

LETTER

PAPR Advantage of Amplitude Clipped OFDM/TDM

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SUMMARY OFDM combined with TDM (OFDM/TDM) can be used to reduce a high peak-to-average power ratio (PAPR) of OFDM, but the PAPR reduction is not sufficient. To further reduce the PAPR, an amplitude clipping can be applied. In this letter, we investigate the effect of clipping on OFDM/TDM with and without channel coding. It is shown that amplitude clipped OFDM/TDM has an advantage over clipped OFDM with respect to the PAPR.

key words: OFDM/TDM, clipping, MMSE-FDE

1. Introduction

Orthogonal frequency division multiplexing (OFDM), which is robust against multipath fading, has a drawback of having a large peak-to-average power ratio (PAPR) [1]. This undesirable feature renders the OFDM particularly sensitive to nonlinear distortions (e.g., amplitude clipping) [1]–[3]. Recently, we proposed OFDM combined with time division multiplexing (OFDM/TDM) to reduce the high PAPR of OFDM, but the PAPR reduction is not sufficient [4]. To further reduce the PAPR of OFDM, an amplitude clipping can be applied; but the bit error rate (BER) performance degrades as shown in [2], [3]. To improve the BER performance, channel coding (e.g., turbo coding [5]) is an effective technique. In this letter, we investigate the effect of clipping on the BER performance of turbo-coded OFDM/TDM. The advantage of amplitude clipped OFDM/TDM over conventional OFDM with respect to the PAPR is evaluated by computer simulation.

The letter is organized as follows. Section 2 presents the OFDM/TDM with amplitude clipping in a frequency-selective fading channel. In Sect. 3, the PAPR advantage of OFDM/TDM is discussed. Section 4 concludes the paper.

2. Amplitude Clipped OFDM/TDM

The transmission system model of amplitude clipped OFDM/TDM with channel coding is illustrated in Fig. 1.

2.1 Mathematical Signal Representation

Information bit sequence is turbo coded, bit interleaved and mapped into the transmit data symbols, corresponding to

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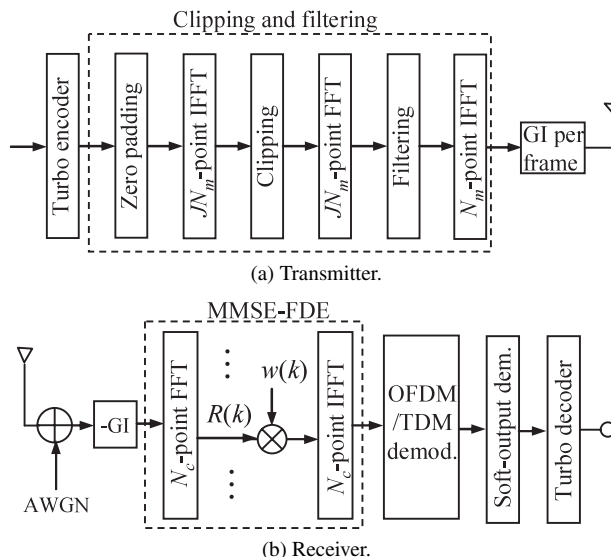


Fig. 1 OFDM/TDM transceiver.

quaternary phase shift keying (QPSK) modulation scheme. This sequence is divided into a sequence of blocks, each having N_c data-modulated symbols $\{d(i); i = 0 \sim N_c - 1\}$ with $E[|d(i)|^2] = 1$, where $E[\cdot]$ denotes the ensemble average operation.

The signaling interval (called OFDM/TDM frame) of OFDM with N_c subcarriers is divided into K slots [4]. $\{d(i)\}$ is divided into K subblocks with $N_m = N_c/K$ symbols for OFDM/TDM modulation. The k -th subblock to be transmitted in the k th OFDM/TDM slot is denoted by $\{d^k(i); i = 0 \sim N_m - 1\}$, where $d^k(i) = d(kN_m + i)$ for $k = 0 \sim K - 1$. First, JN_m -point inverse fast Fourier transform (IFFT) with zero-padding is applied to generate an interpolated time-domain OFDM signal with N_m subcarriers as

$$s^k(t) = \frac{1}{\sqrt{N_m}} \sum_{i=0}^{N_m-1} d^k(i) \exp\left(j2\pi i \frac{t}{JN_m}\right) \quad (1)$$

for $t = 0 \sim JN_m - 1$, where J is the over-sampling ratio. Then, the signal amplitude is clipped as [2], [3]

$$\bar{s}^k(t) = \begin{cases} s^k(t), & |s^k(t)| \leq \beta \\ \beta \frac{s^k(t)}{|s^k(t)|}, & \text{otherwise} \end{cases} \quad (2)$$

for $t = 0 \sim JN_m - 1$, where β denotes the predetermined clipping ratio, which is known at the receiver side. Due to amplitude clipping, undesired out-of-band (OoB) spectrum

grows. A lowpass filtering is applied to suppress the OoB power. Before filtering, the clipped signal is transformed into the JN_m frequency components by applying JN_m -point FFT as

$$\bar{s}^k(i) = \frac{1}{\sqrt{JN_m}} \sum_{t=0}^{JN_m-1} \tilde{s}^k(t) \exp\left(-j2\pi i \frac{t}{JN_m}\right) \quad (3)$$

for $i = 0 \sim JN_m - 1$. $\{\bar{s}^k(i); i = N_m \sim JN_m - 1\}$ are intermodulation products that would appear as OoB spectrum. The first N_m frequency components are picked up and then, N_m -point IFFT is applied to obtain the amplitude clipped OFDM signal as

$$\tilde{s}^k(t) = \frac{1}{\sqrt{N_m}} \sum_{i=0}^{N_m-1} \bar{s}^k(i) \exp\left(j2\pi t \frac{i}{N_m}\right). \quad (4)$$

Finally, the clipped OFDM/TDM signal is expressed using the equivalent lowpass representation as

$$\tilde{s}(t) = \sqrt{\frac{2E_s}{T_c}} \sum_{k=0}^{K-1} \tilde{s}^k(t - kN_m) u(t - kN_m) \quad (5)$$

for $t = 0 \sim N_c - 1$, where $u(t) = 1(0)$ for $t = 0 \sim N_m - 1$ (elsewhere). In Eq. (5), E_s and T_c denote the data-modulated symbol energy and the sampling duration of N_m -point IFFT, respectively.

After insertion of guard interval (GI), the clipped OFDM/TDM signal is transmitted over an L -path frequency-selective fading channel. We assume a block fading where the path gains remain constant over one OFDM/TDM frame and vary frame-by-frame. The channel impulse response is given as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l), \quad (6)$$

where h_l and τ_l denotes the l th path gain and time delay, respectively, with $E[|h_l|^2] = 1/L$. The maximum time delay of the channel is shorter than the GI length.

At the receiver, N_c -point FFT is applied to decompose the received signal into N_c frequency components $\{R(n); n = 0 \sim N_c - 1\}$. Frequency-domain equalization based on minimum mean square error criterion (MMSE-FDE) is applied to $R(n)$ as [6]

$$\hat{R}(n) = R(n)w(n), \quad (7)$$

where $w(n)$ is the MMSE weight. The time-domain OFDM/TDM signal is recovered by applying N_c -point IFFT to $\{\hat{R}(n); n = 0 \sim N_c - 1\}$, and then OFDM demodulation is carried out using N_m -point FFT to obtain decision variables $\{\hat{d}^k(i); i = 0 \sim N_m - 1\}$ [4].

The MMSE weight $w(n)$ is derived bellow. Using the Busgang theorem [7], a clipped OFDM/TDM signal can be expressed as a sum of a useful attenuated input replica and an uncorrelated nonlinear distortion as

$$\tilde{s}(t) = \alpha s(t) + s_c(t), \quad (8)$$

where α and $s_c(t)$, respectively, denote the attenuation constant and the clipping noise. α is chosen to minimize the mean square error (MSE) $E[|\tilde{s}(t) - \alpha s(t)|^2]$ [7], and can be well approximated as [8]

$$\alpha = 1 - \exp\{-\beta^2\} + \frac{\sqrt{\pi}\beta}{2} \operatorname{erfc}(\beta), \quad (9)$$

where $\operatorname{erfc}[x] = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2) dt$ is the complementary error function. Using Eq. (8), $R(n)$ can be expressed as

$$R(n) = \sqrt{\frac{2E_s}{T_c}} [\alpha S(n)H(n) + S_c(n)H(n)] + N(n), \quad (10)$$

where $S(n)$, $H(n)$, $S_c(n)$ and $N(n)$ denote Fourier transforms of $s(t)$, $h(t)$, $s_c(t)$ and $n(t)$, respectively. $n(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density. The MMSE weight minimizes the MSE between $\hat{R}(n)$ and $S(n)$. After some manipulations, we can show that the MMSE weight is given by (see Appendix A)

$$w(n) = \frac{\alpha H^*(n)}{|H(n)|^2 \left[1 - e^{-\beta^2} \right] + \left(\frac{E_s}{N_0}\right)^{-1}}, \quad (11)$$

where $(\cdot)^*$ denotes the complex conjugate operation. Note that the contribution due to clipping noise is included in the denominator as $\exp(-\beta^2)$. For large β (e.g., $\beta > 7$ dB), $\alpha \approx 1$ and $\exp(-\beta^2) \approx 0$ and hence, Eq. (11) reduces to the MMSE equalization weight for unclipped system. In the above expression, β (while α is obtained by Eq. (9)) is the predetermined value and therefore, it is assumed to be known at the receiver (this is practically reasonable assumption).

2.2 Turbo Decoder in a Nonlinear Channel

For turbo decoding, the log-likelihood ratio (LLR) is required [5]. Let us denote the $2N_c$ -bit sequence that constructs the N_c -symbol block $\{d(i); i = 0 \sim N_c - 1\}$, by $\mathbf{b} = \{b_0(0), b_1(0), b_2(1), b_3(1), \dots, b_{2N_c-2}(N_c-1), b_{2N_c-1}(N_c-1)\}$. The LLR $L(b_j(i))$ for the j th bit in the i th QPSK symbol $d(i)$ is given by [5]

$$L(b_j(i)) = \log \frac{\Pr[b_j(i) = 1 | \hat{d}(i)]}{\Pr[b_j(i) = 0 | \hat{d}(i)]}, \quad (12)$$

where $\Pr[b_j(i) = 1 \text{ (or } 0) | \hat{d}(i)]$ is the a posteriori probability of the data bit $b_j(i)$ and $\hat{d}(i) = \hat{d}^k(i - kN_m)$ with $k = \lfloor i/N_m \rfloor$ ($\lfloor x \rfloor$ represents the largest integer smaller than or equal to x). Using Eq. (B1) (see Appendix B), and the well-known approximation $\log \sum_i \exp(x_i) \approx \max_i x_i$ [9], we have

$$L(b_j(i)) \approx \min \frac{|\hat{d}(i) - \alpha \hat{H} \hat{s}_0|^2}{2\sigma^2} - \min \frac{|\hat{d}(i) - \alpha \hat{H} \hat{s}_1|^2}{2\sigma^2}, \quad (13)$$

where \hat{s}_o (or \hat{s}_1) is the candidate symbol with $b_j(i) = 0$ (or 1) for which the Euclidian distance from $\hat{d}(i)$ is minimum. $2\sigma^2$ denotes variance of the sum of AWGN, residual ISI after FDE and clipping noise used for LLR computation (see Appendix B). Turbo decoding is performed using the LLR sequence.

3. Simulation Results

We assume an OFDM/TDM frame size of $N_c = 256$ samples with the GI length of $N_g = 32$ samples. Over-sampling ratio is $J = 8$. The information bit sequence length is $2N_c = 1024$ bits. A rate 1/3 turbo encoder with constraint length 4 and (13, 15) recursive systematic convolutional (RSC) component encoders is applied. The parity bit sequences are punctured to obtain coding rate of 1/2. The turbo coded bit sequence is interleaved before data-modulation. A block bit interleaver is used as channel interleaver. Log-MAP decoding with 8 iterations is carried out at the receiver. The propagation channel is an $L = 16$ -path block Rayleigh fading channel with uniform power delay profile (i.e., $E[|h_l|^2] = 1/L$). We assume ideal channel estimation.

Computer simulation was carried out to measure the PAPR of OFDM/TDM, where the PAPR is defined as the maximum instantaneous peak power, measured over an OFDM/TDM frame, normalized by the average power. Figure 2 illustrates the complementary cumulative distribution function (CCDF) of the PAPR with $\beta = 4$ dB as a function of K for two cases; without clipping (w/o clipping) and with clipping and filtering (w/clipping). Due to amplitude clipping, undesired OoB spectrum grows. The use of amplitude clipping and filtering is effective to suppress OoB spectrum, but the PAPR re-grows. However, the clipped OFDM/TDM still has an advantage over the clipped OFDM, since the PAPR re-growth reduces as K increases. The PAPR level (at CCDF = 10^{-3}) of clipped OFDM/TDM with $K = 16$ (64) is about 1.7 (2.8) dB lower than the conventional OFDM ($K = 1$) when $\beta = 4$ dB. We also evaluated the PAPR for the case of $\beta = 0$ dB and found that the PAPR level (at CCDF = 10^{-3}) of clipped OFDM/TDM with $K = 16$ (64) is about 2.1 (2.5) dB lower than the conventional OFDM.

For higher data rate transmission, a higher level data modulation (e.g., 16QAM) is applied. We have also performed the computer simulation to show the PAPR advantage of clipped OFDM/TDM using 16QAM. According to our results, we found that the PAPR level (at CCDF = 10^{-3}) of clipped OFDM/TDM with $K = 16$ (64) is about 1.5 (2.6) and 1.9 (2.3) dB lower than the conventional OFDM when $\beta = 4$ and 0 dB, respectively.

The impact of the clipping level β on the average BER performance of uncoded and coded amplitude clipped OFDM/TDM with $K=16$ is shown in Fig. 3 as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 ($= 0.25 \times (E_s/N_0) \times (1 + N_g/N_c)$). It is seen that, for $\beta > 4$ dB, the performance degradation due to clipping is small for both uncoded and coded OFDM/TDM.

The required E_b/N_0 for achieving BER = 10^{-3} is shown

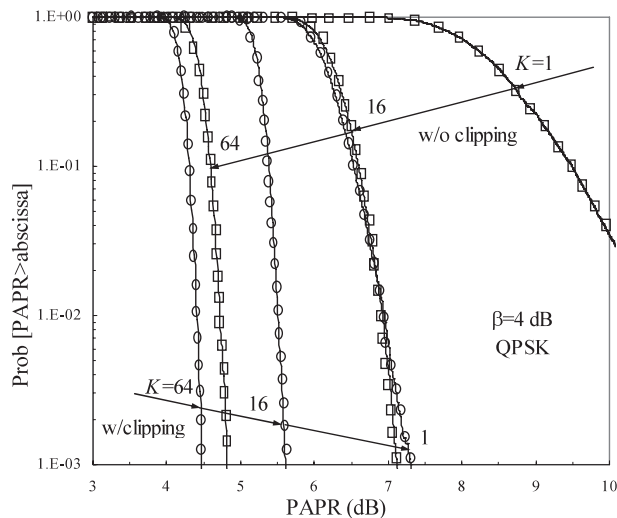


Fig. 2 PAPR distribution.

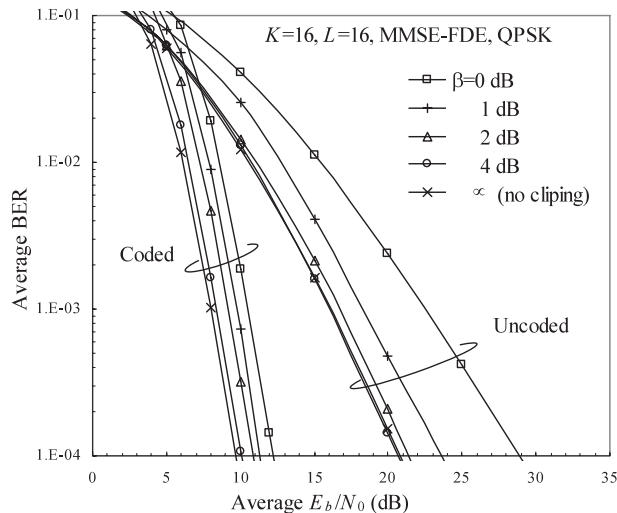


Fig. 3 Average BER performance.

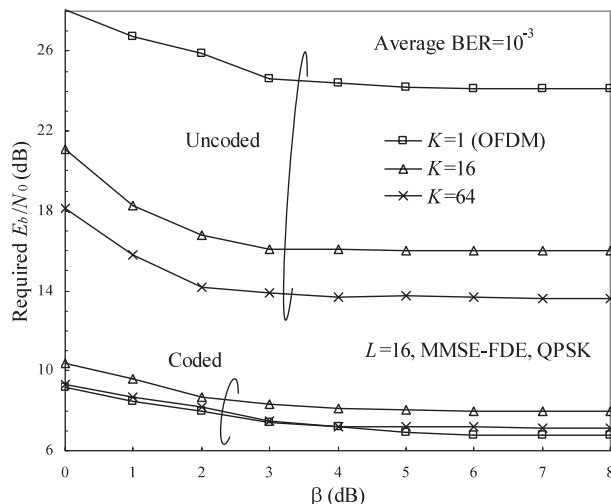


Fig. 4 Required E_b/N_0 as a function of β .

in Fig. 4 as a function of β with K as a parameter. As β reduces, the required E_b/N_0 increases because the clipping noise also increases. However, a small value of β (e.g., 4 dB) can be used for both OFDM/TDM and OFDM.

With $\beta = 4$ dB, without channel coding, OFDM/TDM with $K = 16$ (64) has an E_b/N_0 advantage of 7.5 (10.5) dB over OFDM. With channel coding, OFDM/TDM with $K = 16$ has an E_b/N_0 penalty of 0.9 dB in comparison with OFDM; however, with $K = 64$, almost the same BER can be achieved as OFDM. Consequently, with channel coding, $K = 64$ may be chosen to achieve the same BER as OFDM while still giving the lower PAPR as shown in Fig. 2.

4. Conclusions

In this letter, the effect of clipping on the performance of OFDM/TDM with and without channel coding was studied. It was shown that with channel coding, OFDM/TDM can achieve the same BER performance as OFDM with significantly reduced PAPR in comparison with OFDM.

For additional performance improvement, dynamic resource allocation (DRA) can be applied to OFDM/TDM; but, as K increases, a trade-off between the PAPR improvement and the reduction of degree of freedom for DRA (i.e., the number N_m of subcarriers) is present. Since this paper was intended to evaluate the PAPR benefit of OFDM/TDM, DRA is left as an interesting future work.

References

- [1] S. Hara and R. Prasad, *Multicarrier Techniques for 4G Mobile Communications*, Artech House, 2003.
- [2] X. Li and L.J. Cimini, Jr., "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol.2, no.5, pp.131–133, May 1998.
- [3] J. Armstrong, "New OFDM peak-to-average power reduction scheme," *54th IEEE Vehicular Technology Conference*, vol.1, pp.756–760, Oct. 2001.
- [4] H. Gacanin, S. Takaoka, and F. Adachi, "OFDM combined with TDM using frequency-domain equalization," *JCN, Division II: Wireless Communication*, vol.9, no.1, pp.34–42, March 2007.
- [5] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near optimum error correcting coding and decoding: Turbo codes," *IEEE Trans. Commun.*, vol.44, no.10, pp.1261–1271, Oct. 1996.
- [6] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol.40, no.4, pp.58–66, April 2002.
- [7] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels," *IEEE Trans. Commun.*, vol.48, no.3, pp.430–441, March 2000.
- [8] M. Friese, "On the degradation of OFDM signals due to peak-clipping in optimally predistorted power amplifiers," *IEEE GlobeCom*, vol.2,

pp.939–944, Sydney, Australia, Nov. 1998.

- [9] P. Robertson, E. Vilebrun, and P. Hoeher, "A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain," *Int. Conf. Communications (ICC)*, pp.1009–1013, June 1995.

Appendix A

We define the equalization error $e(n)$ as $e(n) = \hat{R}(n) - S(n)$ and thus, the MSE $E[|e(n)|^2]$ can be expressed as

$$E[|e(n)|^2] = \frac{\alpha^2 E_s}{T_c N_c^2} |H(n)w(n)|^2 + \frac{E_s}{T_c N_c^2} - \frac{2\alpha E_s}{T_c N_c^2} \text{Re}[H^*(n)w(n)] + \frac{N_0}{T_c N_c^2} |w(n)|^2 + \frac{(1 - e^{-\beta^2} - \alpha^2)E_s}{T_c N_c^2} |H(n)w(n)|^2. \quad (\text{A} \cdot 1)$$

Assuming that the clipping noise is a random process, and solving $\partial E[|e(n)|^2]/\partial w(n) = 0$ gives the following MMSE weight:

$$w(n) = \frac{\alpha H^*(n)}{|H(n)|^2 \left[1 - e^{-\beta^2} \right] + \left(\frac{E_s}{N_0} \right)^{-1}}. \quad (\text{A} \cdot 2)$$

Appendix B

Substituting Eq. (10) into Eq. (7) and applying N_c -point IFFT followed by N_m -point FFT, the decision variable is obtained as

$$\hat{d}^k(i) = \alpha \tilde{H} d^k(i) + \mu^k(i), \quad (\text{A} \cdot 3)$$

where $\tilde{H} = \frac{1}{N_c} \sum_{n=0}^{N_c-1} H(n)w(n)$ and $\mu^k(i)$ denotes the composite noise (i.e., the sum of clipping noise, AWGN and residual ISI), which is assumed to be a zero-mean complex-valued Gaussian variable. The variance of $\mu^k(i)$ is given by

$$2\sigma^2 = \sum_{n=0}^{N_c-1} \left[\alpha^2 \left| \hat{H}(n) - \frac{1}{N_c} \sum_{m=0}^{N_c-1} \hat{H}(m) \right|^2 + |w(n)|^2 \right] |\Psi(n; i, k)|^2, \quad (\text{A} \cdot 4)$$

where

$$\Psi(n; i, k) = \frac{1}{N_m} \sum_{t=kN_m}^{(k+1)N_m-1} \exp \left[-j2\pi t \frac{iK - n}{N_c} \right]. \quad (\text{A} \cdot 5)$$