequency-selective Rayleigh fading channel having an exponential power delay profile with decay factor α is assumed. M -antenna diversity reception assuming independent fading is considered. The numerical evaluation of the theoretical average BER for DF-CMLD is done by Monte-Carlo numerical computation as follows. A set of path gains $\{h_{m,l}; m = 0 \sim M - 1 \text{ and } l = 0 \sim L - 1\}$ and spreading sequence $\{c(n); n = kSF \sim (k+1)SF - 1\}$ is generated. The conditional BER is computed a sufficient number of times to obtain the average BER of Eq. (19). The BER performance of DF-CMLD is compared with that of rake combining. In Appendix, the exact conditional BER expression is derived.

Table 1 Numerical and simulation condition.

DS-CDMA	Data modulation	QPSK
	Spreading code	Long PN sequence
Fading channel	Channel model	L -path frequency-selective Rayleigh fading
	Power delay profile	Exponential with decay factor α
Channel estimation		Ideal

Figure 3 plots the upper-bounded average BER performances for DF-CMLD and rake combining with SF as a parameter (the derivation of the conditional BER for rake combining is given in Appendix). No diversity is considered ($M=1$). In a noise dominant low E_b/N_0 region, DF-CMLD is slightly inferior to rake combining. This is due to the power loss produced by subtraction of the signal components belonging to the past symbols. Since the power loss becomes smaller as SF increases, DF-CMLD performance approaches rake combining performance. On the other hand, in a ICI-dominant high E_b/N_0 region, DF-CMLD is superior to rake combining and no error floor is seen. The superiority of DF-CMLD is more significant as SF increases.

The simulated BER performance obtained by computer simulation of DS-CDMA signal transmission with DF-CMLD is also plotted in Fig. 3 to compare with theoretical ones. A fairly good agreement with theoretical and computer simulated results is seen. This confirms the validity of our BER analysis.

As the channel frequency-selectivity increases, the BER performance of rake combining degrades due to large

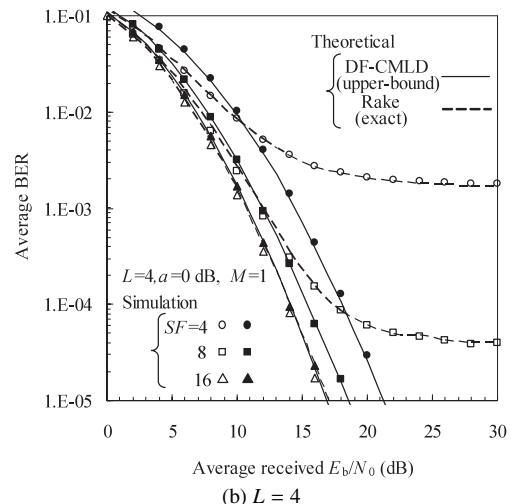
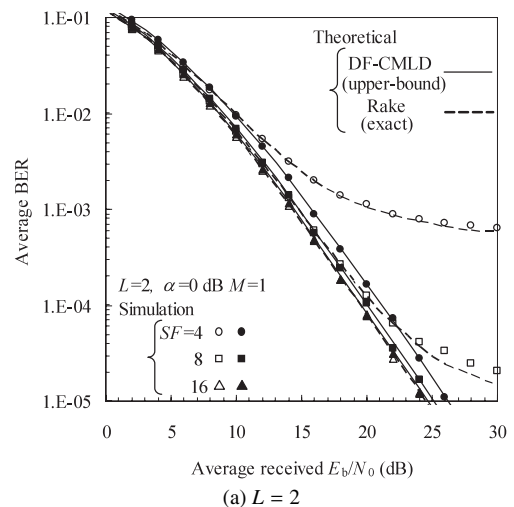
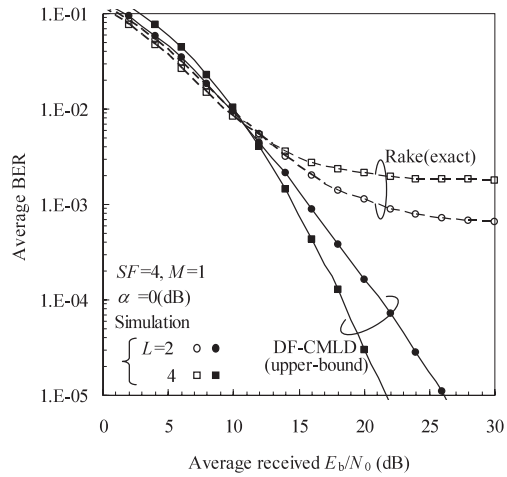
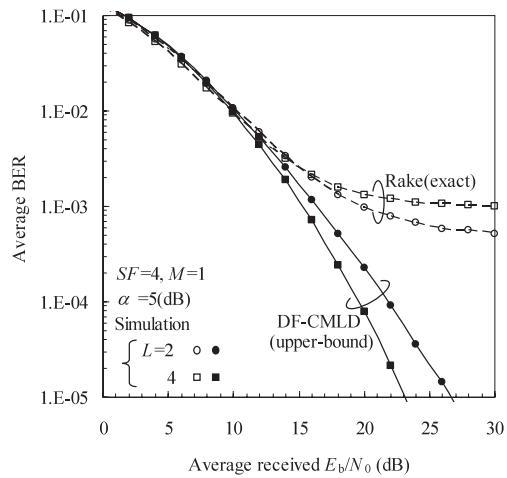


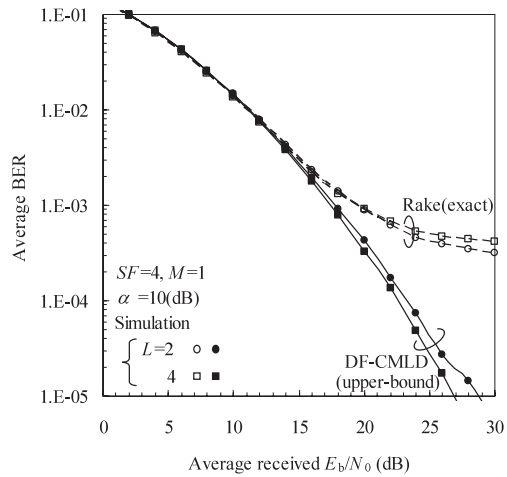
Fig. 3 Average BER performance.



(a) $\alpha = 0$ dB



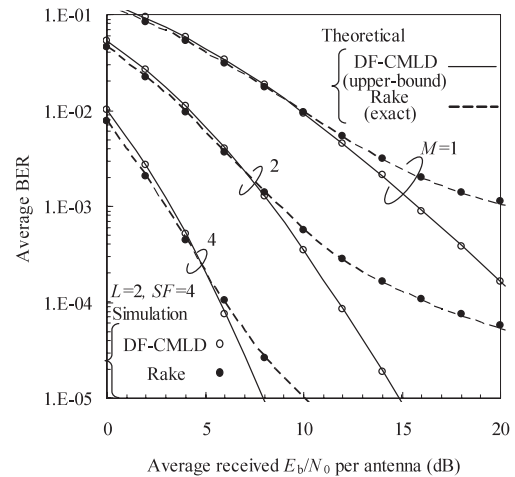
(b) $\alpha = 5$ dB



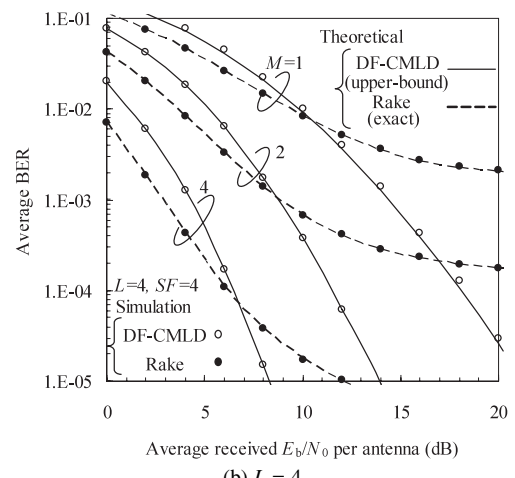
(c) $\alpha = 10$ dB

Fig. 4 Impact of decay factor α when $SF = 4$.

ICI when SF is small. A good indicator of the frequency-selectivity for the given L is the decay factor α of the power delay profile. The performance comparison between DF-CMLD and rake combining is shown in Fig. 4 for the case of $SF=4$. It is seen that rake combining produces an error



(a) $L = 2$



(b) $L = 4$

Fig. 5 Effect of antenna diversity reception when $\alpha = 0$ dB.

floor. However, DF-CMLD produces no error floor and improves the BER performance as L increases.

Antenna diversity reception can be used to further improve the BER performance. Figure 5 plots the average BER performance as a function of the average received E_b/N_0 per antenna with M as a parameter when $SF=4$. For reference, the computer simulated average BER performance is also plotted. It is seen that rake combining produces an error floor even with antenna diversity reception while no BER floor is seen with DF-CMLD.

In DF-CMLD, the decision results of the past data symbols are fed back and used for the log-likelihood computation of the present data symbol decision. Feeding back the decision errors may limit the achievable BER performance improvement of DF-CMLD. To see how the decision errors affect the BER performance achievable with DF-CMLD, the BER performance without feedback errors labeled as “perfect decision feedback” is compared with that with feedback errors labeled as “real decision feedback” in Fig. 6 for $L=2$ - and 4-path uniform power delay profiles ($\alpha=0$ dB). It is seen from Fig. 6 that the BER of the real decision feedback is almost the same as that of perfect decision feedback and the

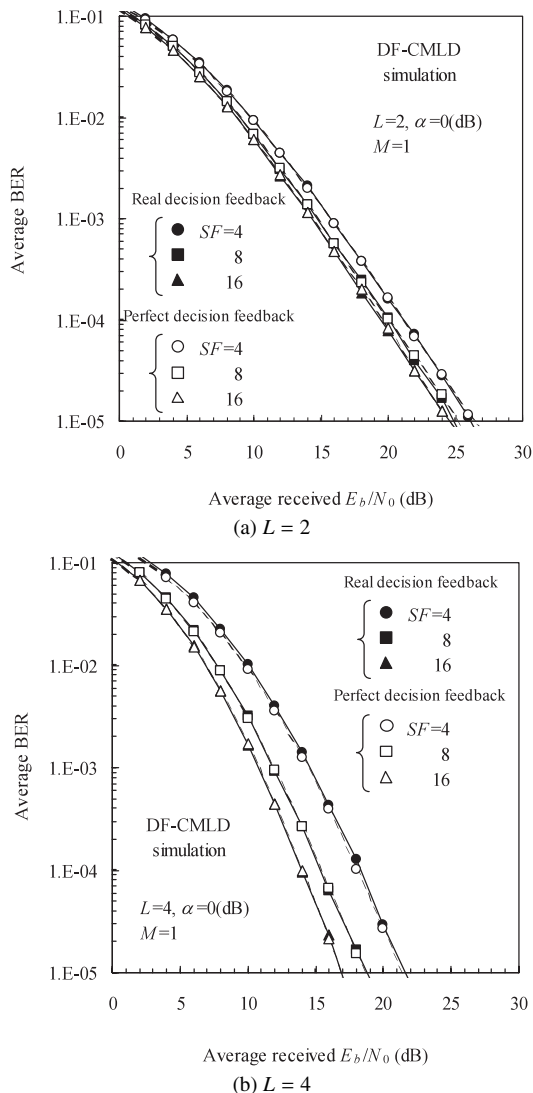


Fig. 6 Impact of decision feedback error when $\alpha = 0$ dB.

impact of the decision error on the BER performance is almost negligible. A possible reason for this is as follows. In DF-CMLD, the previous data symbol only partially contributes to the decision of the present symbol since the maximum time delay among the multipaths is less than one symbol duration.

5. Conclusion

For a low spreading factor, the BER performance with rake combining degrades due to large ICI. In this paper, DF-CMLD was proposed that can suppress the ICI and achieve better BER performance than rake combining. The conditional upper-band BER expression was theoretically derived. The theoretical upper-bounded BER performance with DF-CMLD was evaluated by Monte-Carlo numerical computation using the derived conditional BER expression and compared with that with rake combining. It was confirmed that DF-CMLD can eliminate the error floor which is

produced with rake combining in a high E_b/N_0 region. The theoretical results were confirmed by the computer simulation of signal transmission with DF-CMLD.

In this paper, only the BER performance of rake combining was shown for comparison. However, some techniques to reduce the performance degradation due to the ICI have been proposed for a DS-CDMA system (e.g., a chip equalizer [10], successive and parallel interference cancellers [11]). The performance comparison of DF-CMLD and those techniques is left as an interesting future work.

References

- [1] F. Adachi, "Wireless past and future-evolving mobile communications system," *IEICE Trans. Fundamentals*, vol.E84-A, no.1, pp.55–60, Jan. 2001.
- [2] F. Adachi, "Challenges for broadband mobile technology," *Proc. 12th International Conference on Antennas & Propagation (ICAP 2003)*, pp.1–4, Exeter, U.K., April 2003.
- [3] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next generation mobile communications system," *IEEE Trans. Commun. Mag.*, vol.36, no.9, pp.56–69, Sept. 1998.
- [4] W.C. Jakes Jr., ed., *Microwave mobile communications*, Wiley, New York, 1974.
- [5] J.G. Proakis, *Digital communications*, 4th ed., McGraw-Hill, 2001.
- [6] A.J. Viterbi, *CDMA: Principles of Spread Spectrum Communications*, Addison-Wesley, 1995.
- [7] C. Kchao and G.L. Stuber, "Analysis of a direct-sequence spread-spectrum cellular radio system," *IEEE Trans. Commun.*, vol.41, no.10, pp.1507–1516, Oct. 1993.
- [8] K. Cheun, "Performance of direct-sequence spread-spectrum rake receivers with random spreading sequences," *IEEE Trans. Commun.*, vol.45, no.9, pp.1130–1142, Sept. 1997.
- [9] F. Adachi, "Effects of orthogonal spreading and Rake combining on DS-CDMA forward link in Mobile Radio," *IEICE Trans. Commun.*, vol.E80-B, no.11, pp.1703–1712, Nov. 1997.
- [10] A. Kein, "Data detection algorithm specially designed for the down-link of mobile radio systems," *Proc. IEEE VTC'97*, pp.203–207, Phoenix, May 1997.
- [11] L. Hanzo, L.L. Yang, E.L. Kuan, and K. Yen, *Single and multi-carrier DS-CDMA: Multi-user detection, space-time spreading, synchronization, networking and standards*, John Wiley & Sons Inc., 2003.

Appendix: Exact BER Analysis for Rake Combining

In the rake receiver, the received signal $r_m(n)$ on the m th antenna is despread by multiplying the spreading sequence and summed up over one symbol period (SF chip). Then, the maximal-ratio combining (MRC) of all despread outputs associated with L paths is carried out. Then, M rake receiver outputs are summed to produce the decision variable $z_{\text{rake}}(k)$ for the k th symbol, given by

$$z_{\text{rake}}(k) = \sqrt{2S} d(k) \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} |h_{m,l}|^2 + \mu_{\text{rake}} + w_{\text{rake}}, \quad (\text{A} \cdot 1)$$

where μ_{rake} is the ICI component and w_{rake} is the noise component. μ_{rake} and w_{rake} are given by

$$\left\{ \begin{aligned} \mu_{\text{rake}} &= \frac{\sqrt{2S}}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{\substack{j=0 \\ j \neq l}}^{L-1} h_{m,l}^* h_{m,j} \\ &\times \sum_{n=kSF+l}^{(k+1)SF-1+l} d\left(\left\lfloor \frac{n-j}{SF} \right\rfloor\right) c^*(n-l)c(n-j) \\ w_{\text{rake}} &= \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{n=kSF+l}^{(k+1)SF-1+l} w_m(n) c^*(n-l) h_{m,l}^* \end{aligned} \right. \quad (\text{A} \cdot 2)$$

We derive the conditional BER for the given set of path gains $\{h_{m,l}; m = 0 \sim M-1 \text{ and } l = 0 \sim L-1\}$ and spreading sequence $\{c(n); n = kSF-1 \sim (k+1)SF-1\}$. $z_{\text{rake}}(k)$ is the complex Gaussian variable with mean $\sqrt{2S}d(k) \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} |h_{m,l}|^2 + \mu_{\text{rake}}$ and variance $2\sigma^2$. $2\sigma^2$ given by

$$\begin{aligned} 2\sigma^2 &= E[|w_{\text{rake}}|^2] \\ &= E\left[\left|\frac{1}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{n=kSF+l}^{(k+1)SF-1+l} w_m(n) c^*(n-l) h_{m,l}^*\right|^2\right] \\ &= \frac{1}{SF^2} \left[\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} h_{m,l}^* h_{m',l'} \right. \\ &\quad \times \sum_{\substack{n=kSF+l \\ n'=kSF+l'}}^{(k+1)SF-1+l} \sum_{\substack{n'=kSF+l' \\ n=kSF+l}}^{(k+1)SF-1+l'} E[w_m(n) w_{m'}^*(n')] \\ &\quad \left. \times c^*(n-l) c(n'-l') \right] \\ &= \left(\frac{2N_0}{T_c} \right) \frac{1}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left[|h_{m,l}|^2 + \frac{1}{SF} \right. \\ &\quad \left. \times \sum_{\substack{l'=0 \\ l' \neq l}}^{L-1} \left(\sum_{\substack{n=kSF+l \\ n'=kSF+l'}}^{(k+1)SF-1+l} \sum_{\substack{n'=kSF+l' \\ n=kSF+l}}^{(k+1)SF-1+l'} c^*(n-l) c(n-l') \right) \right] \quad (\text{A} \cdot 3) \end{aligned}$$

Using

$$\begin{aligned} &h_{m,l}^* h_{m,l'} \sum_{n=kSF+\max(l',l)}^{(k+1)SF-1+\min(l',l)} c^*(n-l) c(n-l') \\ &+ h_{m,l'}^* h_{m,l} \sum_{n=kSF+\max(l,l')}^{(k+1)SF-1+\min(l',l)} c^*(n-l') c(n-l) \\ &= 2\text{Re} \left[h_{m,l}^* h_{m,l'} \sum_{n=kSF+l'}^{(k+1)SF-1+l} c^*(n-l) c(n-l') \right], \quad (\text{A} \cdot 4) \end{aligned}$$

Equation (A.3) becomes

$$\begin{aligned} 2\sigma^2 &= \left(\frac{2N_0}{T_c} \right) \frac{1}{SF} \\ &\times \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left[|h_{m,l}|^2 + \frac{1}{SF/2} \right. \\ &\quad \left. \times \sum_{l'=l+1}^{L-1} \text{Re} \left[\sum_{n=kSF+l'}^{(k+1)SF-1+l} h_{m,l}^* h_{m,l'} \right. \right. \\ &\quad \left. \left. \times \sum_{n=kSF+l'}^{(k+1)SF-1+l} c^*(n-l) c(n-l') \right] \right] \quad (\text{A} \cdot 5) \end{aligned}$$

We assume $\tau_l < SF$. Then, μ_{rake} is a contribution from $d(k-1)$, $d(k)$ and $d(k+1)$. Without loss of generality, we assume $d(k) = (1+j)/\sqrt{2}$ and $d(k-1)$ and $d(k+1) = (\pm 1 \pm j)/\sqrt{2}$. Since μ_{rake} is circularly symmetric, the exact conditional BER for QPSK modulation with rake combining can be given by

$$\begin{aligned} &p_{b,\text{exact}} \left(\frac{E_b}{N_0}, \{h_{m,l}\}, \{d(k-1), d(k), d(k+1)\} \right) \\ &= \frac{1}{2} \text{erfc} \left(\frac{\sqrt{S} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} |h_{m,l}|^2 + \text{Re}[\mu_{\text{rake}}]}{\sqrt{2}\sigma} \right) \\ &= \frac{1}{2} \text{erfc} \left(\sqrt{\frac{E_b}{N_0}} \frac{\left[\sum_{m=0}^{M-1} \sum_{l=0}^{L-1} |h_{m,l}|^2 \right. \right. \\ &\quad \left. \left. + \sqrt{2} \text{Re} \left[\frac{1}{SF} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \sum_{\substack{j=0 \\ j \neq l}}^{L-1} h_{m,l}^* h_{m,j} \right. \right. \right. \\ &\quad \left. \left. \times \sum_{n=kSF+l}^{(k+1)SF-1+l} d\left(\left\lfloor \frac{n-j}{SF} \right\rfloor\right) c^*(n-l) c(n-j) \right] \right]}{\sqrt{\sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \left[|h_{m,l}|^2 + \frac{1}{SF/2} \sum_{l'=l+1}^{L-1} \text{Re} \right. \right. \\ &\quad \left. \left. \left[\sum_{n=kSF+l'}^{(k+1)SF-1+l} h_{m,l}^* h_{m,l'} \right. \right. \right. \\ &\quad \left. \left. \times \sum_{n=kSF+l'}^{(k+1)SF-1+l} c^*(n-l) c(n-l') \right] \right]} \right) \quad (\text{A} \cdot 6) \end{aligned}$$

The exact average BER can be numerically evaluated by averaging Eq. (A.6) over $\{c(n); n = kSF \sim (k+1)SF-1\}$ and $\{h_{m,l}; m = 0 \sim M-1 \text{ and } l = 0 \sim L-1\}$.



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