PAPER

Channel Capacity of MC-CDMA and Impact of Residual ICI

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SUMMARY Orthogonal frequency division multiplexing (OFDM), which uses a number of narrowband orthogonal sub-carriers, is a promising transmission technique. Also multi-carrier code division multi-access (MC-CDMA), which combines OFDM and frequency-domain spreading, has been attracting much attention as a future broadband wireless access. It was shown that MC-CDMA has lower channel capacity than OFDM, due to inter-code interference (ICI) resulting from orthogonality distortion caused by frequency-selective fading. Recently, many ICI cancellers have been proposed to mitigate the effect of ICI. In this paper, we derive a channel capacity expression for MC-CDMA assuming perfect ICI cancellation taking into account both frequency diversity gain and space diversity gain and compare it to that of OFDM. Furthermore, we derive a channel capacity expression for the case of imperfect ICI cancellation to discuss the impact of the residual ICI.

key words: *MC-CDMA, channel capacity, ICI cancellation, theoretical study*

1. Introduction

For future wireless communication systems, the development of a broadband wireless access technique is required. In a broadband channel, the frequency-selectivity of the channel gets stronger and the bit error rate (BER) performance of single-carrier transmission is significantly degraded owing to the severe inter-path interference (IPI). Recently, multi-carrier transmission technique, which uses a number of narrowband orthogonal sub-carriers, e.g., such as orthogonal frequency division multiplexing (OFDM) and multi-carrier code division multi-access (MC-CDMA), has been attracting much attention [1].

In MC-CDMA, each data symbol to be transmitted is spread over a number of orthogonal sub-carriers by an orthogonal spreading code. Simple one-tap frequency domain equalization (FDE) per sub-carrier can be used to compensate the amplitude and phase distortions of each sub-carrier. The transmission rate can be increased by using code-multiplexing. However, the transmission performance of MC-CDMA degrades compared to that of OFDM since the inter-code interference (ICI) remains after FDE (this ICI is called the residual ICI) [2]. FDE can also be applied to improve the BER performance of single-carrier transmission and single-carrier direct-sequence code division multiple access (DS-CDMA) [3], [4]; however, the presence of residual ICI limits the BER performance improvement. Recently, several ICI cancellation techniques have been proposed for both MC- and DS-CDMA to reduce the residual ICI and its effectiveness was confirmed by computer simulation [5]–[7].

In [8], the impact of the channel parameters on the MC-CDMA channel capacity was studied assuming Nakagami-m fading. The capacity comparison of MC-CDMA with minimum mean square error (MMSE)-FDE, DS-CDMA with MMSE-FDE, and OFDM was theoretically studied in [9]. The channel capacity of MC-CDMA with matched filter was evaluated in [10]. However, to the best of authors' knowledge, the theoretically achievable upper-bound of MC-CDMA with ICI cancellation has not been studied.

In this paper, we derive a channel capacity expression for MC-CDMA assuming perfect ICI cancellation taking into account both the frequency diversity and space diversity gain and discuss the impact of the channel parameters such as the number of propagation paths and power delay profile. The channel capacity of MC-CDMA assuming perfect ICI cancellation is compared to that of OFDM derived in [11]. Furthermore, in this paper, we discuss the impact of the residual ICI and show that MC-CDMA provides larger channel capacity than OFDM when the residual ICI is sufficiently suppressed.

The rest of the paper is organized as follows. In Sect. 2, the system model is presented and the channel capacities of MC-CDMA assuming perfect ICI cancellation and assuming imperfect ICI cancellation are derived. The numerical computation results are given in Sect. 3. Section 4 concludes the paper.

2. Transmission System Model

2.1 Overall System Model

The transmission system model is shown in Fig. 1. In this paper, the sample-spaced discrete time representation is used. At the transmitter, the binary information sequence is first channel coded, data modulated, and then serial-to-parallel (S/P) converted into U parallel sequences $\{d_u(n); n = 0 \sim \lfloor N_c/SF \rfloor - 1\}$, $u = 0 \sim U-1$, where U is called the code multiplexing order and $\lfloor x \rfloor$ denotes

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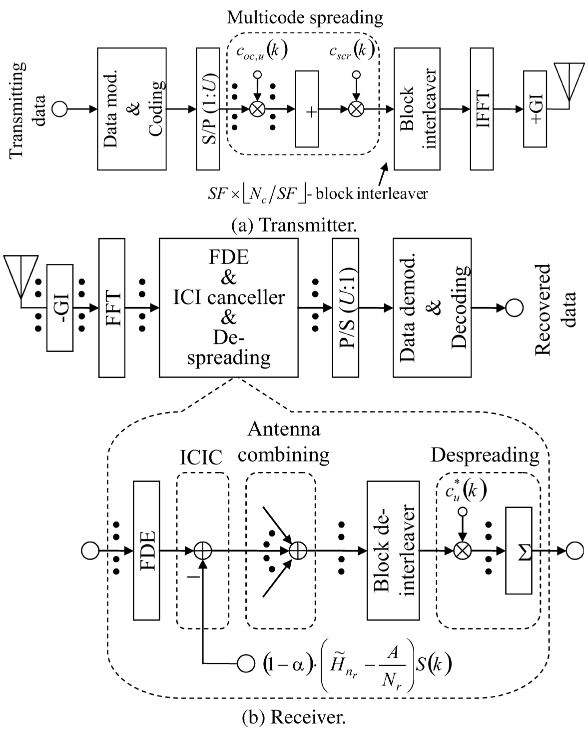


Fig. 1 Transmission system model.

the largest integer smaller than or equal to x. Each data symbol sequence is spread by orthogonal spreading code $\{c_{oc,u}(k); k = 0 \sim SF-1\}$ and then code multiplexed, where SF is called the spreading factor. The code multiplexed chip sequence is multiplied by a common scramble sequence $\{c_{scr}(k); k = 0 \sim N_c-1\}$ to make the resulting multicode signal white Gaussian noise like and then interleaved in the frequency-domain using an $SF \times \lfloor N_c/SF \rfloor$ chip-block interleaver. It has been shown [12] that if the scramble sequence is not used, the transmission performance degrades because of the periodic property of auto-correlation and cross-correlation functions of the orthogonal spreading codes. N_c-point inverse fast Fourier transform (IFFT) is applied to a block of code multiplexed chip sequence of N_c chips to generate an MC-CDMA signal.

Without loss of generality, we consider the transmission of the multi-code MC-CDMA signal $s(t)$ during $t = 0 \sim N_c - 1$. $s(t)$ can be expressed using the equivalent lowpass representation as

$$s(t) = \sum_{k=0}^{N_c-1} S(k)\exp\left(j2\pi\frac{t}{N_c}k\right), \tag{1}$$

with

$$\begin{aligned} S(k) &= \sum_{u=0}^{U-1} S_u(k)\,c_{scr}(k) \\ &= \sqrt{2P}\sum_{u=0}^{U-1} d(\lfloor k/SF\rfloor)\,c_{oc,u}(k \bmod SF)\,c_{scr}(k), \end{aligned} \tag{2}$$

where $P = E_c/(SF\cdot T_c)$ is the transmit power per spreading code with E_c being the signal energy per FFT sample and T_c being the FFT sample length. After the insertion of cyclic prefix (CP) into the guard interval (GI), the multicode MC-CDMA signal, $\{s(t \bmod N_c); t = -N_g \sim N_c - 1\}$, is transmitted, where N_g is the GI length.

The channel is assumed to be an L-path frequency-selective fading channel with each path being subjected to independent fading. The impulse response of the channel associated with the n_r-th receive antenna can be expressed as

$$h_{n_r}(\tau) = \sum_{l=0}^{L-1} h_{n_r,l}\,\delta(\tau-\tau_l), \tag{3}$$

where $h_{n_r,l}$ is the path gain with $E\left[\sum_{l=0}^{L-1}\left|h_{n_r,l}\right|^2\right] = 1$ ($E[.]$ denotes the ensemble average operation and $\delta(.)$ is the delta function) and τ_l is the time delay of the l-th path between the transmit antenna and the n_r-th receive antenna.

After removing the GI, the received signal $\{r_{n_r}(t); t = 0 \sim N_c - 1\}$ on the n_r-th antenna can be expressed as

$$r_{n_r}(t) = \sum_{l=0}^{L-1} h_{n_r,l}\,s((t-\tau_l) \bmod N_c) + n_{n_r}(t), \tag{4}$$

where $n_{n_r}(t)$ is a zero mean additive white Gaussian noise (AWGN) process having variance $2\sigma^2 = 2N_0/T_c$ with N_0 being the single-sided power spectrum density. The received signal is decomposed into N_c frequency components by applying N_c-point FFT. The k-th sub-carrier component of the received signal, $R_{n_r}(k)$, is expressed as

$$\begin{aligned} R_{n_r}(k) &= \frac{1}{N_c}\sum_{t=0}^{N_c-1} r_{n_r}(t)\exp\left(-j2\pi\frac{k}{N_c}t\right) \\ &= \frac{1}{N_c}\sum_{t=0}^{N_c-1}\left(\sum_{l=0}^{L-1} h_{n_r,l}\,s(t-\tau_l) + n_{n_r}(t)\right)\exp\left(-j2\pi\frac{k}{N_c}t\right) \\ &= H_{n_r}(k)\,S(k) + \Pi_{n_r}(k), \end{aligned} \tag{5}$$

where $H_{n_r}(k)$ and $\Pi_{n_r}(k)$ are the Fourier transforms of the channel impulse response and the AWGN, respectively. They are given as

$$\begin{cases} H_{n_r}(k) = \sum_{l=0}^{L-1} h_{n_r,l}\exp\left(-j2\pi\frac{k}{N_c}\tau_l\right) \\ \Pi_{n_r}(k) = \frac{1}{N_c}\sum_{t=0}^{N_c-1} n_{n_r}(t)\exp\left(-j2\pi\frac{k}{N_c}t\right) \end{cases}, \tag{6}$$

Letting $c_u(k) = c_{oc,u}(k)\,c_{scr}(k)$, the k-th sub-carrier component can be rewritten as

$$R_{n_r}(k) = H_{n_r}(k)\left(\sqrt{2P}\sum_{u=0}^{U-1} d_u\left(\left\lfloor\frac{k}{SF}\right\rfloor\right)c_u(k)\right) + \Pi_{n_r}(k). \tag{7}$$

Equation (7) can be further rewritten as

$$\begin{aligned} R_{n_r}(k) &= \sqrt{2P}H_{n_r}(k)c_u(k)d_u(\lfloor k/SF \rfloor) \\ &\quad + \mu_{n_r}(k) + \Pi_{n_r}(k), \end{aligned} \tag{8}$$

where $\mu_{n_r}(k)$ is the ICI component given by

$$\mu_{n_r}(k) = H_{n_r}(k)\left(\sqrt{2P}\sum_{\substack{u'=0 \\ \neq u}}^{U-1} d_{u'}\left(\left\lfloor \frac{k}{SF} \right\rfloor\right) c_{u'}(k)\right). \tag{9}$$

2.2 Channel Capacity Expression Assuming Perfect ICI Cancellation

The received signal can be expressed using the matrix form as

$$\begin{aligned} \mathbf{R}(k) &= \mathbf{H}(k)\,c_u(k)\,d_u(\lfloor k/SF \rfloor) + \boldsymbol{\mu}(k) + \boldsymbol{\Pi}(k) \\ &= \sqrt{2P}\begin{pmatrix} H_0(k) \\ \vdots \\ H_{N_r-1}(k) \end{pmatrix} c_u(k)\,d_u\left(\left\lfloor \frac{k}{SF} \right\rfloor\right) \\ &\quad + \begin{pmatrix} \mu_0(k) \\ \vdots \\ \mu_{N_r-1}(k) \end{pmatrix} + \begin{pmatrix} \Pi_0(k) \\ \vdots \\ \Pi_{N_r-1}(k) \end{pmatrix}, \end{aligned} \tag{10}$$

where $\mathbf{H}(k)$ is the channel gain column vector with the size of $N_r \times 1$. $\mathbf{R}(k)$ is extended to the equivalent received signal vector $\hat{\mathbf{R}}(n)$ with the size of $(N_r \cdot SF) \times 1$ for the n-th data symbol. $\hat{\mathbf{R}}(n)$ can be represented as [13]

$$\begin{aligned} \hat{\mathbf{R}}(n) &= \left(R_0(nSF), \cdots, R_{N_r-1}((n+1)SF-1)\right)^T \\ &= \sqrt{2P}\begin{pmatrix} H_0(nSF)\,c_u(0) \\ \vdots \\ H_0((n+1)SF-1) \\ \times c_u(SF-1) \\ \vdots \\ H_{N_r-1}(nSF)\,c_u(0) \\ \vdots \\ H_{N_r-1}((n+1)SF-1) \\ \times c_u(SF-1) \end{pmatrix} d_u(n) \\ &\quad + \begin{pmatrix} \mu_0(nSF) \\ \vdots \\ \mu_0\begin{pmatrix} (n+1)SF \\ -1 \end{pmatrix} \\ \vdots \\ \mu_{N_r-1}(nSF) \\ \vdots \\ \mu_{N_r-1}\begin{pmatrix} (n+1)SF \\ -1 \end{pmatrix} \end{pmatrix} + \begin{pmatrix} \Pi_0(nSF) \\ \vdots \\ \Pi_0\begin{pmatrix} (n+1)SF \\ -1 \end{pmatrix} \\ \vdots \\ \Pi_{N_r-1}(nSF) \\ \vdots \\ \Pi_{N_r-1}\begin{pmatrix} (n+1)SF \\ -1 \end{pmatrix} \end{pmatrix}. \end{aligned} \tag{11}$$

By defining the diagonal spreading code sequence with the size of $(N_r \cdot SF) \times (N_r \cdot SF)$ as

$$\mathbf{c}_u = diag(c_u(0), \cdots, c_u(SF-1), \cdots, c_u(0), \cdots, c_u(SF-1)), \tag{12}$$

$\hat{\mathbf{R}}(n)$ can be expressed as

$$\begin{aligned} \hat{\mathbf{R}}(n) &= \mathbf{c}_u\begin{pmatrix} H_0(nSF) \\ \vdots \\ H_0((n+1)SF-1) \\ \vdots \\ H_{N_r-1}(nSF) \\ \vdots \\ H_{N_r-1}((n+1)SF-1) \end{pmatrix}\sqrt{2P}d_u(n) \\ &\quad + \hat{\boldsymbol{\mu}}(n) + \hat{\boldsymbol{\Pi}}(n) \\ &= \mathbf{c}_u\hat{\mathbf{H}}(n)\sqrt{2P}d_u(n) + \hat{\boldsymbol{\mu}}(n) + \hat{\boldsymbol{\Pi}}(n). \end{aligned} \tag{13}$$

We use the Gaussian approximation of ICI. Using Eq. (13), the channel capacity, $C(n)$, for the n-th parallel channel is given by [14]

$$C(n) = \log_2 \frac{\det A_s \cdot \det A_r}{\det A_u}, \tag{14}$$

where $\det A_s$, $\det A_r$ and $\det A_u$ are given by

$$\begin{cases} \det A_s = \det E\left[\left(\sqrt{2P}d_u(n)\right)\left(\sqrt{2P}d_u(n)\right)^*\right] \\ \det A_r = \det E\left[\hat{\mathbf{R}}(n)\,\hat{\mathbf{R}}^H(n)\right] \\ \det A_u = \det E\left[\mathbf{u}(n)\,\mathbf{u}^H(n)\right] \end{cases}, \tag{15}$$

and

$$\mathbf{u}(n) = \left(\sqrt{2P}d_u(n), \hat{\mathbf{R}}^T(n)\right)^T. \tag{16}$$

Since we are assuming independent fading and $E\left[|H_{n_r}(k)|^2\right] = 1$, we have

$$\begin{cases} E\left[d_u(n)\,d_u^*(n)\right] = 1 \\ E\left[\hat{\boldsymbol{\mu}}(n)\,\hat{\boldsymbol{\mu}}^H(n)\right] = \dfrac{(U-1)\cdot 2E_c}{SF \cdot T_c} \cdot \mathbf{I}_{(N_r \cdot SF)} \\ E\left[\hat{\boldsymbol{\Pi}}(n)\,\hat{\boldsymbol{\Pi}}^H(n)\right] = \dfrac{2N_0}{N_c \cdot T_c} \cdot \mathbf{I}_{(N_r \cdot SF)} \end{cases}, \tag{17}$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix. Thus, Eq. (15) becomes

$$\begin{cases} \det A_s = \det\left(E\left[\left(\sqrt{2P}d_u(n)\right)\left(\sqrt{2P}d_u(n)\right)^*\right]\right) \\ = \dfrac{2E_c}{T_c \cdot SF} \\ \det A_r = \det\left(E\left[\hat{\mathbf{R}}(n)\,\hat{\mathbf{R}}^H(n)\right]\right) \\ = \det\begin{pmatrix} \dfrac{2E_c}{T_c \cdot SF} \cdot \mathbf{c}_u\hat{\mathbf{H}}(n)\,\hat{\mathbf{H}}^H(n)\,\mathbf{c}_u^H \\ +\left(\dfrac{(U-1)\cdot 2E_c}{T_c \cdot SF} + \dfrac{2N_0}{N_c \cdot T_c}\right) \cdot \mathbf{I}_{(N_r \cdot SF)} \end{pmatrix} \\ \det A_u = \det\left(E\left[\mathbf{u}(n)\,\mathbf{u}^H(n)\right]\right) \\ = \det A_s \cdot \det\left(\left(\dfrac{(U-1)\cdot 2E_c}{T_c \cdot SF} + \dfrac{2N_0}{N_c \cdot T_c}\right) \cdot \mathbf{I}_{(N_r \cdot SF)}\right) \end{cases}. \tag{18}$$

Substitution of Eq. (18) into Eq. (14) gives

$$C(n) = \log_2 \det\left(\mathbf{I}_{(N_r \cdot SF)} + \Gamma \cdot \mathbf{c}_u\hat{\mathbf{H}}(n)\,\hat{\mathbf{H}}^H(n)\,\mathbf{c}_u^H\right). \tag{19}$$

Since $\det\left(\mathbf{I}_{m\times m}+\mathbf{X}_{m\times n}\mathbf{Y}^H_{m\times n}\right)=\det\left(\mathbf{I}_{n\times n}+\mathbf{Y}^H_{m\times n}\mathbf{X}_{m\times n}\right)$ [15], $C(n)$ can be rewritten as

$$\begin{aligned} C(n) &= \log_2\left(1+\Gamma\cdot\hat{\mathbf{H}}^H(n)\,\hat{\mathbf{H}}(n)\right) \\ &= \log_2\left(1+\Gamma\cdot\sum_{n_r=0}^{N_r-1}\sum_{k=nSF}^{(n+1)SF-1}|H_{n_r}(k)|^2\right), \end{aligned} \tag{20}$$

where Γ is the received signal-to-interference plus noise power ratio (SINR) and given by

$$\Gamma=\frac{(1/SF)(E_s/N_0)}{(U-1)(1/SF)(E_s/N_0)+1}. \tag{21}$$

Assuming perfect ICI cancellation, the received SINR reduces to

$$\Gamma=\frac{1}{SF}\cdot\frac{E_s}{N_0}. \tag{22}$$

The channel capacity of MC-CDMA with spreading factor SF and code multiplexing order U is given by

$$\begin{aligned} C_{MC} &= \frac{U}{N_c}\sum_{n=0}^{\lfloor\frac{N_c}{SF}\rfloor-1}C(n) \\ &= \frac{U}{N_c}\sum_{n=0}^{\lfloor\frac{N_c}{SF}\rfloor-1}\log_2\left(1+\frac{1}{SF}\frac{E_s}{N_0}\sum_{n_r=0}^{N_r-1}\sum_{k=nSF}^{(n+1)SF-1}|H_{n_r}(k)|^2\right). \end{aligned} \tag{23}$$

On the other hand, the channel capacity of OFDM is given by [11]

$$\begin{aligned} C_{OFDM} &= \frac{1}{N_c}\sum_{n=0}^{N_c-1}C(n) \\ &= \frac{1}{N_c}\sum_{n=0}^{N_c-1}\log_2\left(1+\frac{E_s}{N_0}\sum_{n_r=0}^{N_r-1}|H_{n_r}(n)|^2\right). \end{aligned} \tag{24}$$

2.3 Channel Capacity Expression Assuming Imperfect ICI Cancellation

First, we derive the conditional received SINR. The ICI remains (i.e., residual ICI) since the equivalent channel gain is different from its average over all sub-carriers. The residual ICI can be expressed as [6], [7]

$$M_{n_r}(k)=\left(\tilde{H}_{n_r}(k)-\frac{A}{N_r}\right)S(k), \tag{25}$$

where $\tilde{H}_{n_r}(k)=w_{n_r}(k)H_{n_r}(k)$ ($w_{n_r}(k)$ is the FDE weight for the k-th sub-carrier at the n_r-th receive antenna and is given in Eq. (28)) and A is the average equivalent channel gain given by

$$A=\frac{1}{SF}\sum_{n_r=0}^{N_r-1}\sum_{k=nSF}^{(n+1)SF-1}\tilde{H}_{n_r}(k). \tag{26}$$

The analysis of channel capacity taking into account the residual ICI is quite difficult if not impossible. In this paper, we introduce a heuristic approach; the degree of ICI cancellation is represented by the parameter α, $\alpha=0$ represents perfect ICI cancellation. The received signal after ICI cancellation is expressed as

$$\begin{aligned} \tilde{R}(k) &= \sum_{n_r=0}^{N_r-1}\left(w_{n_r}(k)R_{n_r}(k)-(1-\alpha)\cdot M_{n_r}(k)\right) \\ &= AS(k)+\alpha\left\{\sum_{n_r=0}^{N_r-1}\left(\tilde{H}_{n_r}(k)-\frac{A}{N_r}\right)\right\}S(k)+\sum_{n_r=0}^{N_r-1}\tilde{\Pi}_{n_r}(k). \end{aligned} \tag{27}$$

There is a tradeoff relationship between orthogonality restoration and noise enhancement. Although perfect orthogonality can be restored by zero-forcing (ZF)-FDE, the noise enhancement results in the performance degradation. MMSE-FDE can restore the orthogonality to some degree while suppressing the noise enhancement. However, the residual ICI is present after MMSE-FDE. Thus, in this paper, assuming MMSE-FDE, the cancellation of the residual ICI is considered. The MMSE equalization weight, taking into account the receive antenna diversity and the residual ICI, can be obtained as [6]

$$w_{n_r}(k)=\frac{H^*_{n_r}(k)}{\alpha^2\cdot\sum_{n_r=0}^{N_r-1}|H_{n_r}(k)|^2+\left(\frac{U}{SF}\cdot\frac{E_s}{N_0}\right)^{-1}}. \tag{28}$$

The despreading operation is carried out to obtain the decision variable as

$$\begin{aligned} \tilde{d}_u(n) &= \frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\tilde{R}(k)\,c^*_u(k) \\ &= \frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\left(\begin{array}{l} AS(k) \\ +\alpha\left(\sum_{n_r=0}^{N_r-1}\left(\tilde{H}_{n_r}(k)-\frac{A}{N_r}\right)\right)S(k) \\ +\sum_{n_r=0}^{N_r-1}\tilde{\Pi}_{n_r}(k) \end{array}\right)c^*_u(k) \\ &= \sqrt{2P}Ad_u(n)+\mu_{ICI}+\mu_{noise}, \end{aligned} \tag{29}$$

where the 1st term represents the desired signal component, the 2nd term the residual ICI, and the 3rd term the noise component. μ_{ICI} and μ_{noise} are given as

$$\begin{cases} \mu_{ICI} \\ \quad =\alpha\sum_{\substack{u'=0\\ \neq u}}^{U-1}\left(\sqrt{2P}d_{u'}(n)\left(\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\left(\begin{array}{l}\left(\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right. \\ \left. -A\right) \\ \times c_{u'}(k)\,c^*_u(k)\end{array}\right)\right)\right) \\ \mu_{noise}=\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\tilde{\Pi}_{n_r}(k)\,c^*_u(k) \end{cases}. \tag{30}$$

The sub-carrier spacing is $1/T_s$ (where $T_s = T_c \cdot N_c$ is the effective OFDM symbol length). The instantaneous received SINR is the function of the received E_s/N_0 and the set of instantaneous channel gains, $\{H_{n_r}(k); k = 0 \sim N_c - 1\}$. The averaging operation over SF sub-carriers is carried out in the despreading process. $\lfloor N_c/SF \rfloor$ data symbols are transmitted during one MC-CDMA symbol interval. Here, the instantaneous received SINR of the n-th symbol ($n = 0 \sim \lfloor N_c/SF \rfloor - 1$) is denoted as $\gamma_n(E_s/N_0, \{H(k)\})$. Since μ_{ICI} is the sum of many components, the sum of μ_{ICI} and μ_{noise} can be treated as a new zero mean complex valued Gaussian variable μ according to the central limit theorem. The variance of μ is given by

$$\begin{aligned} 2\sigma_\mu^2 &= E\left[|\mu|^2\right] = 2\sigma_{ICI}^2 + 2\sigma_{noise}^2 \\ &= E\left[|\mu_{ICI}|^2\right] + E\left[|\mu_{noise}|^2\right], \end{aligned} \tag{31}$$

Since the transmit signal is uncorrelated to each other and spreading sequence is orthogonal to each other, i.e.,

$$\begin{cases} E\left[S_{u'}(n)S_{u''}^*(n)\right] = \dfrac{2E_c}{SF \cdot T_c}\delta(u' - u'') \\ E\left[c_{u'}(k)c_u^*(k)c_{u''}^*(k')c_u(k')\right] = \delta(k - k')\delta(u' - u'') \end{cases}, \tag{32}$$

the variance of μ_{ICI} and μ_{noise} can be given by (see Appendix)

$$\begin{cases} 2\sigma_{ICI}^2 = \\ \dfrac{\alpha^2}{SF} \cdot \dfrac{2E_c \cdot (U-1)}{SF \cdot T_c}\left(\begin{array}{l} \dfrac{1}{SF}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\left|\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right|^2 \\ -|A|^2 \end{array}\right) \\ 2\sigma_{noise}^2 = \dfrac{1}{SF^2} \cdot \dfrac{2N_0}{N_c \cdot T_c}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\left|w_{n_r}(k)\right|^2 \end{cases}. \tag{33}$$

The conditional received SINR, $\gamma_n(E_s/N_0, \{H(k)\})$, is given by

$$\begin{aligned} \gamma_n\left(\frac{E_s}{N_0}, \{H(k)\}\right) &= \frac{\left|\sqrt{\dfrac{E_c}{SF \cdot T_c}}Ad_u(n)\right|^2}{\sigma_\mu^2} \\ &= \frac{\dfrac{E_s}{N_0}\left|\dfrac{1}{SF}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right|^2}{\left(\begin{array}{l} \alpha^2 \cdot \dfrac{E_s}{N_0} \cdot \dfrac{(U-1)}{SF} \cdot \left(\begin{array}{l} \dfrac{1}{SF}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\left|\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right|^2 \\ -\left|\dfrac{1}{SF}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right|^2 \end{array}\right) \\ +\dfrac{1}{SF}\displaystyle\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\left|w_{n_r}(k)\right|^2 \end{array}\right)}. \end{aligned} \tag{34}$$

Since the signal bandwidth is $1/T_c$, the normalized channel capacity (bps/Hz) can be expressed as

$$C_{MC} = \frac{U}{N_c}\sum_{n=0}^{\lfloor N_c/SF \rfloor - 1}\log_2\left(1 + \gamma_n\left(\frac{E_s}{N_0}, \{H(k)\}\right)\right). \tag{35}$$

By substituting Eq. (34) into Eq. (35), the channel capacity of MC-CDMA assuming imperfect ICI cancellation can be obtained.

3. Numerical Results

The condition for numerical evaluation is summarized in Table 1. The number of sub-carriers is set to $N_c = 256$. The spreading factor SF is varied from 1 to 256 and the full code multiplexing is assumed, i.e., $U = SF$. The channel is assumed to be an L-path block Rayleigh fading channel with exponential power delay profile with the decay factor γ dB. The time delay of the l-th path is $\tau_l = l$ FFT samples. The receive antenna diversity is used with the number of receive antennas of $N_r = 1 \sim 6$. The performance loss owing to the insertion of GI is not taken into account in the paper. To obtain the diversity gain, a block interleaver with the size of $SF \times \lfloor N_c/SF \rfloor$ is used except for Fig. 2 and Fig. 3(a).

3.1 Perfect ICI Cancellation Case

The channel capacity of MC-CDMA assuming perfect ICI cancellation, which is given by Eq. (23), is compared to that of OFDM.

The channel capacity of MC-CDMA is plotted in Fig. 2 as a function of the spreading factor SF for both cases of interleaving and no interleaving. It can be seen from the figure that as spreading factor SF increases, the channel capacity increases owing to the increased frequency diversity gain if no frequency-domain block interleaving is used. However, if the interleaving is applied, even when the spreading factor is small (such as $SF = 16$), a sufficient frequency diversity gain can be obtained. It should be worth noticing that when receive antenna diversity is used (e.g., $N_r = 4$), the channel

Table 1 Simulation condition.

		MC-CDMA	OFDM
Number of sub-carriers		$N_c = 256$	
Spreading factor		$SF = 1 \sim 256$	
Code multiplex		$U = SF$	
Channel interleaver		Block interleaver	
Channel model	Fading		Block Rayleigh fading
	Number of paths		$L = 1 \sim 16$
	Decay factor		$\gamma = 0 \sim 6$ dB
	Maximum Doppler frequency		$f_D \cong 0$Hz
Number of receive antennas		$N_r = 1 \sim 6$	

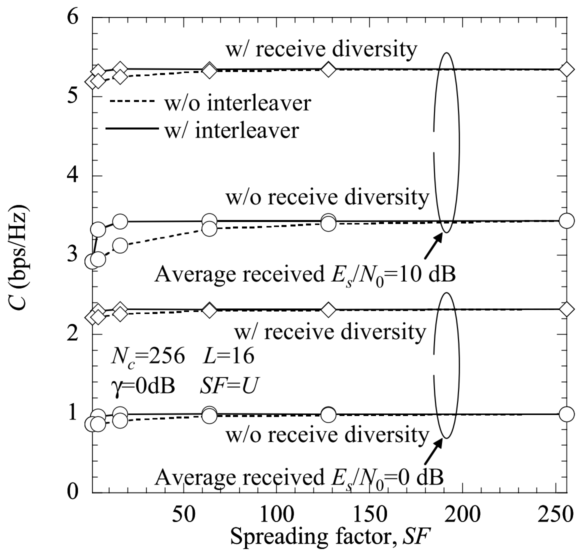
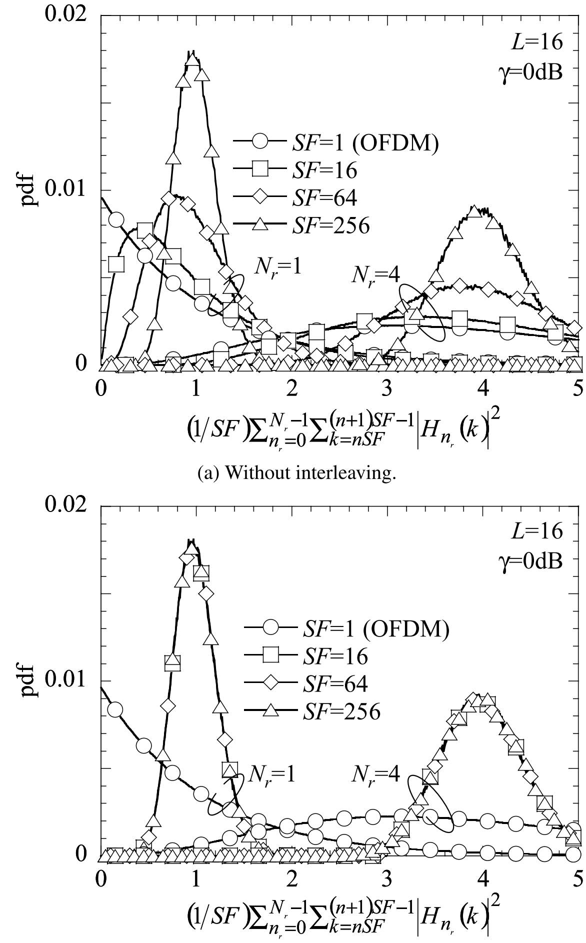


Fig. 2 Impacts of SF and interleaver on the channel capacity of MC-CDMA.

capacity is almost insensitive to SF except for the case of $SF = 1$. This is because a sufficient space diversity gain is obtained and therefore, an additional gain due to frequency diversity is small. The channel capacity of MC-CDMA depends on the power gain due to spreading as shown in Eq. (23). This means that the variability of the power gain affects the channel capacity. The probability density function (pdf) of power gain $(1/SF)\sum_{n_r=0}^{N_r-1}\sum_{k=nSF}^{(n+1)SF-1}\left|H_{n_r}(k)\right|^2$ is plotted in Fig. 3. First, we consider the case of no antenna diversity. The variability of power gain can significantly be reduced by increasing SF when no interleaving is used. However, when interleaving is used, almost the same power gain variability is seen irrespective of SF (except for $SF = 1$). This suggests that the achievable channel capacity increases as SF, but almost the same capacity can be obtained with interleaving. Next, we consider the case of antenna diversity. Similar trends to the no antenna diversity case can be observed; but the average power gain is much higher than no diversity case and therefore, much larger capacity can be achieved with antenna diversity. These observations support the results shown in Fig. 2.

The channel capacity of MC-CDMA and that of OFDM are plotted in Fig. 4 with the number of receive antennas, N_r, as a parameter. The difference between the channel capacity of MC-CDMA and that of OFDM becomes smaller owing to the increased space diversity gain as N_r increases.

The impact of channel parameters (the number of multi-paths and decay factor) is shown in Fig. 5. The channel capacity of OFDM is insensitive to the channel parameters and is only sensitive to the received signal power as discussed in [11]. On the other hand, the channel capacity of MC-CDMA increases owing to the increased frequency diversity gain as the number L of paths increases and/or the decay factor γ reduces. The channel capacity of MC-CDMA



(a) Without interleaving.

(b) With interleaving.

Fig. 3 Average equivalent channel gain.

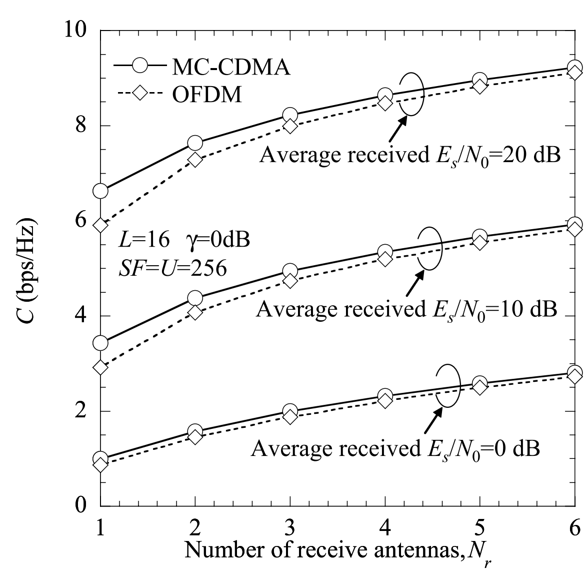
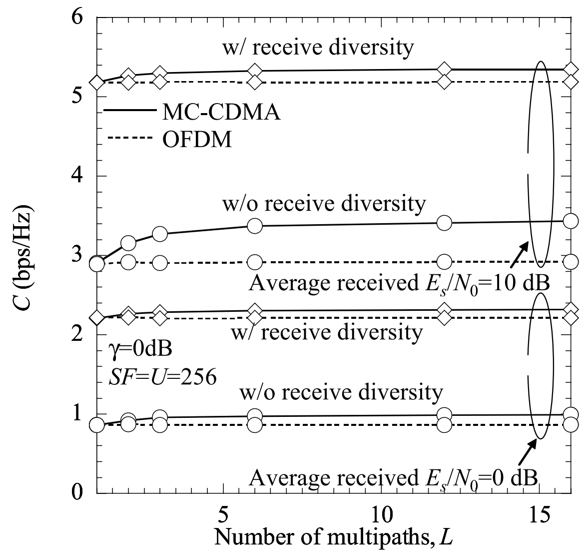
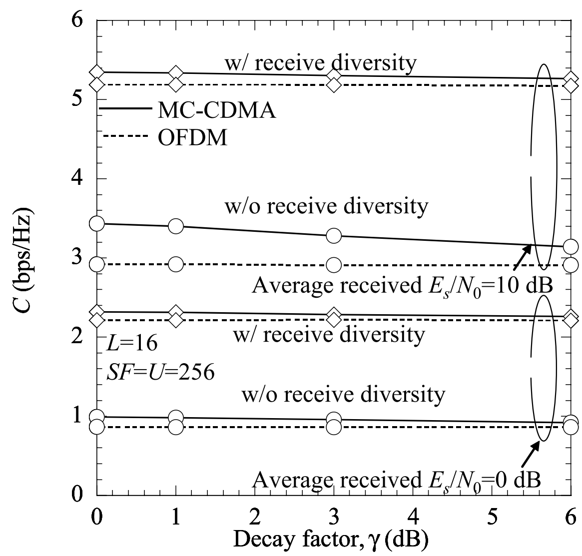


Fig. 4 Impact of antenna diversity.

is almost insensitive to the channel parameters when $N_r = 4$.



(a) Number of paths, L.



(b) Decay factor, γ.

Fig. 5 Impact of channel parameters.

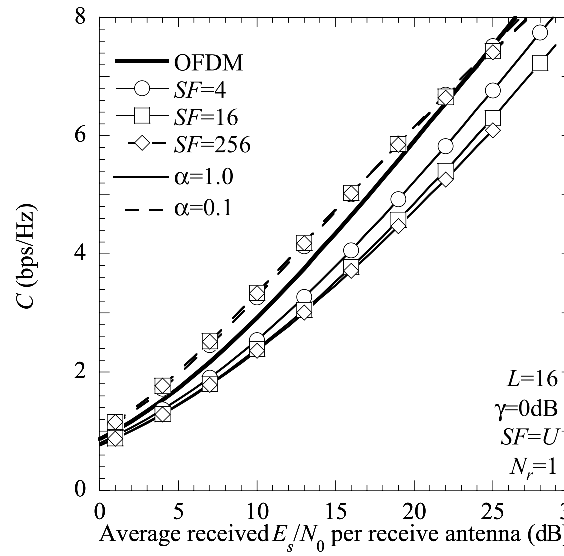


Fig. 6 Impact of ICI cancellation factor α.

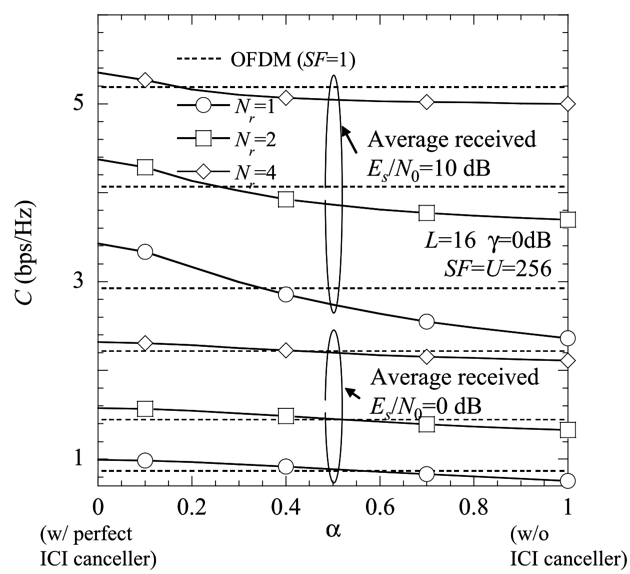


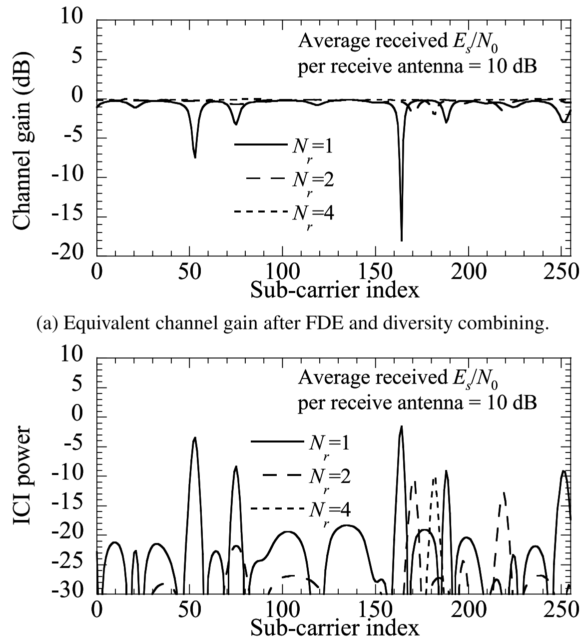
Fig. 7 Impact of ICI cancellation factor when receive antenna diversity is used.

3.2 Imperfect ICI Cancellation Case

The channel capacity of MC-CDMA assuming imperfect ICI cancellation, which is given by Eq. (35), is compared to that of OFDM.

The channel capacity of MC-CDMA is plotted in Fig. 6 as a function of the average received E_s/N_0 per receive antenna with the spreading factor SF and the ICI cancellation factor α as parameters. As can be seen from the figure, the channel capacity of MC-CDMA with $\alpha = 1$ (without ICI cancellation) is always smaller than that of OFDM irrespective of SF. On the other hand, the channel capacity of MC-CDMA with $\alpha = 0.1$ is larger than that of OFDM for the average received $E_s/N_0 < 20$ dB, but smaller than that of OFDM for the average received $E_s/N_0 > 20$ dB.

The impact of α on the channel capacity of MC-CDMA is plotted in Fig. 7 with the number N_r of receive antenna as a parameter for the average received $E_s/N_0 =$ 0 dB and 10 dB. The difference between the channelcapacity of MC-CDMA with $\alpha = 1$ and that of OFDM gets smaller as N_r increases. This can be explained below. As discussed in Sect. 2.3, the residual ICI is present since the equivalent channel gain after diversity combining $\sum_{n_r=0}^{N_r-1} \tilde{H}_{n_r}(k) = \sum_{n_r=0}^{N_r-1} w_{n_r}(k) H_{n_r}(k)$ after FDE differs from the average equivalent channel gain A. However, the variability of $\sum_{n_r=0}^{N_r-1} \tilde{H}_{n_r}(k)$ gets smaller and approaches the value of A given by Eq. (26) as N_r increases as shown in Fig. 8(a). As a consequence, the residual ICI reduces as shown in Fig. 8(b), resulting in the larger channel capacity. The channel capacity of MC-CDMA increases as α gets



(a) Equivalent channel gain after FDE and diversity combining.

(b) ICI power.

Fig. 8 Equivalent channel gain after FDE and ICI power.

smaller and almost the same channel capacity as OFDM can be achieved if $\alpha = 0.4 \sim 0.5$. Here, only receive antenna diversity was considered. However, space-time transmit diversity (STTD) [16] and site diveristy [17] can also be used to mitigate the adverse effect of residual ICI.

4. Conclusion

In the previous literature [8]–[10], it has been shown that the channel capacity of MC-CDMA is worse than that of OFDM. However, the performance degradation of MC-CDMA compared to OFDM is owing to the presence of residual ICI. In this paper, we derived a channel capacity expression for MC-CDMA with ICI cancellation taking into account both the frequency diversity gain and the space diversity gain. Furthermore, a channel capacity expression for MC-CDMA assuming imperfect ICI cancellation was derived and the impact of residual ICI on the channel capacity was discussed. The numerical computation results showed that if the residual ICI can be reduced to about 40% ~ 50%, MC-CDMA provides the channel capacity larger than OFDM.

In this paper, we took a heuristic approach and introduced the ICI cancellation factor to discuss the impact of residual ICI on the channel capacity of MC-CDMA. Next step is to find the value of α for a practical ICI cancellation technique. To what extent the practical ICI cancellation technique can mitigate the residual ICI is left as an interesting future study topic.

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References

[1] S. Hara and R. Prasad, "Overview of multicarrier CDMA," IEEE Commun., Mag., vol.35, no.12, pp.126–133, Dec. 1997.
[2] F. Adachi, D. Garg, S. Takaoka, and K. Takeda, "Broadband CDMA techniques," IEEE Wireless Commun. Mag., vol.12, no.2, pp.8–18, April 2005.
[3] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyarand, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., vol.40, no.4, pp.58–66, April 2002.
[4] F. Adachi, T. Sao, and T. Itagaki, "Performance of multicode DS-CDMA using frequency domain equalization in frequency selective fading channel," Electron. Lett., vol.39, no.2, pp.239–241, Jan. 2003.
[5] R. Dinis, P. Silva, and A. Gusmao, "An iterative frequency-domain decision-feedback receiver for MC-CDMA schemes," Proc. IEEE VTC'05-spring, pp.271–275, May-June 2005.
[6] K. Takeda, K. Ishihara, and F. Adachi, "Frequency-domain ICI cancellation with MMSE equalization for DS-CDMA downlink," IEICE Trans. Commun., vol.E89-B, no.12, pp.3335–3343, Dec. 2006.
[7] K. Ishihara, K. Takeda, and F. Adachi, "Iterative frequency-domain soft interference cancellation for multicode DS- and MC-CDMA transmission and performance comparison," IEICE Trans. Commun., vol.E89-B, no.12, pp.3344–3355, Dec. 2006.
[8] Z. Kang and K. Yao, "On the capacity and asymptotic performance of MC-CDMA system over Nakagami-m fading channels," Proc. IEEE VTC'04-spring, vol.2, pp.1153–1157, May 2004.
[9] M. Ma, Y. Yang, H. Cheng, and B. Jiao, "A capacity comparison between MC-CDMA and CP-CDMA," Proc. IEEE VTC'06-fall, pp.1–4, Sept. 2006.
[10] M. Debbah, "Capacity of a downlink MC-CDMA multi-cell network," IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), vol.4, pp.761–764, May 2004.
[11] H. Bölcskei, D. Gesbert, and A.J. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," IEEE Trans. Commun., vol.50, no.2, pp.225–234, Feb. 2002.
[12] D. Garg, High Speed Packet Access Techniques for Broadband CDMA Mobile Radio, Doctoral Thesis, Tohoku University, March 2005.
[13] K. Adachi, F. Adachi, and M. Nakagawa, "A study on channel capacity of MC-CDMA," IEICE Technical Report, RCS2007-91, Oct. 2007.
[14] G.J. Foschini and M. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," Wirel. Pers. Commun., vol.6, no.3, pp.311–335, March 1998.
[15] J.N. Franklin, Matrix theory, pp.80–81, Prentice-Hall, 1968.
[16] S.M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Sel. Areas Commun., vol.6, no.8, pp.1481–1458, Oct. 1998.
[17] T. Inoue, S. Takaoka, and F. Adachi, "Frequency-domain equalization for MC-CDMA downlink site diversity and performance evaluation," IEICE Trans. Commun., vol.E88-B, no.1, pp.84–92, Jan. 2005.

Appendix: Derivation of $2\sigma^2_{ICI}$ and $2\sigma^2_{noise}$.

The variance of residual ICI, μ_{ICI}, is given by

$$
\begin{aligned}
2\sigma_{ICI}^2 &= E\left[\left|\mu_{ICI}\right|^2\right] \\
&= E\left[\left|\alpha \sum_{\substack{u'=0 \\ \neq u}}^{U-1}\left(\frac{S_{u'}(n)}{SF}\sum_{k=nSF}^{(n+1)SF-1}\left(\begin{array}{l}\left(\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)-A\right) \\ \times c_{u'}(k)\, c_u^*(k)\end{array}\right)\right)\right|^2\right].
\end{aligned} \tag{A$\cdot$1}
$$

Since the transmitted signals are mutually uncorrelated and the spreading sequences are orthogonal to each other as

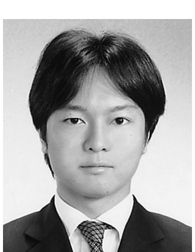
$$
\begin{cases} E\left[S_{u'}(n)\, S_{u''}^*(n)\right] = \dfrac{2E_c}{SF\cdot T_c}\delta\left(u'-u''\right) \\ E\left[c_{u'}(k)\, c_u^*(k)\, c_{u''}^*(k')\, c_u(k')\right] \end{cases}, \tag{A$\cdot$2}
$$

$2\sigma_{ICI}^2$ becomes

$$
\begin{aligned}
&2\sigma_{ICI}^2 \\
&= \left(\frac{\alpha}{SF}\right)^2\cdot\frac{2E_c\cdot(U-1)}{SF\cdot T_c}\sum_{k=nSF}^{(n+1)SF-1}\left|\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)-A\right|^2 \\
&= \frac{\alpha^2}{SF}\cdot\frac{2E_c\cdot(U-1)}{SF\cdot T_c}\left(\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\left|\sum_{n_r=0}^{N_r-1}\tilde{H}_{n_r}(k)\right|^2-|A|^2\right).
\end{aligned} \tag{A$\cdot$3}
$$

Since the noise components of different subcarriers are uncorrelated to each other, the variance of μ_{noise} is given by

$$
\begin{aligned}
&2\sigma_{noise}^2 = E\left[\left|\mu_{noise}\right|^2\right] \\
&= E\left[\left|\frac{1}{SF}\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\tilde{\Pi}_{n_r}(k)\, c_u^*(k)\right|^2\right] \\
&= \frac{1}{SF^2}\sum_{k=nSF}^{(n+1)SF-1}\sum_{k'=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\sum_{n_r'=0}^{N_r-1}E\left[\begin{array}{l} w_{n_r}(k)\,\Pi_{n_r}(k)\, c_u^*(k) \\ \times w_{n_r'}^*(k')\,\Pi_{n_r'}^*(k')\, c_u(k')\end{array}\right] \\
&= \frac{1}{SF^2}\cdot\frac{2N_0}{N_c\cdot T_c}\sum_{k=nSF}^{(n+1)SF-1}\sum_{n_r=0}^{N_r-1}\left|w_{n_r}(k)\right|^2.
\end{aligned} \tag{A$\cdot$4}
$$



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