

# RLS Channel Estimation with Adaptive Forgetting Factor for DS-CDMA Frequency-Domain Equalization

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**SUMMARY** Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can increase the downlink bit error rate (BER) performance of DS-CDMA beyond that possible with conventional rake combining in a frequency-selective fading channel. FDE requires accurate channel estimation. Recently, we proposed a pilot-assisted channel estimation (CE) based on the MMSE criterion. Using MMSE-CE, the channel estimation accuracy is almost insensitive to the pilot chip sequence, and a good BER performance is achieved. In this paper, we propose a channel estimation scheme using one-tap recursive least square (RLS) algorithm, where the forgetting factor is adapted to the changing channel condition by the least mean square (LMS) algorithm, for DS-CDMA with FDE. We evaluate the BER performance using RLS-CE with adaptive forgetting factor in a frequency-selective fast Rayleigh fading channel by computer simulation.

**key words:** DS-CDMA, frequency-domain equalization, channel estimation, RLS algorithm

## 1. Introduction

The 4th generation (4G) mobile communication systems [1] which provide broadband wireless services of e.g. 100 Mbps to 1 Gbps are expected around 2015. In the present 3rd generation (3G) systems, direct sequence-code division multiple access (DS-CDMA) is adopted as the wireless access technique [2]. However, since the broadband wireless channel is severely frequency-selective, the bit error rate (BER) performance of DS-CDMA with rake combining significantly degrades. The use of frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can provide a better BER performance of DS-CDMA than the rake combining [3], [4].

FDE requires accurate estimation of the channel transfer function. Pilot-assisted channel estimation (CE) can be used. Time-domain pilot-assisted CE was proposed for single-carrier transmission in [5]. After the channel impulse response is estimated according to the least-sum-of-squared-error (LSSE) criterion, the channel transfer function is obtained by applying fast Fourier transform (FFT). Frequency-domain pilot-assisted CE was proposed in [6], [7]. The received pilot signal is transformed into the frequency-domain pilot signal and then the pilot modulation is removed using zero forcing (ZF) or least square (LS) technique. As the pilot

signal, the Chu sequence [8] that has the constant amplitude in both time- and frequency-domain is used. However, the number of Chu sequences is limited. For example, it is only 128 for the case of 256-bit period [8].

PN sequences can be used for the pilot. Using a partial sequence taken from a long PN sequence, a very large number of pilots can be generated. However, since the frequency spectrum of the partial PN sequence is not constant, the use of ZF-CE produces noise enhancement [9]. The noise enhancement can be mitigated by the minimum mean square error channel estimation (MMSE-CE) [9]. Using MMSE-CE, the channel estimation accuracy is made almost insensitive to the pilot chip sequence. Recently, we proposed a 2-step maximum likelihood channel estimation (MLCE) to further improve the estimation accuracy [10]. However, the 2-step MLCE has the tracking ability problem in a fast fading environment since it assumes a block fading in which the channel gains stay constant over a frame. Channel estimation using recursive least square (RLS) algorithm was proposed to track the time-varying channels [11]. In [11], RLS algorithm and the superimposed training sequences are applied for channel estimation in orthogonal frequency division multiplexing (OFDM). However, the forgetting factor of RLS algorithm was not adapted in [11] and needs to be set according to the channel condition.

In this paper, we propose a channel estimation scheme using one-tap RLS algorithm, where the forgetting factor is adapted to the changing channel condition by the least mean square (LMS) algorithm, for DS-CDMA with FDE. We evaluate, by computer simulation, the BER performance of DS-CDMA using RLS-CE with adaptive forgetting factor in a frequency-selective fast Rayleigh fading channel. The achievable BER performance is compared with those using 2-step MLCE and using MMSE-CE with 1st order interpolation.

## 2. Transmission System Model

### 2.1 Overall Transmission System Model

The transmission system model for multicode DS-CDMA with FDE is illustrated in Fig. 1. Throughout the paper, the chip-spaced discrete-time signal representation is used. At the transmitter, a binary data sequence is transformed into data-modulated symbol sequence and then converted to  $U$  parallel streams by serial-to-parallel (S/P) conversion. Then,

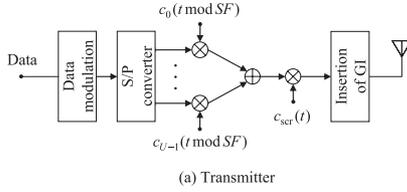
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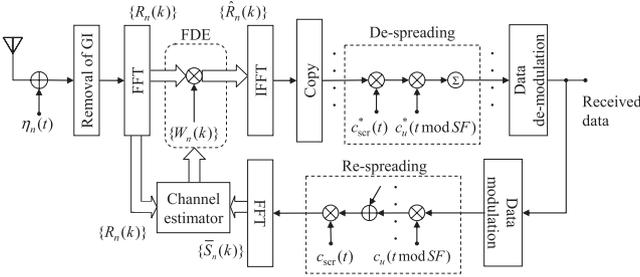
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(a) Transmitter



(b) Receiver

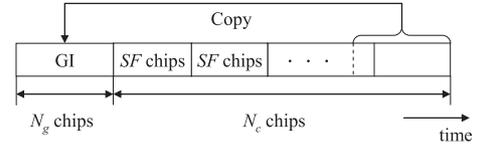
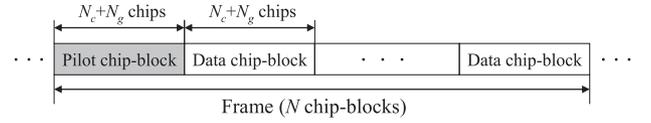
**Fig. 1** Transmitter/receiver structure for DS-CDMA with FDE.

each parallel stream is divided into a sequence of blocks of  $N_c/SF$  symbols each, where  $N_c$  and  $SF$  denote the size of the chip-block and the spreading factor, respectively. The  $m$ th data symbol of the  $n$ th chip-block ( $n = 0 \sim N-1$ ) in the  $u$ th stream is represented by  $d_{n,u}(m)$ ,  $m = 0 \sim N_c/SF - 1$ .  $d_{n,u}(m)$  is spread by multiplying it with an orthogonal spreading sequence  $\{c_u(t); t = 0 \sim SF - 1\}$ . The resultant  $U$  chip-blocks of  $N_c$  chips each are added and further multiplied by a common scramble sequence  $\{c_{scr}(t); t = \dots, 1, 0, 1, \dots\}$  to make the resultant multicode DS-CDMA chip-block like white-noise. The last  $N_g$  chips of each  $N_c$  chip-block is copied as a cyclic prefix and inserted into the guard interval (GI) placed at the beginning of each chip-block, as illustrated in Fig. 2. For channel estimation, one pilot chip-block is transmitted every  $N-1$  data chip-blocks to constitute a frame of  $N$  chip-blocks, as shown in Fig. 3.

The GI-inserted chip-block is transmitted over a frequency-selective fading channel and is received at a receiver. After removal of the GI, the received chip-block is decomposed by  $N_c$ -point FFT into  $N_c$  frequency components. The channel estimation using RLS-CE is performed as follows. The RLS-CE is carried out using the received pilot chip-block. Using the channel estimate, a series of MMSE-FDE,  $N_c$ -point inverse FFT (IFFT), de-spreading and data de-modulation is performed on the 1st data chip-block in the frame. Then, the chip-block replica is regenerated and the RLS-CE is carried out using the chip-block replica as a pilot. The forgetting factor used in the RLS algorithm is updated using the LMS algorithm (see Sect. 3). This is repeated until the reception of the last data chip-block in the frame.

## 2.2 Signal Representation

The  $n$ th chip-block  $\{\tilde{s}_n(t); t = 0 \sim N_c - 1\}$  can be expressed, using the equivalent lowpass representation, as

**Fig. 2** Chip-block structure.**Fig. 3** Frame structure.

$$\tilde{s}_n(t) = \sqrt{2P} s_n(t) \quad (1)$$

with

$$s_n(t) = \left\{ \sum_{u=0}^{U-1} d_{n,u} \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) c_u(t \bmod SF) \right\} c_{scr}(t), \quad (2)$$

where  $P$  is the transmit power and  $\lfloor x \rfloor$  represents the largest integer smaller than or equal to  $x$ . After inserting the GI of  $N_g$  chips, the  $n$ th chip-block is transmitted. The propagation channel is assumed to be a frequency-selective fading channel having chip-spaced  $L$  discrete paths, each subjected to independent fading. The channel impulse response  $h_n(\tau)$  can be expressed as

$$h_n(\tau) = \sum_{l=0}^{L-1} h_{n,l} \delta(\tau - \tau_l), \quad (3)$$

where  $h_{n,l}$  and  $\tau_l$  are the complex-valued path gain and time delay of the  $l$ th path ( $l = 0 \sim L-1$ ), respectively, with  $\sum_{l=0}^{L-1} E[|h_{n,l}|^2] = 1$  ( $E[\cdot]$  denotes the ensemble average operation). In this paper, we assume that the maximum time delay difference  $\tau_{L-1} - \tau_0$  of the channel is shorter than the GI length. We assume that the path gains stay constant over one chip-block but change block by block.

The  $n$ th received chip-block  $\{r_n(t); t = 0 \sim N_c - 1\}$  can be expressed as

$$r_n(t) = \sum_{l=0}^{L-1} h_{n,l} \tilde{s}_n(t - \tau_l) + \eta_n(t), \quad (4)$$

where  $\eta_n(t)$  is a zero-mean complex Gaussian process with variance  $2N_0/T_c$  with  $T_c$  and  $N_0$  being respectively the chip duration and the single-sided power spectrum density of the additive white Gaussian noise (AWGN).

## 2.3 MMSE-FDE

After the removal of the GI, the received chip-block is decomposed by an  $N_c$ -point FFT into  $N_c$  frequency components. The  $k$ th frequency component of the  $n$ th chip-block ( $n = 0 \sim N-1$ ) can be written as

$$\begin{aligned} R_n(k) &= \sum_{t=0}^{N_c-1} r_n(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ &= H_n(k)S_n(k) + \Pi_n(k), \end{aligned} \quad (5)$$

where  $H_n(k)$  is the channel gain,  $S_n(k)$  is the signal component, and  $\Pi_n(k)$  is the noise due to zero-mean AWGN. They are given by

$$\begin{cases} S_n(k) = \sum_{t=0}^{N_c-1} s_n(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ H_n(k) = \sqrt{2P} \sum_{l=0}^{L-1} h_{n,l} \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right) \\ \Pi_n(k) = \sum_{t=0}^{N_c-1} \eta_n(t) \exp\left(-j2\pi k \frac{t}{N_c}\right). \end{cases} \quad (6)$$

One-tap MMSE-FDE is carried out as

$$\hat{R}_n(k) = W_n(k)R_n(k), \quad (7)$$

where  $W_n(k)$  is the MMSE-FDE weight and is given by [3], [4]

$$W_n(k) = \frac{H_n^*(k)}{UN_c |H_n(k)|^2 + 2\sigma^2} \quad (8)$$

with  $2\sigma^2 (= 2N_0N_c/T_c)$  being the variance of  $\Pi_n(k)$  and  $*$  denoting the complex conjugate operation.  $H_n(k)$  and  $\sigma^2$  are unknown to the receiver and need to be estimated. In this paper,  $H_n(k)$  is estimated by RLS-CE.  $\sigma^2$  can be estimated according to [9].

$N_c$ -point IFFT is applied to transform the frequency-domain signal  $\{\hat{R}_n(k); k = 0 \sim N_c - 1\}$  into the time-domain chip-block  $\{\hat{r}_n(t); t = 0 \sim N_c - 1\}$  as

$$\hat{r}_n(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{R}_n(k) \exp\left(j2\pi t \frac{k}{N_c}\right). \quad (9)$$

Finally, de-spreading is carried out on  $\{\hat{r}_n(t)\}$ , giving

$$\hat{d}_{n,u}(m) = \frac{1}{SF} \sum_{t=mSF}^{(m+1)SF-1} \hat{r}_n(t)c_u^*(t \bmod SF)c_{scr}^*(t), \quad (10)$$

which is the decision variable for data de-modulation on  $\hat{d}_{n,u}(m)$ .

### 3. RLS Channel Estimation Using Adaptive Forgetting Factor

The channel estimate of  $H_n(k)$  is denoted by  $\tilde{H}_n(k)$ . In Sect. 3.1, we present the one-tap RLS algorithm for the channel estimation. Then, the forgetting factor adaptation is described in Sect. 3.2. The chip-block replica is generated to be used as a pilot in the channel estimation for MMSE-FDE of the next chip-block. The noise reduction in the channel estimate is done by applying the delay-time

domain windowing technique [12], [13]. These are presented in Sect. 3.3. In Sect. 3.4, the computational complexity of the proposed RLS-CE is compared with 2-step MLCE [10], MMSE-CE with decision-feedback [9], and MMSE-CE with 1st order interpolation.

#### 3.1 One-Tap RLS Algorithm

One-tap RLS algorithm is illustrated in Fig. 4. We use the following cost function for the RLS algorithm [14]:

$$\varepsilon_n(k) = \sum_{i=1}^n \lambda^{n-i} |e_i(k)|^2, \quad (11)$$

where  $e_i(k)$  is given by

$$e_i(k) = R_i(k) - \tilde{H}_n(k)S_i(k), \quad (12)$$

where  $\lambda$  ( $0 < \lambda \leq 1$ ) is the forgetting factor. The channel estimate  $\tilde{H}_n(k)$  is the one that minimizes  $\varepsilon_n(k)$ . Solving  $\partial\varepsilon_n(k)/\partial\tilde{H}_n(k) = 0$  gives

$$\tilde{H}_n(k) = Z_n(k)/\Phi_n(k), \quad (13)$$

where  $Z_n(k)$  and  $\Phi_n(k)$  are respectively given by

$$\begin{cases} Z_n(k) = \sum_{i=1}^n \lambda^{n-i} R_i(k)S_i^*(k) \\ \Phi_n(k) = \sum_{i=1}^n \lambda^{n-i} |S_i(k)|^2. \end{cases} \quad (14)$$

To update  $Z_n(k)$  and  $\Phi_n(k)$  recursively, they are rewritten as

$$\begin{cases} Z_n(k) = \lambda Z_{n-1}(k) + R_n(k)S_n^*(k) \\ \Phi_n(k) = \lambda \Phi_{n-1}(k) + |S_n(k)|^2. \end{cases} \quad (15)$$

Substituting Eq. (15) into Eq. (13) gives the following update equation based on the RLS algorithm:

$$\tilde{H}_n(k) = \tilde{H}_{n-1}(k) + G_n(k)\xi_n(k), \quad (16)$$

where

$$\begin{cases} G_n(k) = S_n^*(k)/\Phi_n(k) \\ \xi_n(k) = R_n(k) - \tilde{H}_{n-1}(k)S_n(k). \end{cases} \quad (17)$$

The optimum forgetting factor  $\lambda$  changes according to the change in the channel statistical property (i.e., fading rate and fading type). In this paper, assuming that channel statistical property does not change rapidly,  $\lambda$  is adapted by the LMS algorithm [14].

#### 3.2 Adaptive Algorithm of Forgetting Factor $\lambda$

The following cost function is used:

$$J_n(k) = \frac{1}{2} E[|\xi_n(k)|^2]. \quad (18)$$

Finding  $\lambda$  that minimizes Eq. (18) corresponds to the steepest descent method as

$$\lambda_n(k) = \lambda_{n-1}(k) + \mu(-\nabla\lambda_n(k)), \quad (19)$$

where  $\mu$  is the step size and the gradient vector  $\nabla\lambda_n(k)$  is the differentiation of the cost function  $J_n(k)$  with respect to  $\lambda$ .  $\nabla\lambda_n(k)$  is given as

$$\nabla\lambda_n(k) = \frac{\partial J_n(k)}{\partial \lambda} = -\text{Re}[E[\Psi_{n-1}(k)S_n(k)\xi_n^*(k)]], \quad (20)$$

where

$$\Psi_n(k) = \frac{\partial \tilde{H}_n(k)}{\partial \lambda}. \quad (21)$$

Substituting Eqs. (16) and (18) into (21) gives

$$\begin{aligned} \Psi_n(k) &= (1 - G_n(k)S_n(k))\Psi_{n-1}(k) \\ &\quad + \frac{\partial \Phi_n^{-1}(k)}{\partial \lambda} S_n^*(k)\xi_n(k). \end{aligned} \quad (22)$$

Differentiation of inverse of Eq. (15) with respect to  $\lambda$  gives the updating equation for  $\partial\Phi_n^{-1}(k)/\partial\lambda$  as

$$\begin{aligned} \frac{\partial \Phi_n^{-1}(k)}{\partial \lambda} &= \lambda^{-1} \left\{ (1 - G_n(k)S_n(k))^2 \frac{\partial \Phi_{n-1}^{-1}(k)}{\partial \lambda} \right. \\ &\quad \left. + |G_n(k)|^2 + \Phi_n^{-1}(k) \right\}. \end{aligned} \quad (23)$$

The above mentioned steepest descent method requires the gradient vector  $\nabla\lambda_n(k)$  at each iteration  $n$ . However,  $\nabla\lambda_n(k)$  is unknown and must be estimated using the available data. The instantaneous estimate of  $\nabla\lambda_n(k)$  on the basis of Eq. (20) is

$$\hat{\nabla}\lambda_n(k) = -\text{Re}[\Psi_{n-1}(k)S_n(k)\xi_n^*(k)]. \quad (24)$$

Replacing  $\nabla\lambda_n(k)$  in Eq. (19) by Eq. (24), we obtain the following LMS algorithm for updating the forgetting factor:

$$\lambda_n(k) = \lambda_{n-1}(k) + \mu \text{Re}[\Psi_{n-1}(k)S_n(k)\xi_n^*(k)]. \quad (25)$$

The forgetting factor  $\lambda_n(k)$  depends on statistical characteristics of the fading channel. Statistical characteristics are identical for all frequencies. Therefore, in this paper,  $\lambda_n = (1/N_c) \sum_{k=0}^{N_c-1} \lambda_n(k)$  is used to suppress the noise.

The proposed RLS-CE requires the knowledge of the transmitted chip-block  $\{S_n(k); k = 0 \sim N_c - 1\}$ ,  $n \geq 1$ . However, since  $\{S_n(k); k = 0 \sim N_c - 1\}$  is unknown at the receiver, the transmitted chip-block replica  $\{\tilde{S}_n(k); k = 0 \sim N_c - 1\}$  needs to be generated by the decision-feedback. This is done as follows. First, MMSE-FDE (Eqs. (7) and (8)) is carried out using the channel estimate for the  $(n-1)$ th chip-block. After performing a series of MMSE-FDE,  $N_c$ -point IFFT, de-spreading and data de-modulation on the  $n$ th chip-block, the tentatively detected symbol sequence  $\{\tilde{d}_{n,u}(m); m = 0 \sim N_c/SF - 1\}$ ,  $u = 0 \sim U - 1$ , is spread to generate the transmitted chip-block replica  $\{\tilde{s}_n(t); t = 0 \sim N_c - 1\}$ :

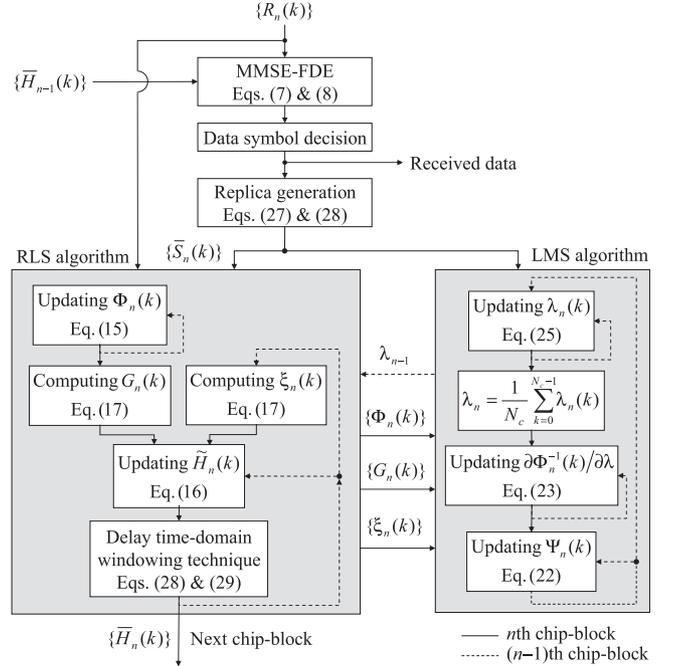


Fig. 4 One-tap RLS algorithm using adaptive forgetting factor.

$$\tilde{s}_n(t) = \left\{ \sum_{u=0}^{U-1} \tilde{d}_{n,u} \left( \left\lfloor \frac{t}{SF} \right\rfloor \right) c_u(t \bmod SF) \right\} c_{scr}(t). \quad (26)$$

The chip-block replica is transformed by an  $N_c$ -point FFT into  $N_c$  frequency components  $\{\tilde{S}_n(k); k = 0 \sim N_c - 1\}$ . The  $k$ th frequency component  $\tilde{S}_n(k)$  of the transmitted chip-block replica is obtained as

$$\tilde{S}_n(k) = \sum_{t=0}^{N_c-1} \tilde{s}_n(t) \exp(-j2\pi k \frac{t}{N_c}). \quad (27)$$

Using  $\{\tilde{S}_n(k); k = 0 \sim N_c - 1\}$  instead of  $\{S_n(k); k = 0 \sim N_c - 1\}$ , the channel estimate  $\{\tilde{H}_n(k); k = 0 \sim N_c - 1\}$  for the  $n$ th chip-block is obtained by RLS algorithm (Eqs. (16) and (17)). The forgetting factor  $\lambda_n$  is updated by LMS algorithm (Eqs. (22), (23) and (25)).

The flowchart of the above-mentioned one-tap RLS algorithm using adaptive forgetting factor is illustrated in Fig. 4.

### 3.3 Further Improvement by Delay Time-Domain Windowing Technique

The instantaneous channel estimate  $\{\tilde{H}_n(k); k = 0 \sim N_c - 1\}$  obtained by RLS-CE is perturbed by the noise due to the AWGN. In this paper, the delay time-domain windowing technique [12], [13] is introduced to reduce the noise. The instantaneous channel estimate  $\{\tilde{H}_n(k); k = 0 \sim N_c - 1\}$  is transformed by  $N_c$ -point IFFT into the instantaneous channel impulse response  $\{\tilde{h}_n(\tau); \tau = 0 \sim N_c - 1\}$  as

$$\tilde{h}_n(\tau) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_n(k) \exp(j2\pi\tau \frac{k}{N_c}). \quad (28)$$

The actual channel impulse response is present only within the GI length, while the noise is spread over an entire delay-time range. Replacing  $\tilde{h}_n(\tau)$  with zero's for  $N_g \leq \tau \leq N_c - 1$  and applying  $N_c$ -point FFT, the improved channel estimate  $\{\tilde{H}_n(k); k = 0 \sim N_c - 1\}$  is obtained as

$$\tilde{H}_n(k) = \sum_{\tau=0}^{N_g-1} \tilde{h}_n(\tau) \exp\left(-j2\pi k \frac{\tau}{N_c}\right). \quad (29)$$

The MMSE-FDE weight of the  $(n+1)$ th chip-block is computed using Eq. (8) by replacing  $H_n(k)$  by  $\tilde{H}_n(k)$ . The channel estimate of the  $(n+1)$ th chip-block is updated using  $\tilde{H}_n(k)$ .

### 3.4 Complexity Comparison

The computational complexity of the proposed RLS-CE is compared with 2-step MLCE [10], MMSE-CE with decision-feedback [9], and MMSE-CE with 1st order interpolation in terms of the number of complex multiplication operations (CMOs) per frame. Noting that  $N_c$ -point FFT (and also IFFT) operation requires  $(N_c/2) \log_2 N_c$  CMOs, Table 1 compares four channel estimation schemes. For  $SF = 16$ ,  $U = 16$ ,  $N_c = 256$ , and  $N = 16$ , it can be shown that the complexity of RLS-CE is approximately 1.2 times higher than MMSE-CE with decision-feedback and is approximately 1.9 times higher than MMSE-CE with 1st order interpolation. However, the complexity of RLS-CE is

**Table 1** CMOs of four channel estimation schemes.

No. of CMOs per frame	RLS-CE	2-step MLCE [10]
RLS-CE	$N(16N_c + 1)$	—
MMSE-CE	—	$3N_c$
MLCE	—	$(N + 1)N_c$
Delay time-domain windowing	$NN_c \log_2 N_c$	$2N_c \log_2 N_c$
FDE & IFFT & de-spreading	$(N - 1)\{3N_c + (N_c/2) \log_2 N_c + U(N_c/SF)(2SF + 1)\}$	$2(N - 1)\{3N_c + (N_c/2) \log_2 N_c + U(N_c/SF)(2SF + 1)\}$
Data chip-block replica generation	$(N - 1)\{N_c(U + 1) + (N_c/2) \log_2 N_c\}$	$(N - 1)\{N_c(U + 1) + (N_c/2) \log_2 N_c\}$
No. of CMOs per frame	MMSE-CE with decision-feedback [9]	MMSE-CE with 1st order interpolation
MMSE-CE	$3NN_c$	$6N_c$
1st order interpolation	—	$8N_c$
Delay time-domain windowing	$NN_c \log_2 N_c$	$2N_c \log_2 N_c$
1st order filtering	$2(N - 1)N_c$	—
FDE & IFFT & de-spreading	$(N - 1)\{3N_c + (N_c/2) \log_2 N_c + U(N_c/SF)(2SF + 1)\}$	$(N - 1)\{3N_c + (N_c/2) \log_2 N_c + U(N_c/SF)(2SF + 1)\}$
Data chip-block replica generation	$(N - 1)\{N_c(U + 1) + (N_c/2) \log_2 N_c\}$	—

approximately 0.8 times that of 2-step MLCE.

## 4. Computer Simulation

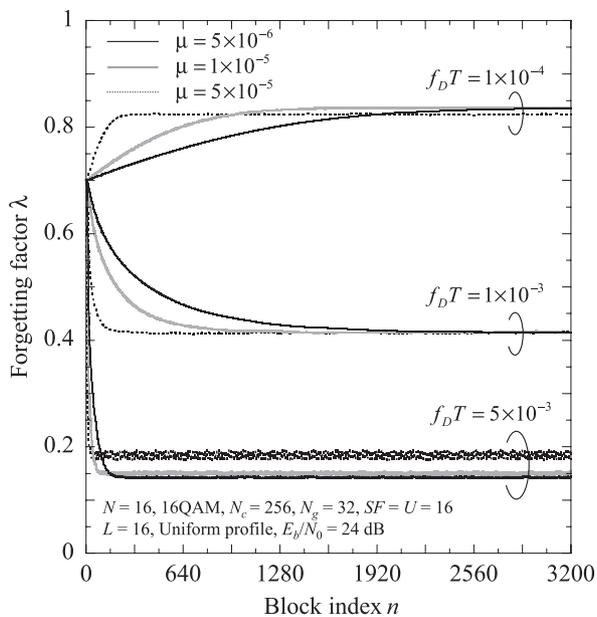
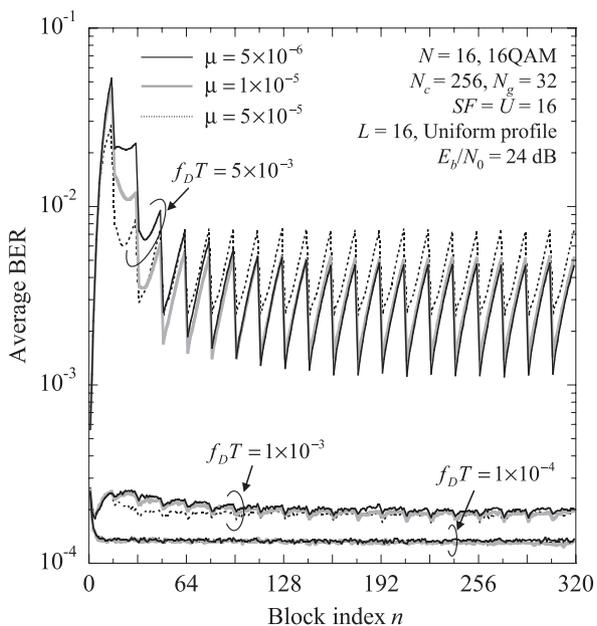
The simulation condition is shown in Table 2. We assume 16QAM data modulation, an FFT block size of  $N_c = 256$  chips and a GI of  $N_g = 32$  chips. One pilot chip-block is transmitted every 15 data chip-blocks (i.e.,  $N = 16$ ). We assume the spreading factor  $SF = 16$  and an  $L = 16$ -path frequency-selective Rayleigh fading channel having uniform power delay profile. The initial value of  $\lambda$  is set to  $\lambda_{-1} = 0.7$ .

Figures 5(a) and 5(b) show the convergence performance of forgetting factor  $\lambda$  and that of block-average BER, respectively, with the normalized Doppler frequency  $f_D T$  (where  $T = (N_c + N_g)T_c$ ) is the chip-block length) as a parameter for the full code-multiplexing case ( $U = SF = 16$ ) at  $E_b/N_0 = 24$  dB. As seen from Fig. 5(a), the convergence rate of  $\lambda$  tends to become slower when smaller value of  $\mu$  is used for  $f_D T = 1 \times 10^{-4}$ ,  $1 \times 10^{-3}$  and  $5 \times 10^{-3}$ . However, it can be seen from Fig. 5(b) that the convergence rate of the block-average BER is similar even if which value of  $\mu = 5 \times 10^{-6}$ ,  $1 \times 10^{-5}$  or  $1 \times 10^{-4}$  is used when  $f_D T = 1 \times 10^{-4}$  and  $1 \times 10^{-3}$ . On the other hand, when  $f_D T = 5 \times 10^{-3}$  (fast fading), the convergence rate of the block-average BER is different for a different value of  $\mu$ . Because of limitation in the tracking capability against fast fading, the block-average BER periodically varies at the pilot block insertion cycle (or frame period) even if which value of  $\mu$  is used. However, it can be seen that the use of  $\mu = 5 \times 10^{-6}$  provides the minimum BER. Below,  $\mu = 5 \times 10^{-6}$  is used.

Figure 6 shows the impact of fading rate on the achievable BER as a function of the normalized Doppler frequency  $f_D T$  at  $E_b/N_0 = 24$  dB for the full code-multiplexing case ( $U = SF = 16$ ). It is seen from Fig. 6 that the forgetting

**Table 2** Simulation condition.

Transmitter	Data modulation	16QAM
	FFT block size	$N_c = 256$
	Guard interval length	$N_g = 32$
	Spreading sequence	Product of Walsh sequence and PN sequence
	Spreading factor	$SF = 16$
	Code multiplexing order	$U = 1, 16$
	Pilot chip sequence	PN sequence
	No. of chip-blocks/frame	$N = 16$
	Channel	Fading
Power delay profile		$L = 16$ -path uniform power delay profile
Receiver	Frequency-domain equalization	MMSE
	Channel estimation	RLS-CE ( $\mu = 5 \times 10^{-6}$ )

(a) Forgetting factor  $\lambda$ .

(b) Block-average BER.

**Fig. 5** Convergence performance.

factor  $\lambda$  can be optimally adjusted by LMS algorithm over all  $f_D T$ 's.

The simulated BER performance of multicode DS-CDMA using RLS-CE is plotted in Fig. 7 for  $U = 1$  and 16 as a function of the average received bit energy-to-AWGN noise power spectrum density ratio  $E_b/N_0$  ( $= 0.25(P \cdot SF \cdot T_c/N_0)(1 + N_g/N_c)N/(N-1)$ ). The BER performances using 2-step MLCE, pilot-assisted MMSE-CE with decision-feedback, MMSE-CE with 1st order interpolation and ideal CE are also plotted for comparison.

First, the  $U = 1$  case is discussed. It is seen from

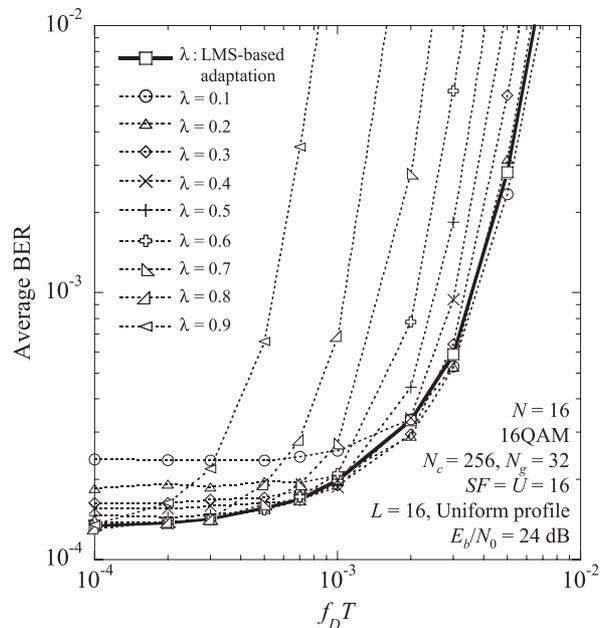
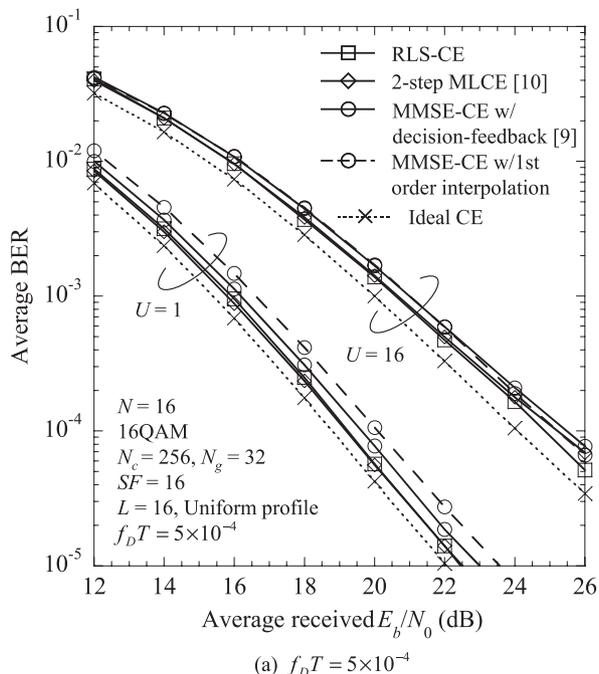
**Fig. 6** Impact of fading rate.

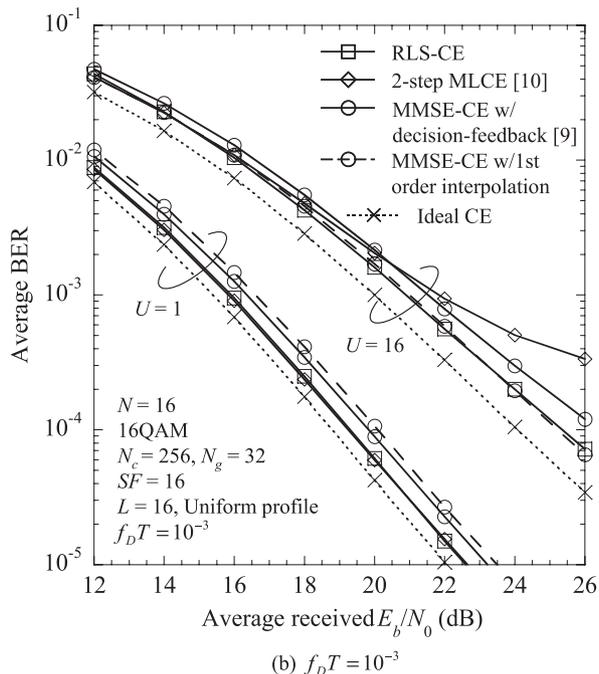
Fig. 7 that RLS-CE provides a better BER performance than MMSE-CE with decision-feedback and MMSE-CE with 1st order interpolation. The  $E_b/N_0$  loss from the ideal CE case for BER =  $10^{-4}$  is about 0.8 (1) dB for MMSE-CE with decision-feedback when  $f_D T = 5 \times 10^{-4}$  ( $10^{-3}$ ), but about 1.3 dB for MMSE-CE with 1st order interpolation when  $f_D T = 5 \times 10^{-4}$  and  $10^{-3}$ . The  $E_b/N_0$  loss includes a pilot insertion loss of 0.28 dB. RLS-CE gives the  $E_b/N_0$  loss of about 0.4 (0.5) dB which is the same as 2-step MLCE when  $f_D T = 5 \times 10^{-4}$  ( $10^{-3}$ ).

Next, the  $U = 16$  case is discussed. When  $f_D T = 5 \times 10^{-4}$  (see Fig. 7(a)), RLS-CE provides a better BER performance than 2-step MLCE, MMSE-CE with decision-feedback and MMSE-CE with 1st order interpolation. When  $f_D T = 10^{-3}$  (see Fig. 7(b)), RLS-CE provides a better BER performance than 2-step MLCE and MMSE-CE with decision-feedback. It provides a same BER performance as MMSE-CE with 1st order interpolation.

Figure 8 shows the impact of fading rate on the achievable BER at  $E_b/N_0 = 24$  dB as a function of the normalized Doppler frequency  $f_D T$  for the full code-multiplexing case ( $U = SF = 16$ ). For comparison, the BER performance using 2-step MLCE, pilot-assisted MMSE-CE with decision-feedback and MMSE-CE with 1st order interpolation are also plotted. It is seen from Fig. 8 that RLS-CE always provides a better BER performance than 2-step MLCE and MMSE-CE with decision-feedback. However, RLS-CE is inferior to MMSE-CE with 1st order interpolation if  $f_D T \geq 10^{-3}$ . As an example, assume a CDMA system of chip-rate  $1/T_c = 100$  Mcps (bandwidth of 100 MHz) and  $SF = 16$  using 3.5 GHz carrier frequency (the global use of 3.4~3.6 GHz has been allocated for IMT advanced systems in Nov. 2007 by ITU-R [15]),  $f_D T$  becomes  $10^{-3}$  when the moving speed reaches  $v = 107$  km/h. Therefore, RLS-CE



(a)  $f_D T = 5 \times 10^{-4}$



(b)  $f_D T = 10^{-3}$

Fig. 7 BER performance comparison.

can be superior to MMSE-CE with 1st order interpolation if  $v \leq 107$  km/h. If the terminal speed is above this speed, then MMSE-CE with 1st order interpolation should be used.

### 5. Conclusions

In this paper, we proposed a one-tap RLS-CE with adaptive forgetting factor for multicode DS-CDMA with FDE. It was shown by computer simulation that the proposed RLS-CE improves the BER performance compared to 2-step MLCE

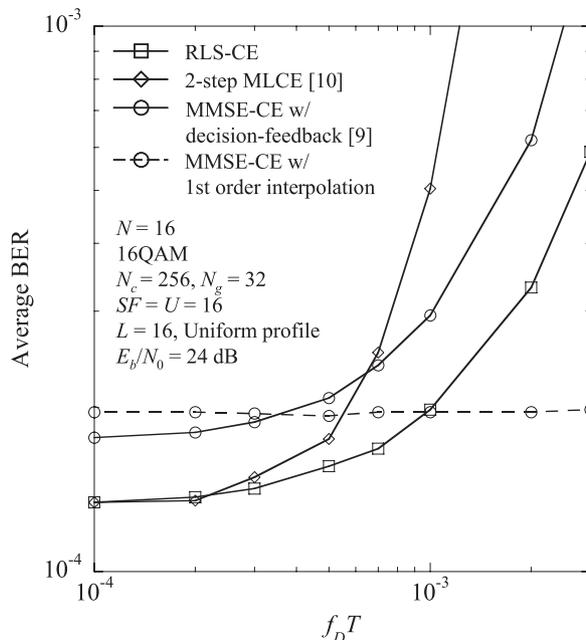


Fig. 8 Impact of fading rate.

and pilot-assisted MMSE-CE with decision-feedback. RLS-CE with adaptive forgetting factor has a better tracking ability against the fading variation and provides a better BER performance than MMSE-CE with 1st order interpolation for the normalized Doppler frequency  $f_D T \leq 10^{-3}$ .

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### Appendix A: Derivation of Eq. (22)

Substituting Eq. (16) into Eq. (21) gives

$$\Psi_n(k) = \frac{\partial \tilde{H}_{n-1}(k)}{\partial \lambda} + \frac{\partial G_n(k)}{\partial \lambda} \xi_n(k) + G_n(k) \frac{\partial \xi_n(k)}{\partial \lambda}. \quad (\text{A} \cdot 1)$$

By substituting Eq. (17) into Eq. (A·1), we have

$$\Psi_n(k) = (1 - G_n(k)S_n(k)) \Psi_{n-1}(k) + \frac{\partial \Phi_n^{-1}(k)}{\partial \lambda} S_n^*(k) \xi_n(k). \quad (\text{A} \cdot 2)$$

### Appendix B: Derivation of Eq. (23)

$\Phi_n^{-1}(k)$  of Eq. (15) can be rewritten as

$$\Phi_n^{-1}(k) = \lambda^{-1} \Phi_{n-1}^{-1}(k) - \frac{\lambda^{-2} (\Phi_{n-1}^{-1}(k))^2 |S_n(k)|^2}{1 + \lambda^{-1} \Phi_{n-1}^{-1}(k) |S_n(k)|^2}. \quad (\text{A} \cdot 3)$$

$\partial \Phi_n^{-1}(k) / \partial \lambda$  is given by

$$\frac{\partial \Phi_n^{-1}(k)}{\partial \lambda} = -\lambda^{-2} \Phi_{n-1}^{-1}(k) + \lambda^{-1} \frac{\partial \Phi_{n-1}^{-1}(k)}{\partial \lambda} - \frac{\partial}{\partial \lambda} \left( \frac{\lambda^{-2} (\Phi_{n-1}^{-1}(k))^2 |S_n(k)|^2}{1 + \lambda^{-1} \Phi_{n-1}^{-1}(k) |S_n(k)|^2} \right). \quad (\text{A} \cdot 4)$$

The 3rd term for Eq. (A·4) can be rewritten as

$$\frac{\partial}{\partial \lambda} \left( \frac{\lambda^{-2} (\Phi_{n-1}^{-1}(k))^2 |S_n(k)|^2}{1 + \lambda^{-1} \Phi_{n-1}^{-1}(k) |S_n(k)|^2} \right) = \lambda^{-1} \left\{ (2G_n(k)S_n(k) - |G_n(k)|^2 |S_n(k)|^2) \right.$$

$$\left. \times \frac{\partial \Phi_{n-1}^{-1}(k)}{\partial \lambda} - |G_n(k)|^2 \frac{\lambda^{-2} (\Phi_{n-1}^{-1}(k))^2 |S_n(k)|^2}{1 + \lambda^{-1} \Phi_{n-1}^{-1}(k) |S_n(k)|^2} \right\}. \quad (\text{A} \cdot 5)$$

Hence, we have

$$\frac{\partial \Phi_n^{-1}(k)}{\partial \lambda} = \lambda^{-1} \left\{ (1 - G_n(k)S_n(k))^2 \frac{\partial \Phi_{n-1}^{-1}(k)}{\partial \lambda} + |G_n(k)|^2 + \Phi_n^{-1}(k) \right\}. \quad (\text{A} \cdot 6)$$



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