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LETTER

# Joint Frequency-Domain STTD and Antenna Diversity Reception Based on MMSE Criterion for OFDM/TDM

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**SUMMARY** In this letter, we introduce frequency-domain space-time transmit diversity (STTD) encoding/decoding to orthogonal frequency division multiplexing combined with time division multiplexing (OFDM/TDM) on a frame-by-frame basis (i.e., over several concatenated OFDM signals in the frequency-domain) to achieve both spatial and frequency diversity gains and improve the bit error rate (BER) performance. The theoretical BER performance is evaluated by numerical computation using the derived conditional BER and confirmed by computer simulation.

***key words:*** *OFDM/TDM, frequency-domain STTD*

## 1. Introduction

Space-time transmit diversity (STTD) [1] in combination with OFDM [2] and single carrier (SC) [3] has been gaining much attention for high data rate transmission. The STTD encoding for OFDM is performed on each subcarrier independently to achieve spatial diversity gain, but frequency diversity gain cannot be obtained. In [3], modified Alamouti's STTD encoding is applied to SC transmission with frequency domain equalization (FDE) on a block-by-block basis to exploit both the channel frequency-selectivity and spatial diversity and thereby improve the bit error rate (BER) performance. Recently, we proposed to apply FDE based on minimum mean square error (MMSE) criterion to OFDM combined with time division multiplexing (OFDM/TDM) [4]. To further improve the BER performance, transmit and receive antenna diversity can be used. In this letter, we present joint frequency-domain STTD and antenna diversity reception based on MMSE criterion for OFDM/TDM as an extension of work done in [3]. Unlike conventional STTD for OFDM that encodes each *OFDM subcarrier* independently, we present STTD encoding on a frame-by-frame basis in the frequency-domain over each *OFDM/TDM frequency component* similar to [3]. At a receiver, frame-by-frame based joint frequency-domain STTD decoding and antenna diversity reception based on MMSE criterion is carried out to achieve both spatial and frequency diversity gains. In Sect. 2, the frame-by-frame frequency-domain STTD encoding/decoding and an expression for the conditional BER is derived. In Sect. 3, the theoretical BER performance is evaluated. Section 4 concludes the paper.

## 2. Joint Frequency-Domain STTD and Antenna Diversity

The OFDM/TDM transmitter/receiver with joint STTD and antenna diversity reception is shown in Fig. 1 with $N_t$=2 transmit antennas and $N_r$ receive antennas.

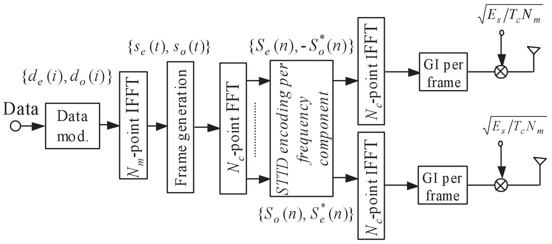
### 2.1 OFDM/TDM Frame Generation

The $q$th OFDM/TDM frame of the data-modulated symbol sequence is denoted as $\{d_q(i);\ i = 0 \sim N_c - 1\}$. Then, the $q$th frame sequence is divided into $K$ blocks of $N_m$ symbols. The $k$-th block symbol sequence is denoted as $\{d_q^k(i);\ i = 0 \sim N_m - 1\}$, where $d_q^k(i) = d_q(kN_m + i)$ with $|d_q^k(i)| = 1$. The $N_m$-point IFFT is applied to each data block to generate a sequence of $K$ OFDM signals with $N_m = N_c/K$ subcarriers. The OFDM/TDM signal of the $q$th modulated-data frame can be expressed using equivalent lowpass representation as

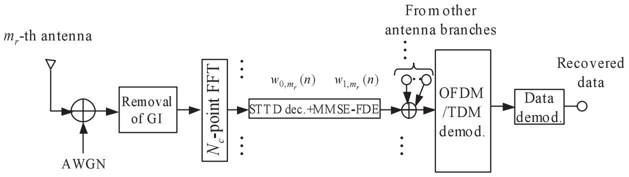
$$s_q(t) = s_q^{\lfloor t/N_m \rfloor}(t - kN_m) \tag{1}$$

for $t = 0 \sim N_c - 1$, where $\lfloor x \rfloor$ represents the largest integer smaller than or equal to $x$ and $s_q^{\lfloor t/N_m \rfloor}(t)$ is the OFDM signal with $N_m$ subcarriers, for $t = 0 \sim N_m - 1$ given by [4]

$$s_q^{\lfloor t/N_m \rfloor}(t) = \sqrt{2P} \sum_{i=0}^{N_m-1} d_q^{\lfloor t/N_m \rfloor}(i) \exp\left[ j2\pi t \frac{i}{N_m} \right], \tag{2}$$



(a) Transmitter



(b) Receiver

**Fig. 1** OFDM/TDM transmitter/receiver structure.

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**Table 1** STTD encoding for OFDM/TDM.

| Frame interval | Antenna | |
|---|---|---|
| | #0 | #1 |
| Even ($q$=2$u$) | $\sqrt{1/2}\,S_e(n)$ | $\sqrt{1/2}\,S_o(n)$ |
| Odd ($q$=2$u$+1) | $-\sqrt{1/2}\,S_o^*(n)$ | $\sqrt{1/2}\,S_e^*(n)$ |

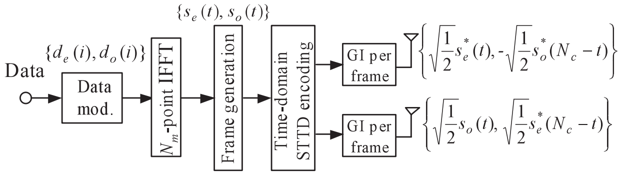

**Fig. 2** Equivalent time-domain STTD encoding for OFDM/TDM.

where $P$ is the transmit signal power. Note that OFDM/TDM becomes conventional OFDM when $K$=1 and becomes SC when $K = N_c$.

### 2.2 Frame-by-Frame Frequency-Domain STTD Encoding for OFDM/TDM

At the transmitter (see Fig. 1(a)), $N_c$-point FFT is applied to decompose each OFDM/TDM signal frame to $N_c$ frequency components. The frequency-domain OFDM/TDM signal for $n = 0 \sim N_c - 1$ is given as

$$S_{e(\text{or}\,o)}(n) = \sum_{t=0}^{N_c-1} s_{e(\text{or}\,o)}(t)\exp\left[-j2\pi n\frac{t}{N_c}\right]. \tag{3}$$

The Alamouti's STTD encoding rule is directly applied to each *frequency component* as shown in Table 1 and then, $N_c$-point IFFT is applied to obtain the STTD encoded OFDM/TDM signal frames. In the even OFDM/TDM frame interval ($q$=2$u$), the STTD encoded OFDM/TDM signals, to be transmitted from the first and second antennas, are $\sqrt{1/2}s_e(t)$ and $\sqrt{1/2}s_o(t)$ for $t = 0 \sim N_c - 1$, respectively. In the odd OFDM/TDM frame interval ($q = 2u + 1$), the STTD encoded signals, to be transmitted from the first and second antennas, are given for $t = N_c \sim 2N_c - 1$ as

$$\begin{cases} \text{Antenna 0}: \dfrac{1}{N_c}\displaystyle\sum_{n=0}^{N_c-1}\left\{-\sqrt{\frac{1}{2}}S_e^*(n)\right\}\exp\left[j2\pi t\frac{n}{N_c}\right] \\ \text{Antenna 1}: \dfrac{1}{N_c}\displaystyle\sum_{n=0}^{N_c-1}\left\{\sqrt{\frac{1}{2}}S_o^*(n)\right\}\exp\left[j2\pi t\frac{n}{N_c}\right] \end{cases}. \tag{4}$$

The above STTD encoding requires FFT/IFFT operation, but a STTD encoding without FFT/IFFT can be derived. Using Eq. (1), we can show

$$\begin{aligned} &\frac{1}{N_c}\sum_{n=0}^{N_c-1}\sqrt{\frac{1}{2}}S^*_{e(\text{or}\,o)}(n)\exp\left[j2\pi t\frac{n}{N_c}\right] \\ &\quad = \sqrt{\frac{1}{2}}s^*_{e(\text{or}\,o)}(N_c-t), \end{aligned} \tag{5}$$

which is a time reversed and conjugate version of $s_{e(\text{or}\,o)}(t)$. Therefore, STTD encoded OFDM/TDM signals, to be transmitted from the first and second antennas in the odd OFDM/TDM frame interval, are $-\sqrt{1/2}s_o^*(N_c - t)$ and $\sqrt{1/2}s_e^*(N_c - t)$ for $t = N_c \sim 2N_c - 1$, respectively. The transmitter structure using the above time-domain STTD encoding for OFDM/TDM is illustrated in Fig. 2. After inserting the cyclic prefix into the GI at the beginning of the frame, the OFDM/TDM signals are transmitted.

### 2.3 Joint STTD Decoding and Antenna Diversity Reception Based on MMSE Criterion

At the receiver (see Fig. 1(b)), by $N_c$-point FFT, even and odd OFDM/TDM frame signals received on the $m_r$-th antenna are decomposed on their corresponding frequency components as

$$\begin{cases} R_{e,m_r}(n) = \sqrt{P}H_{0,m_r}(n)S_e(n) \\ \qquad\qquad + \sqrt{P}H_{1,m_r}(n)S_o(n) + N_{e,m_r}(n) \\ R_{o,m_r}(n) = \sqrt{P}H_{1,m_r}(n)S_e^*(n) \\ \qquad\qquad - \sqrt{P}H_{0,m_r}(n)S_o^*(n) + N_{o,m_r}(n) \end{cases}, \tag{6}$$

where $S_{e(\text{or}\,o)}(n)$, $H_{m_t,m_r}(n) = FFT[h^l_{m_t,m_r}]$ and $N_{e(\text{or}\,o),m_r}(n) = FFT[n_{e(\text{or}\,o)}]$ are the $n$th frequency component of the transmitted OFDM/TDM signal given by Eq. (1), the channel gain between the $m_t$-th transmit antenna and the $m_r$-th receive antenna and the additive white Gaussian noise (AWGN) component, respectively. The joint STTD decoding and antenna diversity reception is carried out on each frequency component as

$$\begin{cases} \hat{R}_e(n) = \displaystyle\sum_{m_r=0}^{N_r-1}\left\{w^*_{0,m_r}(n)R_{e,m_r}(n) + w_{1,m_r}(n)R^*_{o,m_r}(n)\right\} \\ \hat{R}_o(n) = \displaystyle\sum_{m_r=0}^{N_r-1}\left\{w^*_{1,m_r}(n)R_{e,m_r}(n) - w_{0,m_r}(n)R^*_{o,m_r}(n)\right\} \end{cases}, \tag{7}$$

where [5]

$$w_{0(\text{or}\,1),m_r}(n) = \frac{H_{0(\text{or}\,1),m_r}(n)}{\displaystyle\sum_{m_t=0}^{1}\sum_{m_r=0}^{N_r-1}\left|H_{m_t,m_r}(n)\right|^2 + \left(\frac{E_s}{2N_0}\right)^{-1}} \tag{8}$$

with $N_0$ being the single-sided power spectrum density of AWGN. By applying $N_c$-point IFFT, the time-domain OFDM/TDM signal is recovered as

$$\hat{r}_{e(\text{or}\,o)}(t) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{R}_{e(\text{or}\,o)}(n)\exp\left[j2\pi t\frac{n}{N_c}\right] \tag{9}$$

for $t = 0 \sim N_c - 1$. Then, $N_m$-point FFT is applied to obtain the decision variable for data demodulation [4].

### 2.4 BER Analysis

Substituting Eq. (7) into Eq. (9), after some manipulations, based on the Gaussian approximation of the residual ISI after FDE, the total variance $2\sigma^2$ of the ISI plus noise is given as

$$2\sigma^2 = \frac{N_0}{T_c}\frac{1}{N_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1} \left[\begin{array}{l} \dfrac{1}{2}\dfrac{E_s}{N_0}\left|\begin{array}{c} \hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n) \\ -\dfrac{1}{N_c}\displaystyle\sum_{m=0}^{N_c-1}\left[\begin{array}{c}\hat{H}_{0,m_r}(m)\\ +\hat{H}_{1,m_r}(m)\end{array}\right]\end{array}\right|^2 \\ +\left[\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\right]\end{array}\right]|\Psi(n)|^2. \quad (10)$$

The conditional SINR is given as

$$\gamma\left(\frac{E_s}{N_0},\{H_{m_t,m_r}(n)\}\right) = \frac{\dfrac{E_s}{N_0}\left|\dfrac{1}{N_c}\displaystyle\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\right]\right|^2}{\dfrac{1}{K}\displaystyle\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\begin{array}{l}\dfrac{1}{2}\dfrac{E_s}{N_0}\left|\begin{array}{l}\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)-\dfrac{1}{N_c}\\ \times\displaystyle\sum_{m=0}^{N_c-1}\left[\hat{H}_{0,m_r}(m)+\hat{H}_{1,m_r}(m)\right]\end{array}\right|^2 \\ +\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\end{array}\right]|\Psi(n)|^2}. \quad (11)$$

Assuming QPSK data modulation, the conditional BER for the given set of channel gains can be expressed as

$$p_b\left(\frac{E_s}{N_0},\{H_{m_r,m_t}(n)\}\right) = \frac{1}{2}erfc\left[\sqrt{\frac{1}{4}\gamma\left(\frac{E_s}{N_0},\{H_{m_r,m_t}(n)\}\right)}\right], \quad (12)$$

where *erfc*$[x]$ is the complementary error function [6].

## 3. Numerical and Simulation Results

We assume QPSK data-modulation, OFDM/TDM frame length of $N_c$=256 and the GI length of $N_g$=32. The channel is assumed to be an $L$=16-path frequency-selective block Rayleigh fading channel having power delay profile with decay factor $\beta$. Ideal channel estimation is assumed.

The average BER is computed using Monte-Carlo numerical computation by averaging Eq. (12) for different channel gains a sufficient number of times. Figure 3 shows the theoretical and computer simulated BER performance as a function of the average received signal energy per bit-to-AWGN power spectrum density ratio $E_b/N_0 = 0.5(E_s/N_0)\times(1+N_g/N_c)$, where $E_s = PT_cN_m$ with $T_c$ being the sampling time of FFT. From Fig. 3, it can be seen that $K$=16 using frequency-domain STTD with $N_r$=2, for the average BER=$10^{-4}$, achieves diversity gain of about 8.2 and 3 dB in comparison to $K$=1 and 16 without STTD ($N_r$=2 only), respectively. Note that slopes of the curves for $K$=1 (OFDM) and $K$=16 are not the same since frequency-domain STTD encoding/decoding for OFDM/TDM achieves both spatial diversity and frequency diversity gains. Figure 4 shows the impact of channel frequency-selectivity on the BER of OFDM/TDM with joint STTD and antenna diversity for $K$=16 with $\beta$=0, 8 and 16 dB. When $\beta$=8 and 16 dB, the MMSE-FDE antenna diversity gain becomes smaller due to weaker channel frequency-selectivity and the BER performance degrades. For $K$=16 using STTD with $N_r$=2, antenna diversity gain of about 3.4, 5 and 5.6 dB is achieved for the average BER=$10^{-4}$ over $\beta$=0, 8 and 16 dB without STTD ($N_r$=2 only), respectively. A fairly good agreement between the theoretical results and computer simulation results is observed.

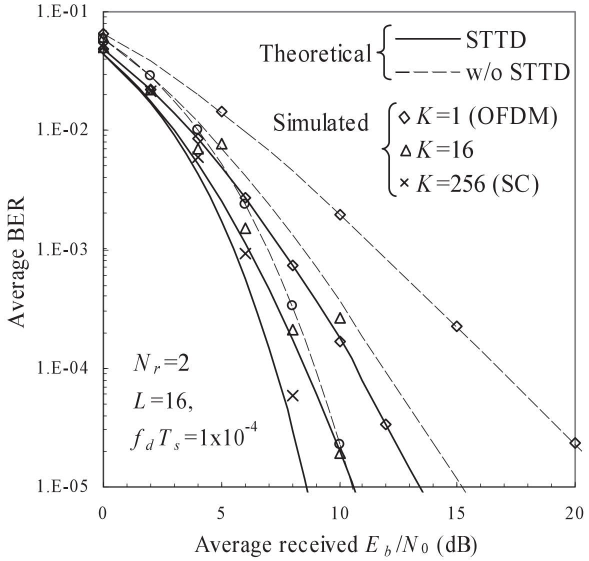

**Fig. 3** Effect of joint STTD and $N_r$-receive diversity.

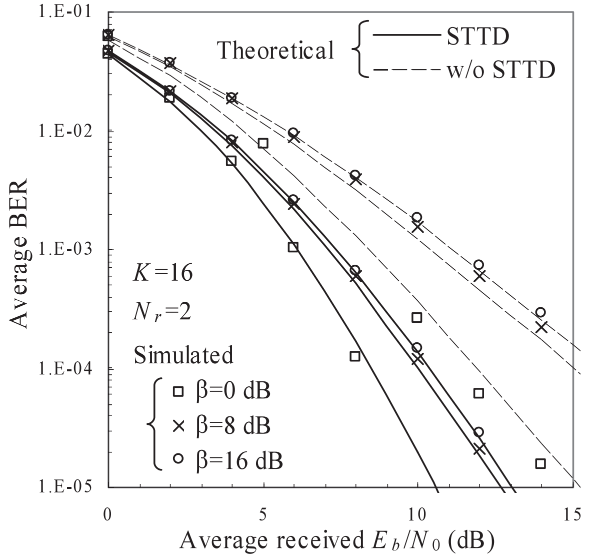

**Fig. 4** Impact of channel decay factor $\beta$.

## 4. Conclusions

In this letter, we presented frequency-domain STTD encoding/decoding for OFDM/TDM transmission. The STTD encoding/decoding for OFDM/TDM is performed on each *OFDM/TDM frequency component* on a frame-by-frame basis and achieves both spatial and frequency diversity gains. It was shown, by theoretical analysis and simulation, that frequency-domain STTD improves the BER performance of OFDM/TDM.

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